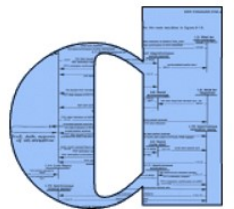


Markov Automata

The state of affairs

Holger Hermanns

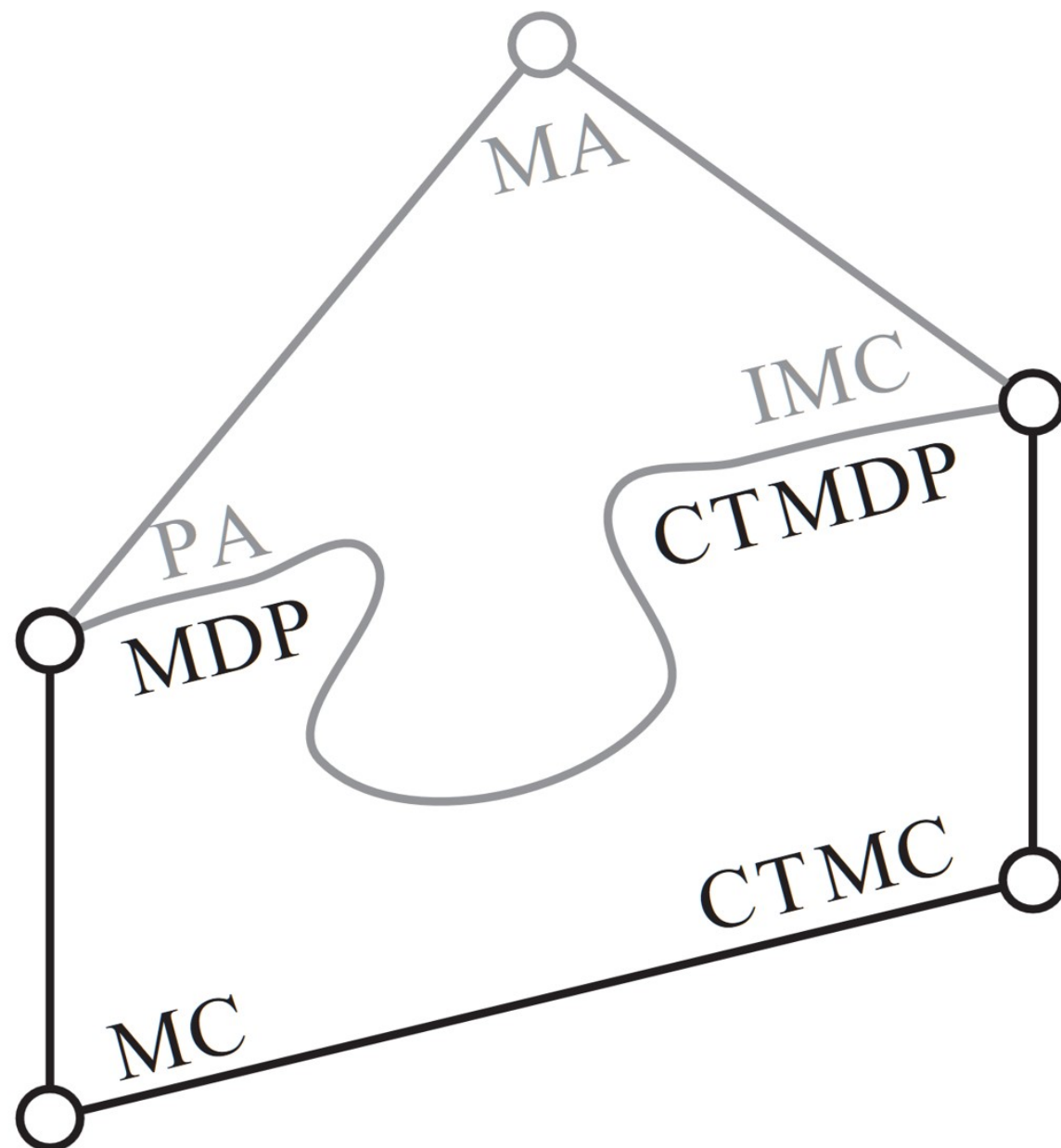
Saarland University

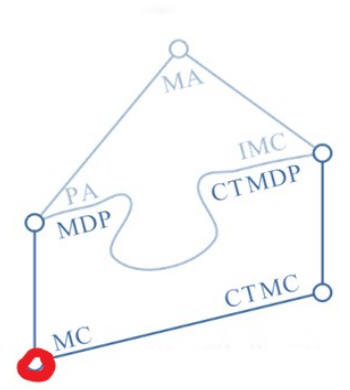
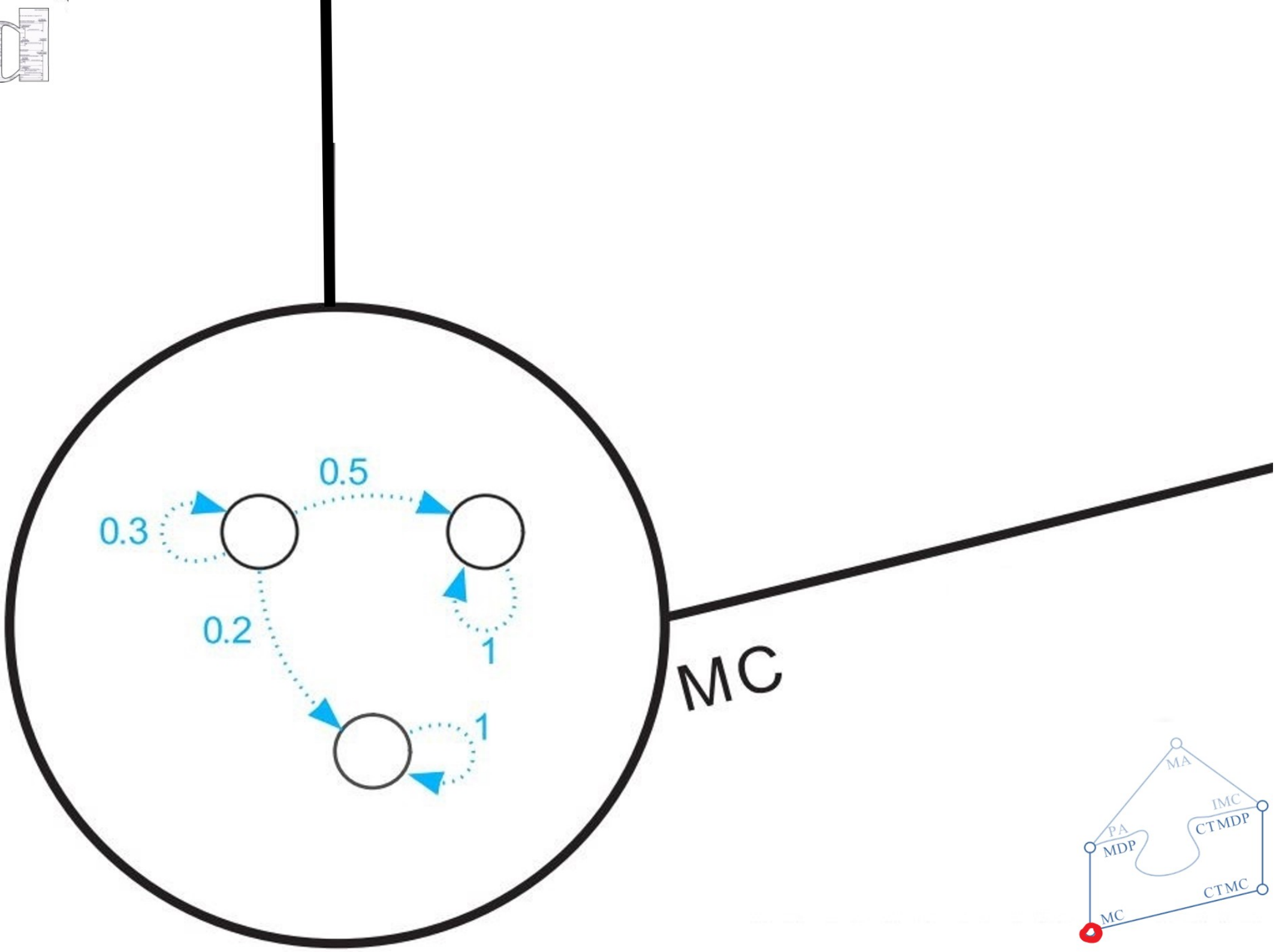


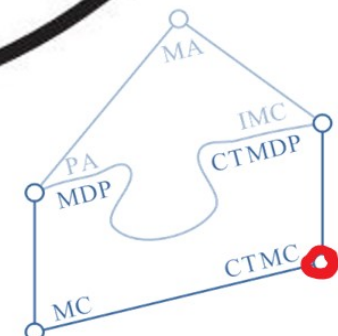
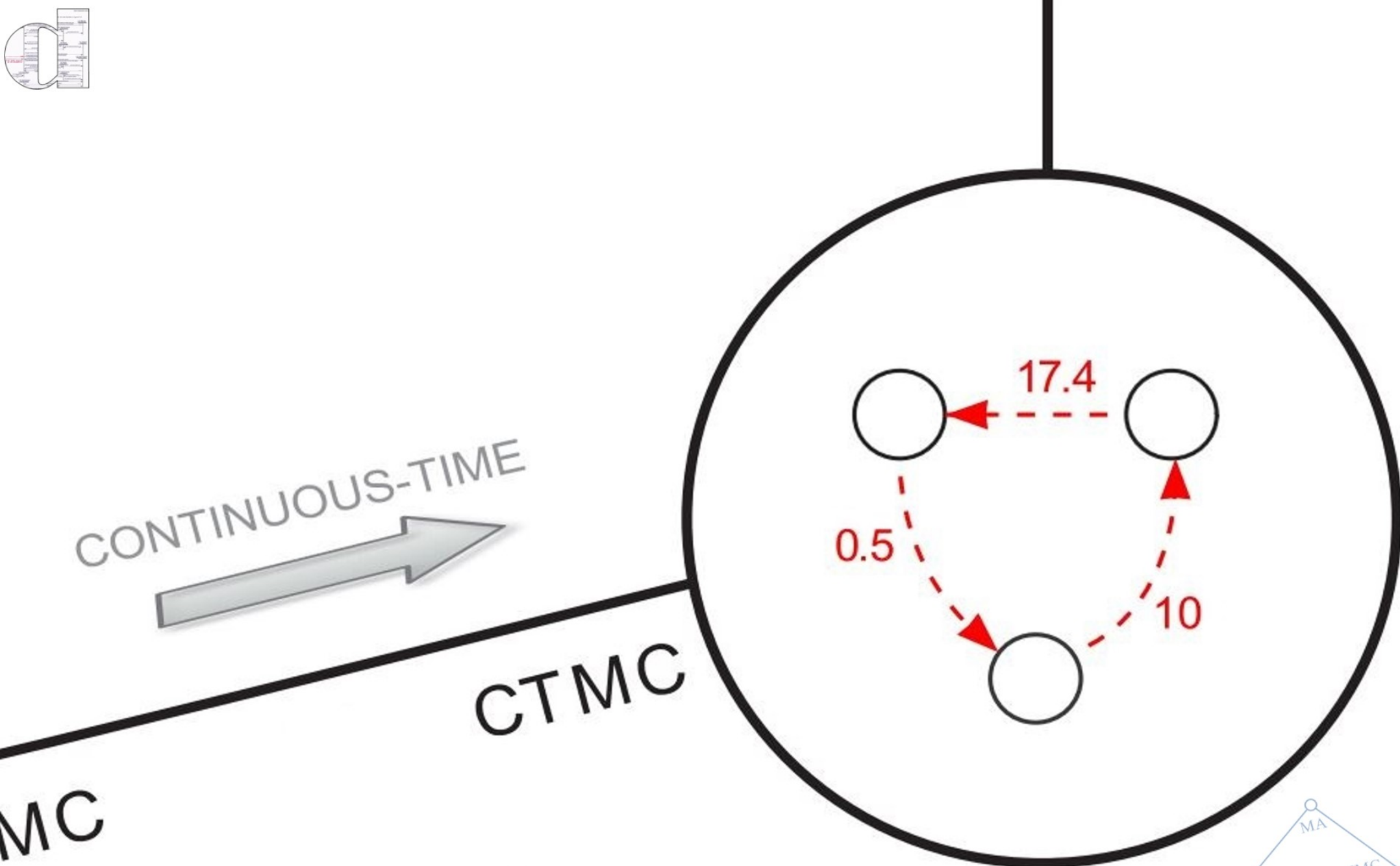


Markov Automata?

- What?
- Why?
- How?
 - Construction
 - Compression
 - Verification
 - Extension
- Open Challenges





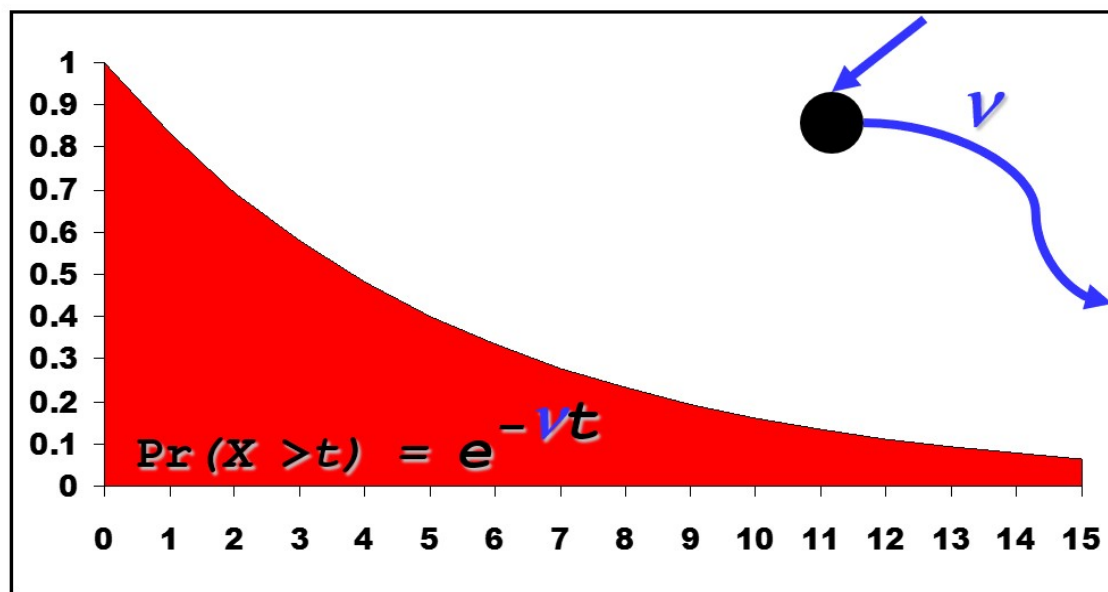




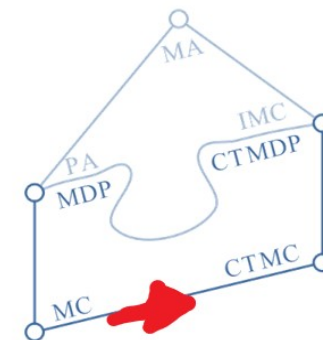
Continuous Time?

“CTMC”

- basically transition-weighted automata.
- all times are exponentially distributed;



- sojourn time in states are memory-less;

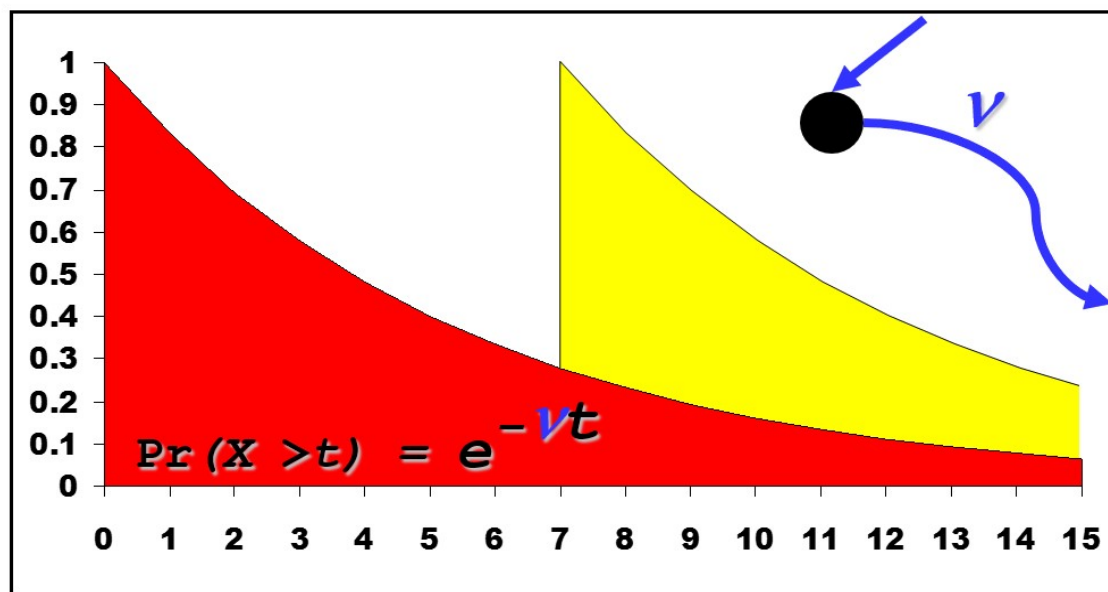




Continuous Time?

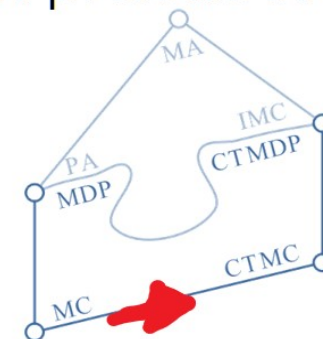
“CTMC”

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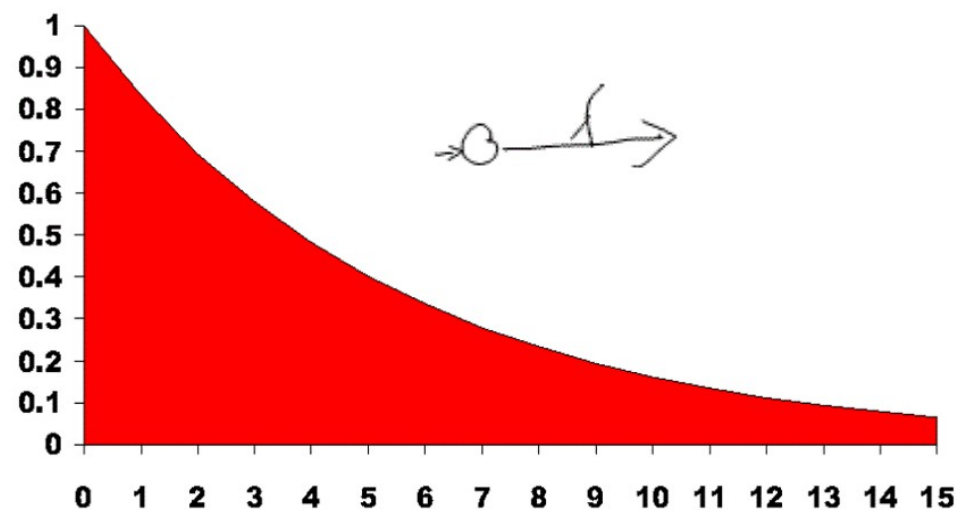
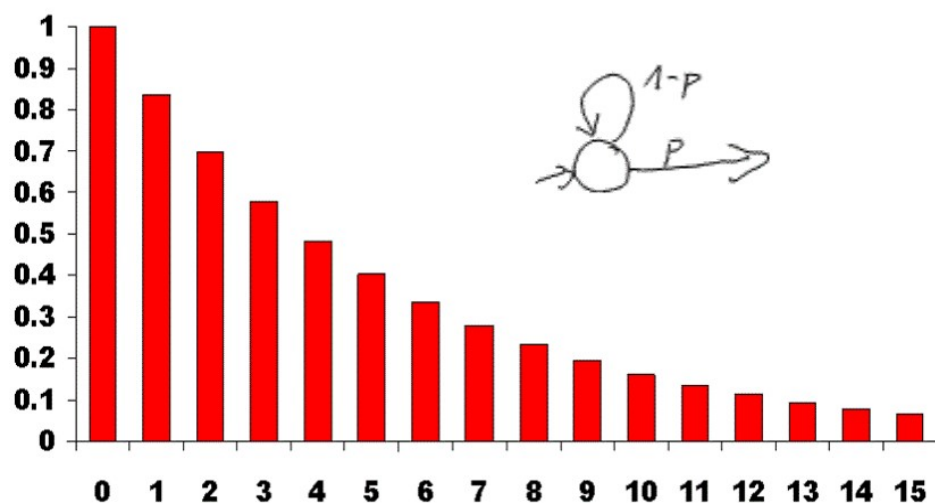
- sojourn time in states are memory-less;

- γ very well investigated class of stochastic processes,
- efficient and numerically stable analysis algorithms available;
- best guess, if only mean values are known;
- very widely used in practice.



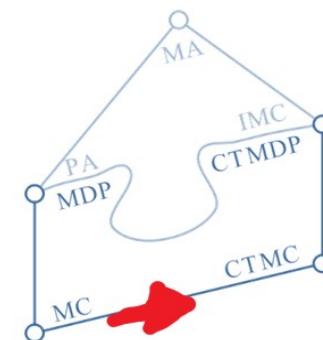


Discrete vs. Continuous Time?



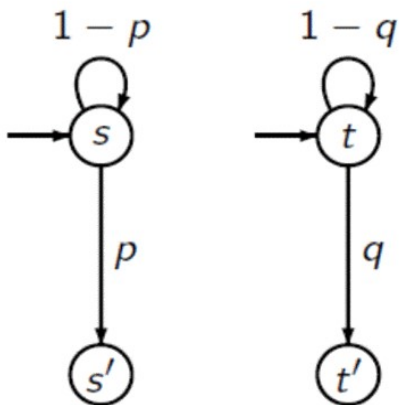
- For a given time step Δ a discretised exponential distribution is a geometric distribution.
- In the limit $\Delta \rightarrow 0$ a geometric distribution is an exponential distribution.
- This can be lifted to MCs:

For each CTMC \mathcal{M} and time step Δ ,
there is discretised DTMC \mathcal{D}_Δ .

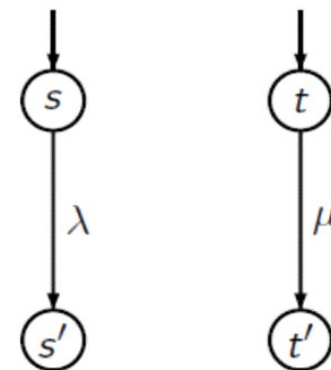




Discrete vs. Continuous Time?

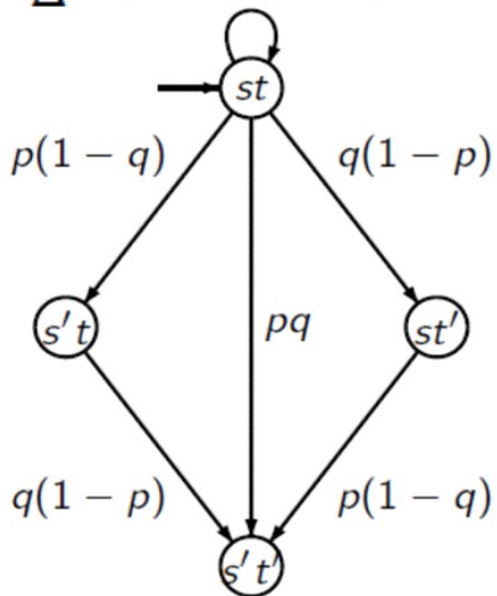


$$Q^{\parallel} = \lim_{\Delta \rightarrow 0} (P_{\Delta}^{\otimes} - I) / \Delta$$



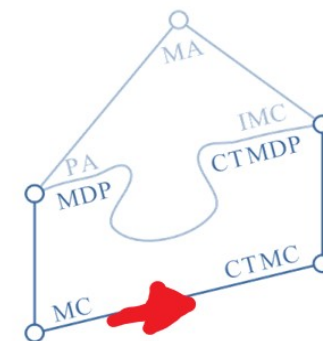
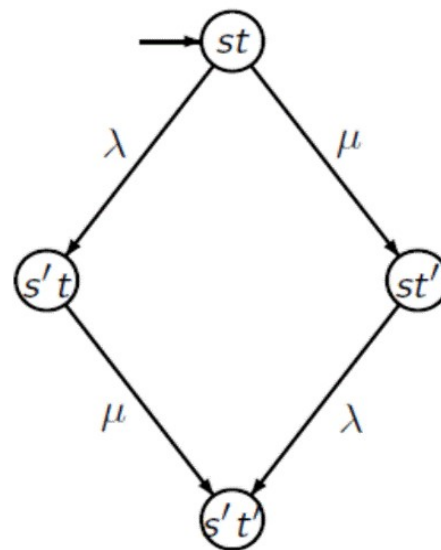
Synchronous Composition for MC

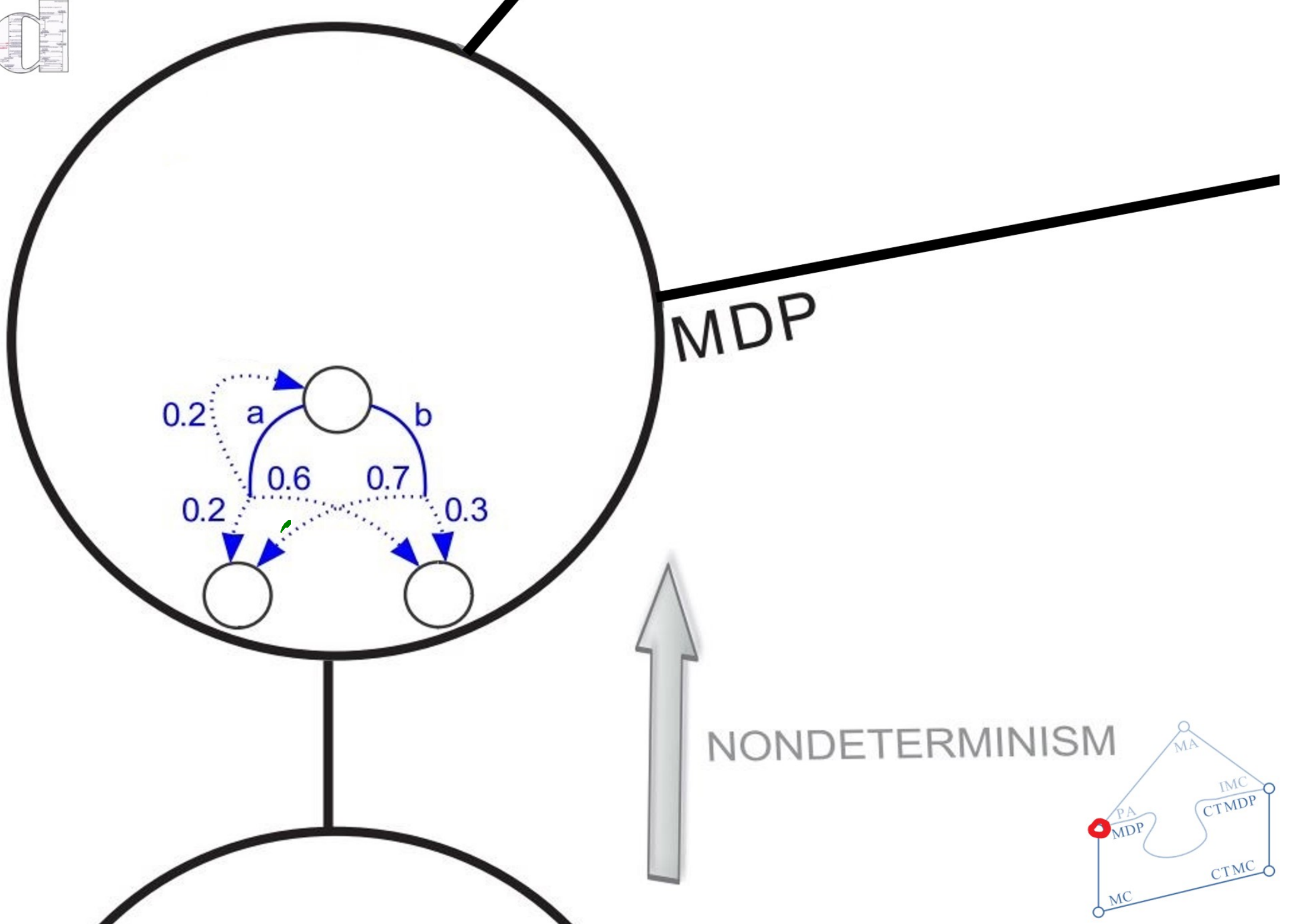
$$\mathcal{D}_{\Delta} \otimes \mathcal{D}'_{\Delta} \quad (1-p)(1-q)$$



Interleaving for CTMC

$$\mathcal{M}^{\parallel} \mathcal{M}'$$

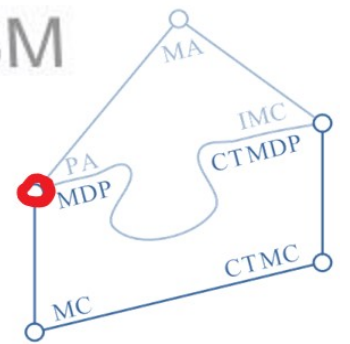


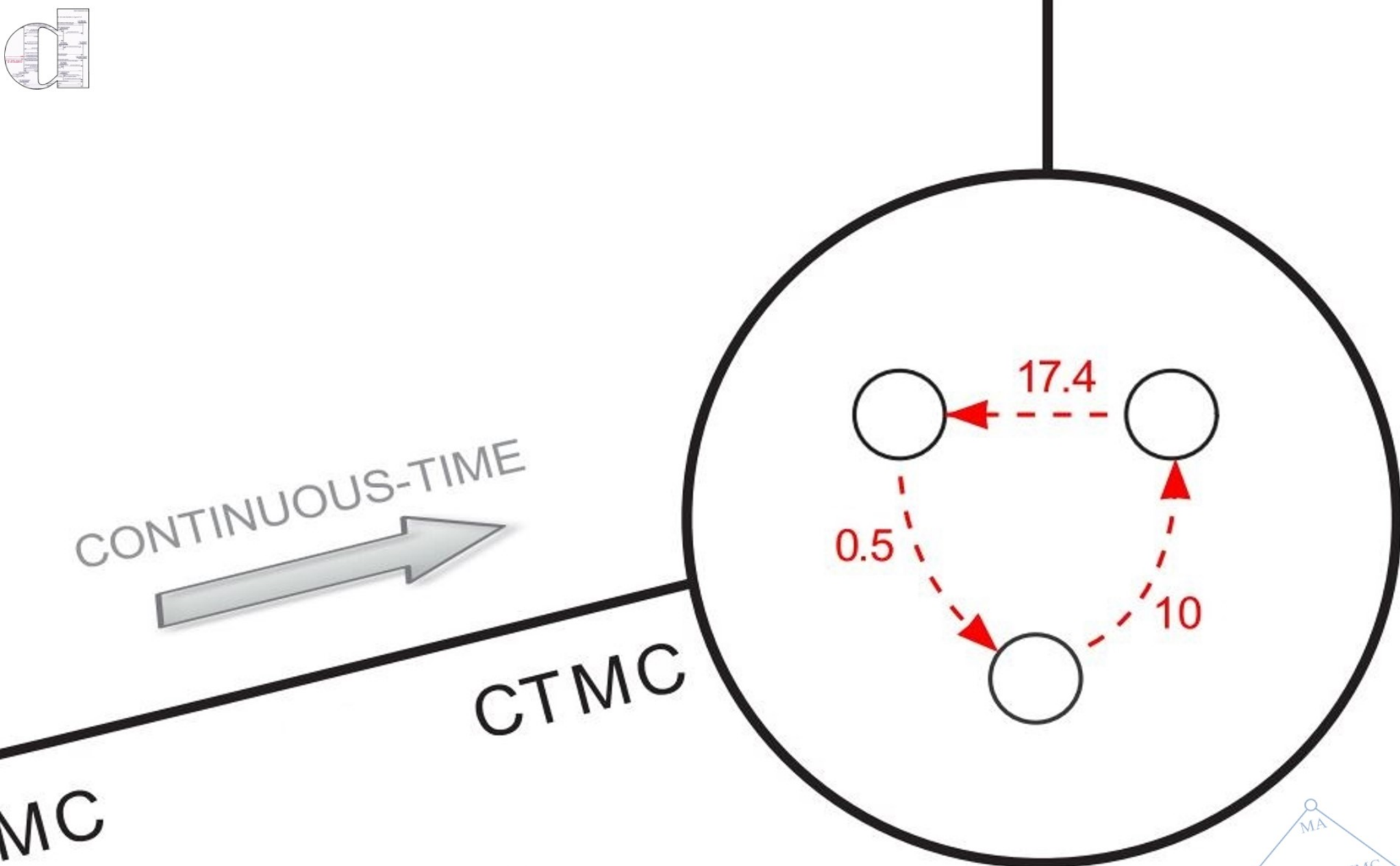


MDP



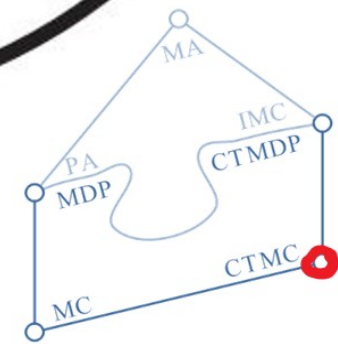
NONDETERMINISM





MC

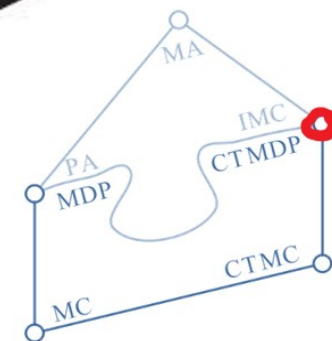
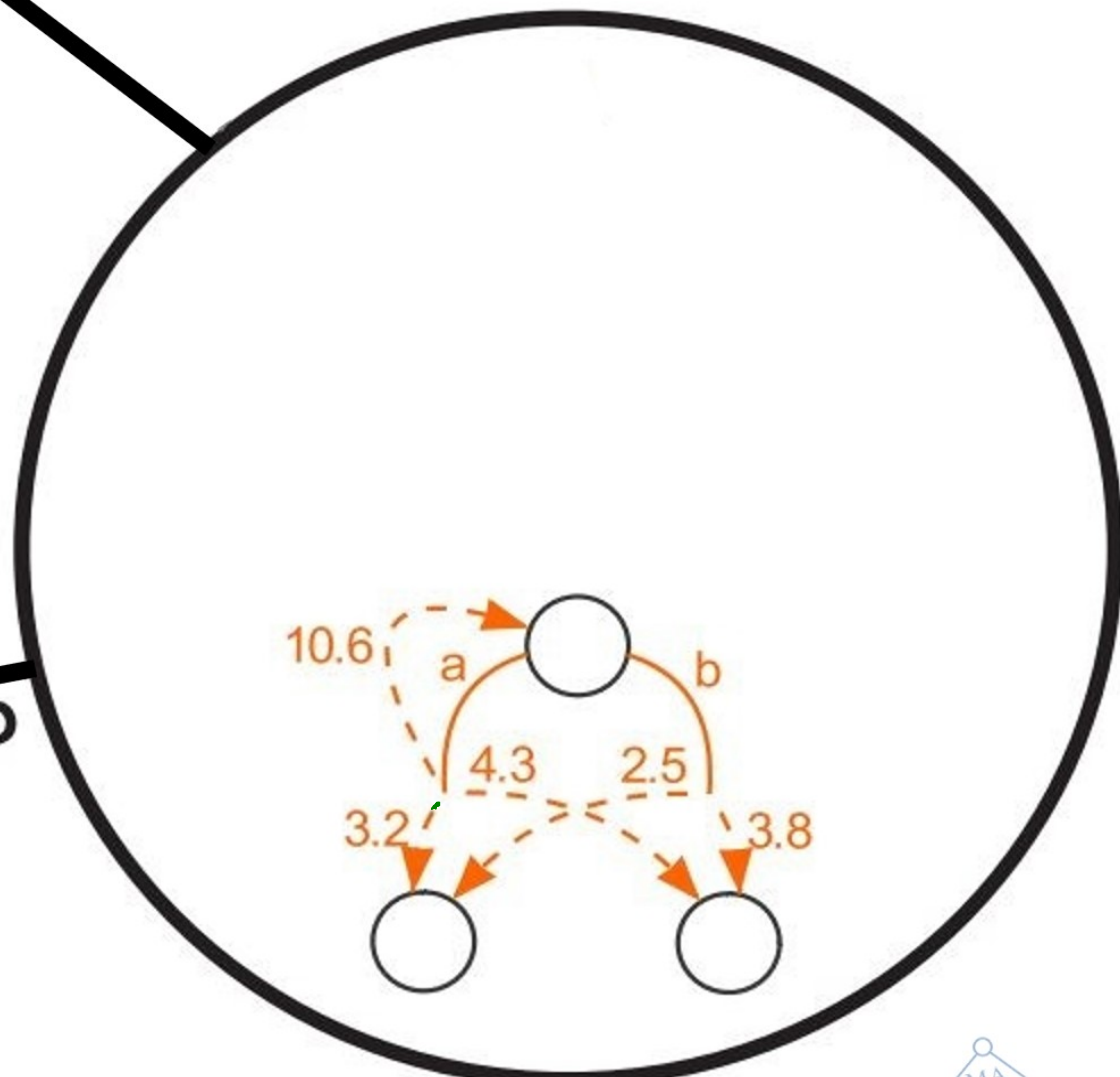
CTMC





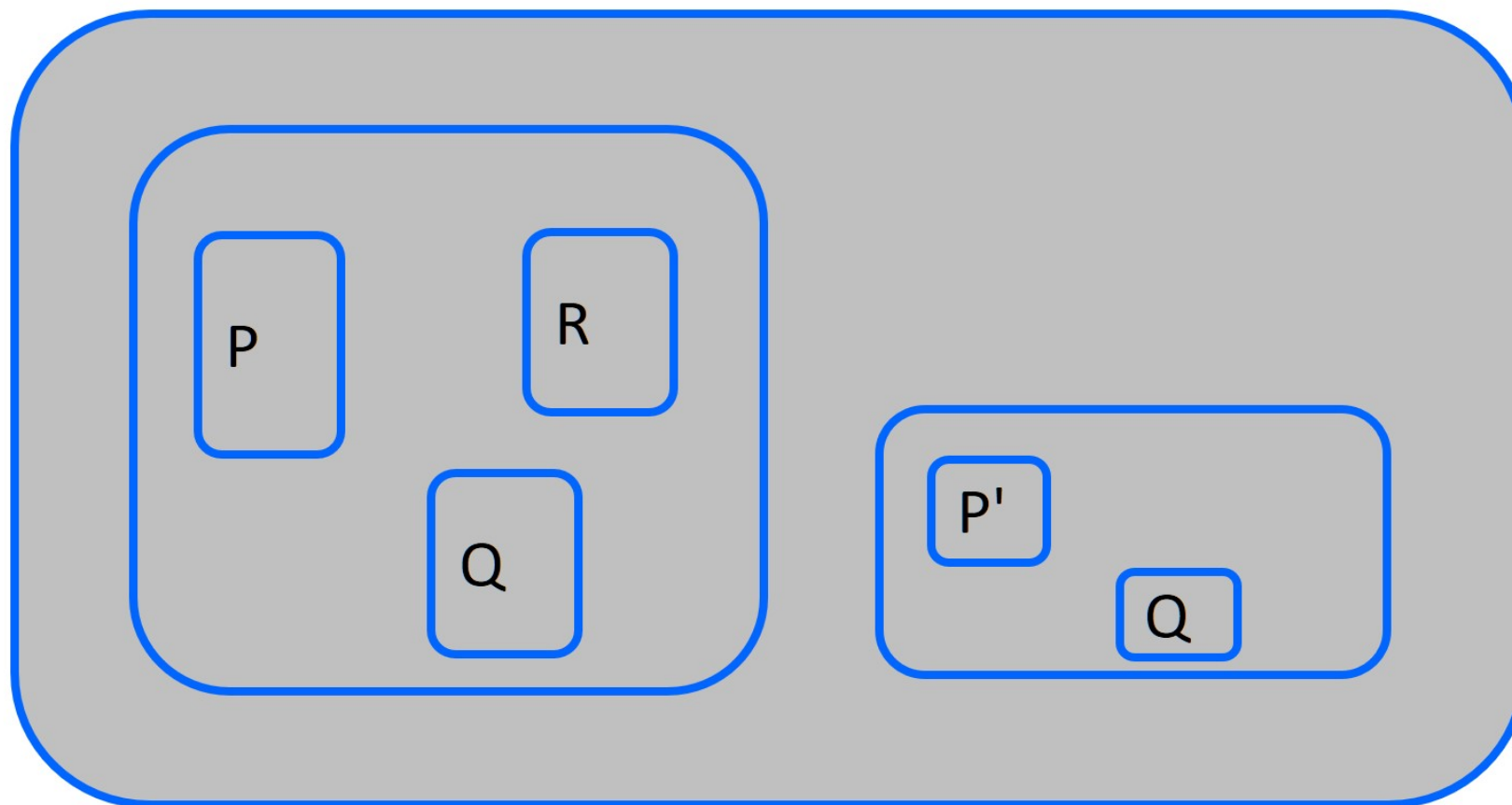
MDP

CTMDP

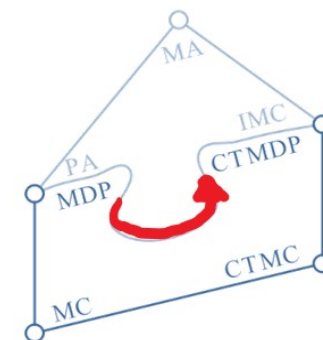




Compositionality?

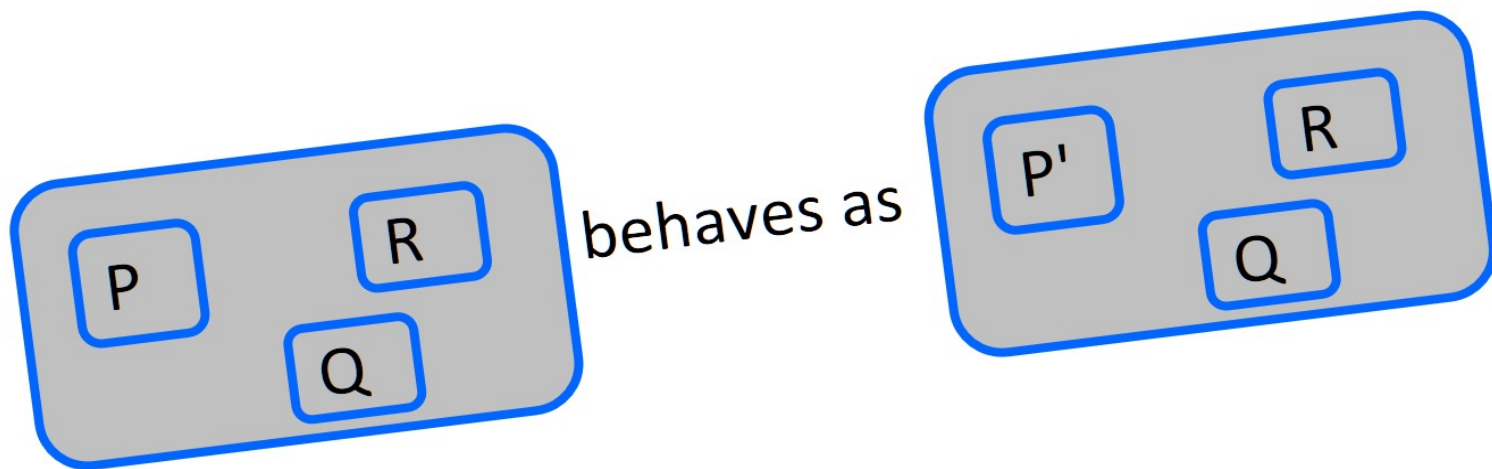


Process Algebraic Composition Operators.





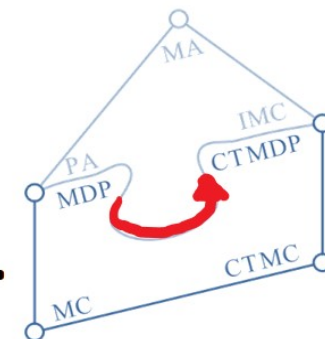
Compositionality?

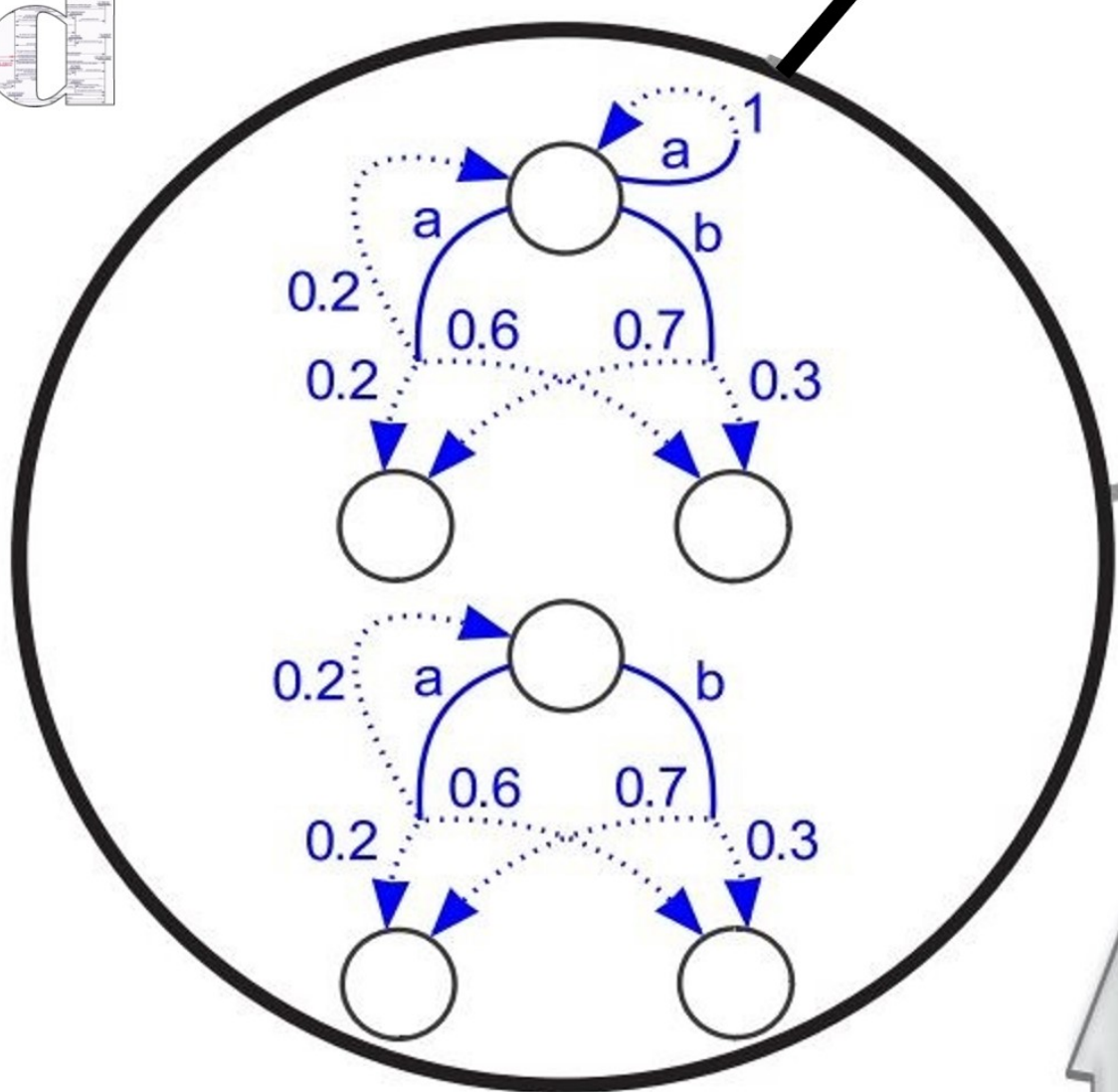


if P behaves as P'

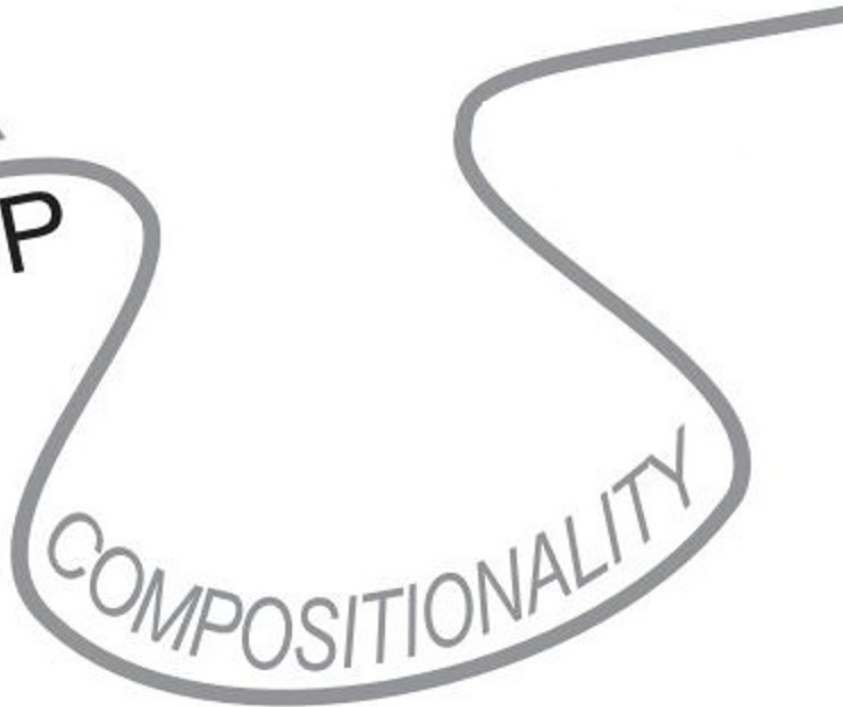
Process Algebraic Composition Operators.

Natural Notions of Bisimulation, are Congruences.

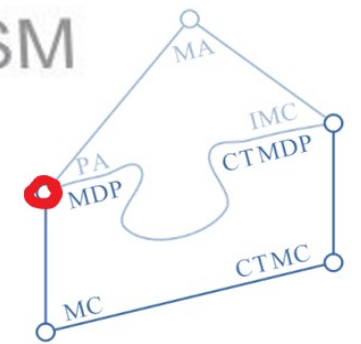




PA
MDP



NONDETERMINISM

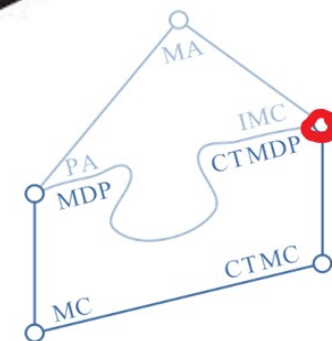
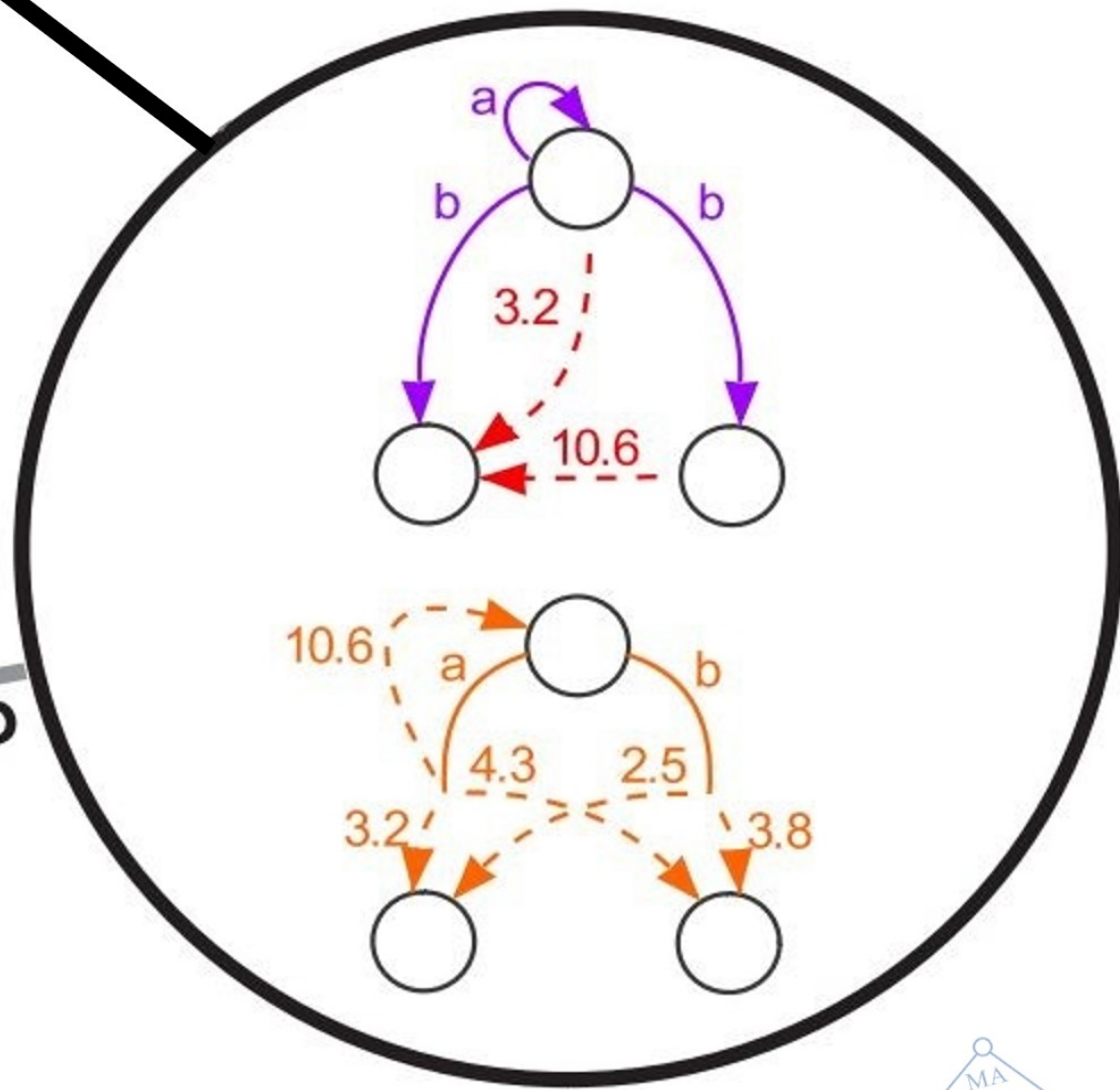


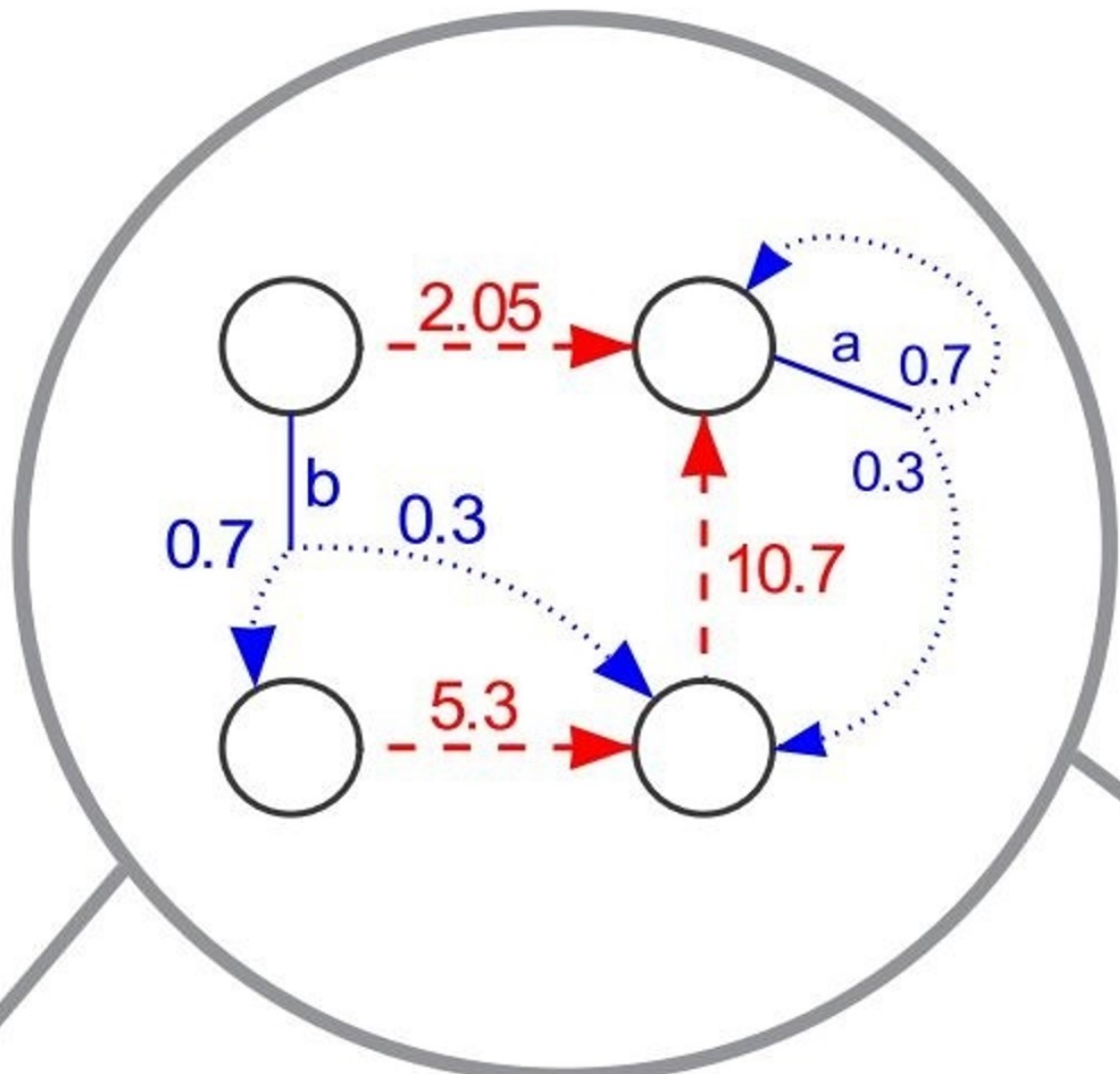


PA
MDP

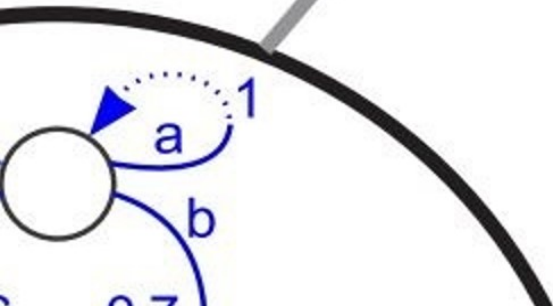
IMC
CTMDP

COMPOSITIONALITY

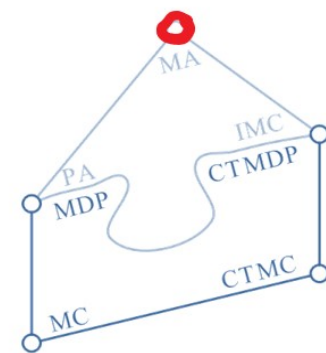




MA



IMC





Christian Eisentraut, Holger Hermanns, Lijun Zhang:

On Probabilistic Automata in Continuous Time. LICS 2010: 342-351

Christian Eisentraut, Holger Hermanns, Lijun Zhang:

Concurrency and Composition in a Stochastic World. CONCUR 2010: 21-39

Markov Automata

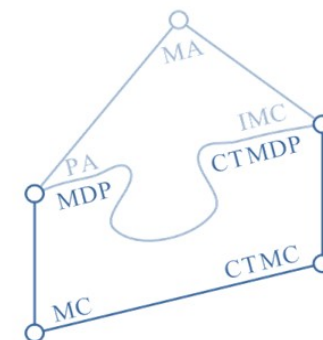
Semantics

Yuxin Deng, Matthew Hennessy:

On the semantics of Markov automata. Inf. Comput. 222: 139-168 (2013)

Christian Eisentraut, Holger Hermanns, Joost-Pieter Katoen, Lijun Zhang:

A Semantics for Every GSPN. Petri Nets 2013: 90-109





Markov Automata

Definition 1. A Markov automaton MA is a quintuple $(S, Act, \rightarrow, \twoheadrightarrow, s_o)$, where

- S is a nonempty finite set of states,
- Act is a set of actions containing the internal action τ ,
- $\rightarrow \subset S \times Act \times Dist(S)$ is a set of probabilistic transitions, and
- $\twoheadrightarrow \subset S \times \mathbb{R}_{\geq 0} \times S$ is a set of Markov timed transitions, and
- $s_o \in S$ is the initial state.

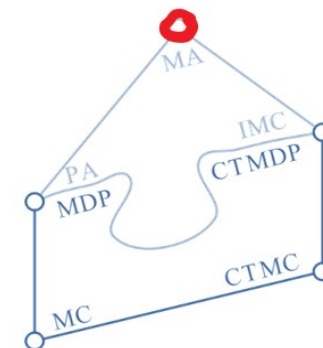
Labelled Transition Systems: If $\twoheadrightarrow = \emptyset$ and \rightarrow is Dirac.

Discrete-time Markov chains: If $\twoheadrightarrow = \emptyset$ and $|Act| = 1$ and \rightarrow is deterministic.

Continuous-time Markov chains: If $\rightarrow = \emptyset$.

Probabilistic Automata: If $\twoheadrightarrow = \emptyset$.

Interactive Markov chains: If \rightarrow is Dirac.





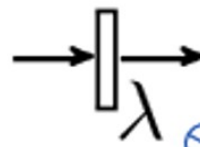
Generalized Stochastic Petri Net

Very popular.

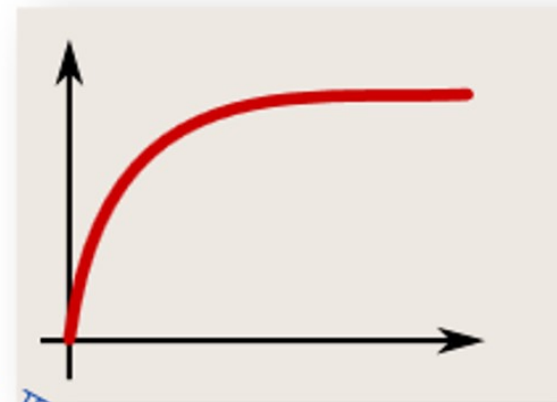
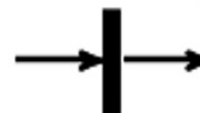
target formalism
for XYZML

Tokens ●
Places ○
Transitions

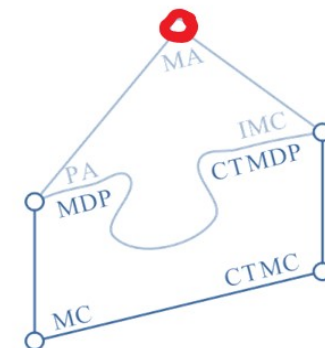
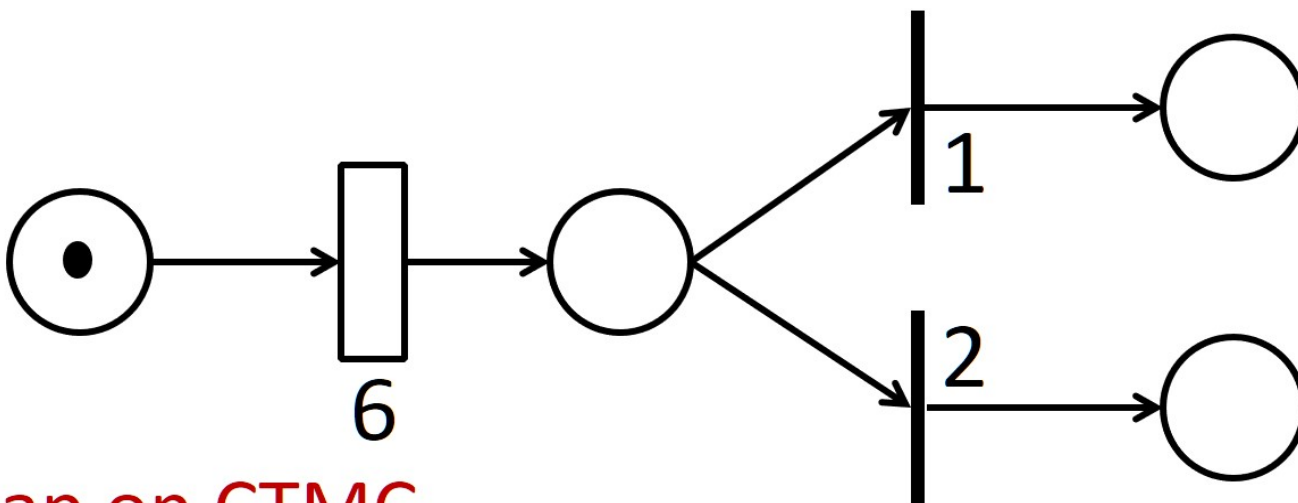
timed



immediate



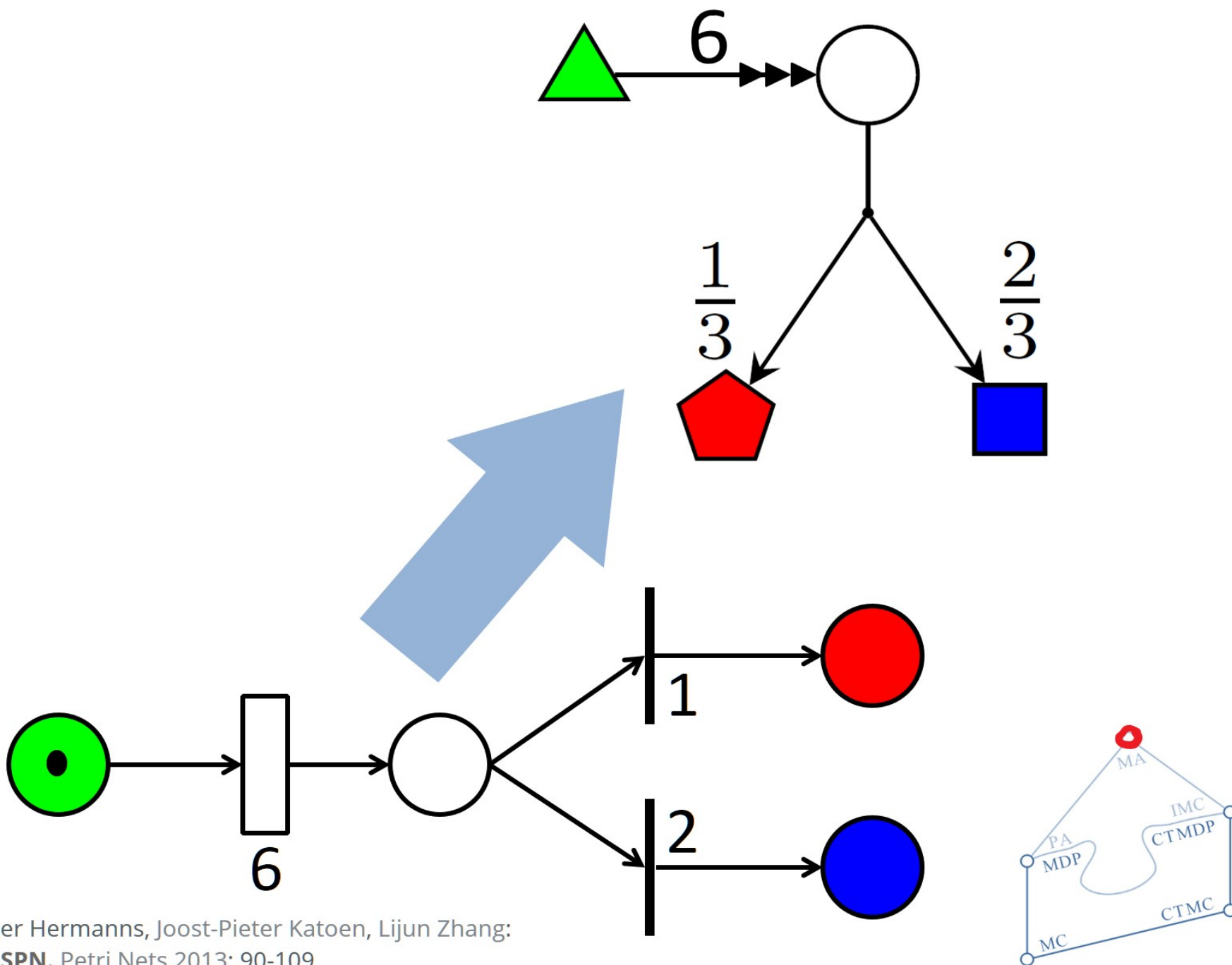
Incomplete
semantics
for more
than 25
years.



Meant to map on CTMC.

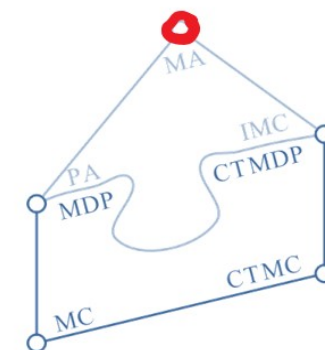
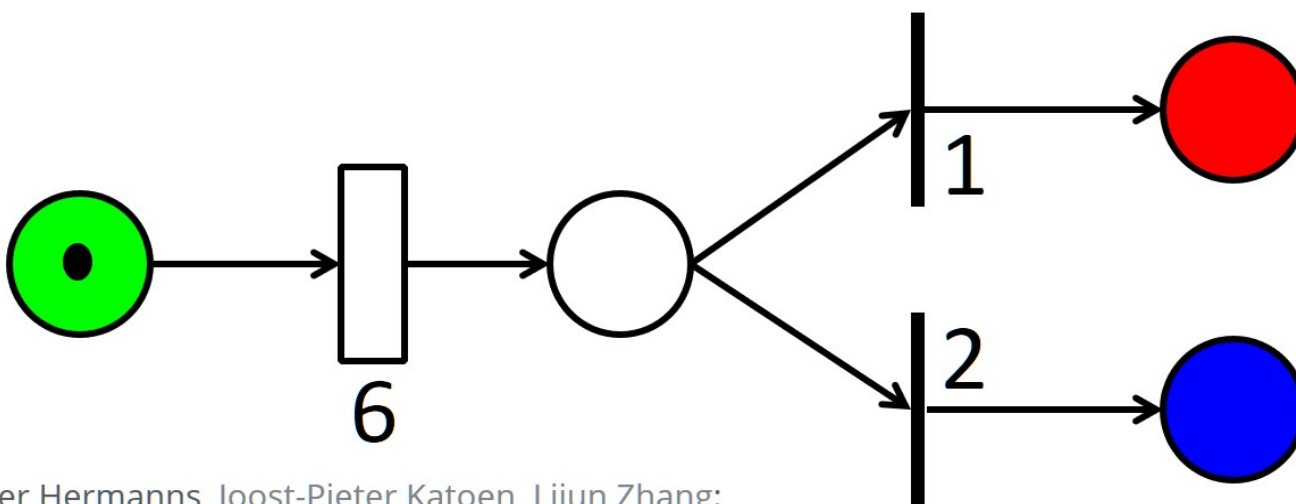
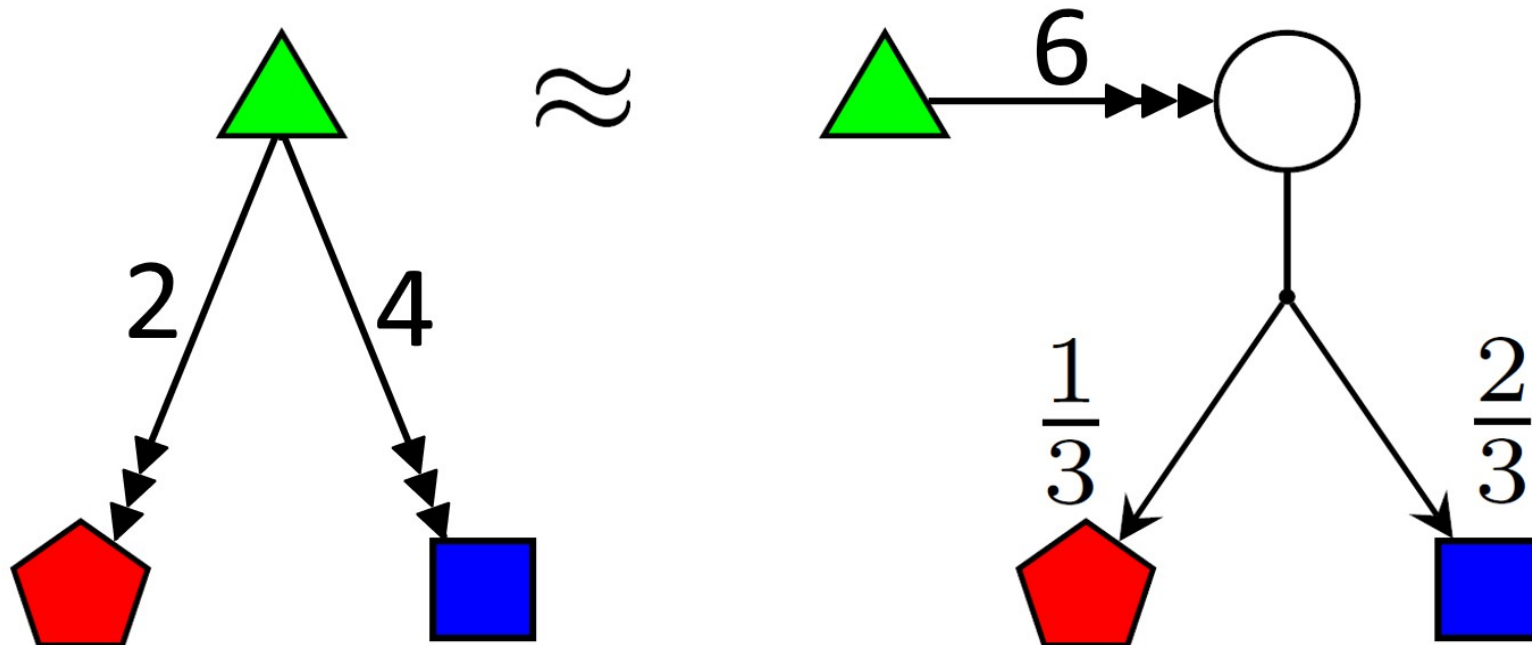


GSPN Semantics: Markov Automata



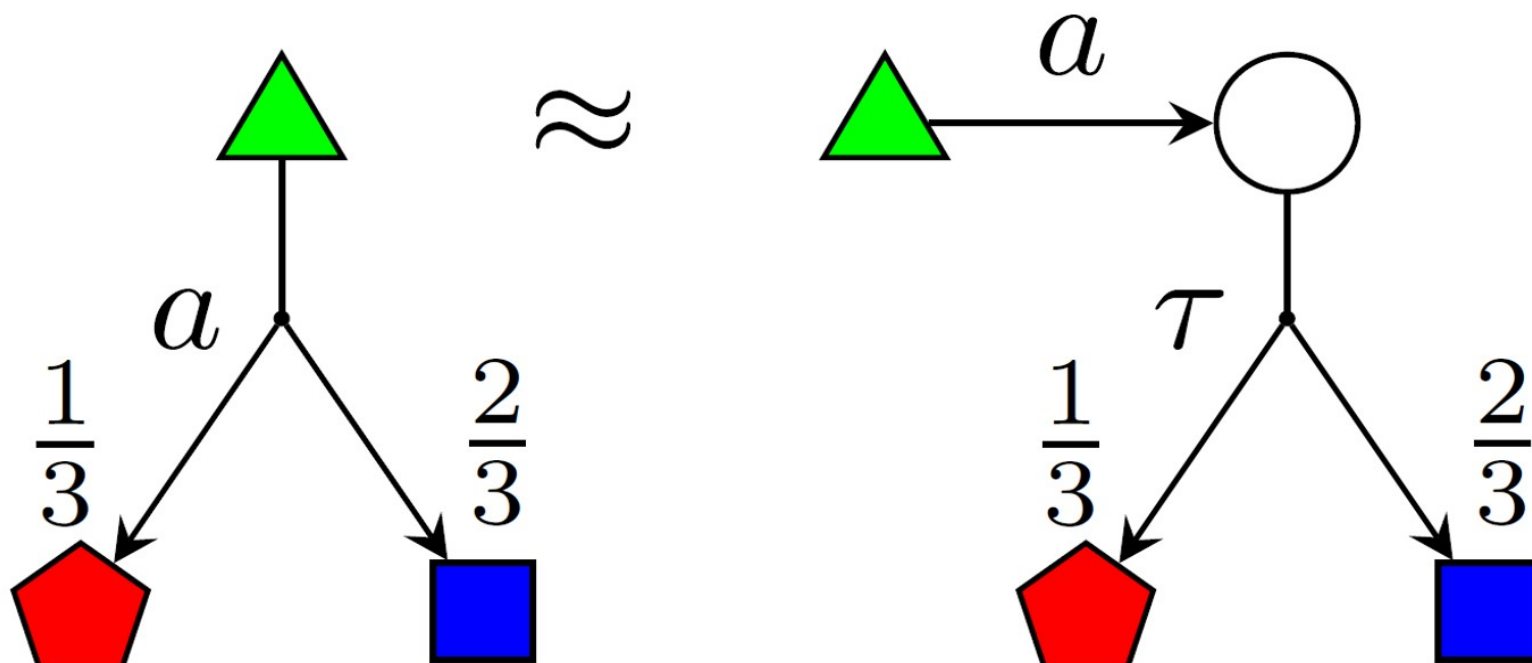


MA Weak Bisimulation





MA Weak Bisimulation



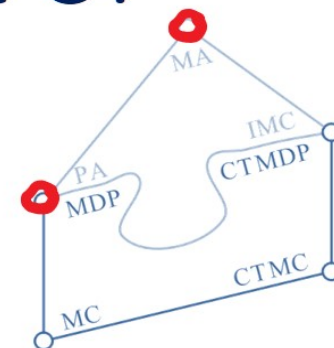
- explained on PA -

How can this work?

There is no state on the left that matches state \circ .

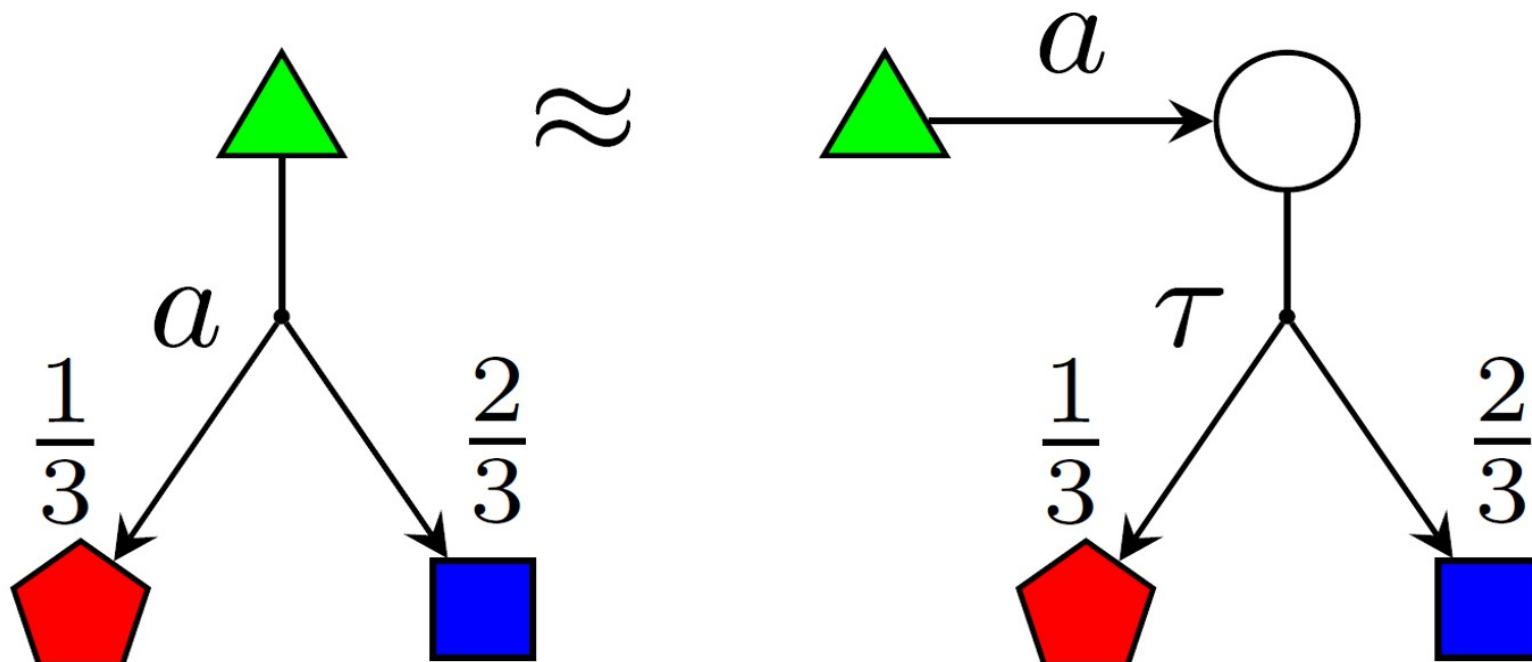
Well, there is a matching distribution.

Bisimulations on distributions!





Weak Distribution Bisimulation



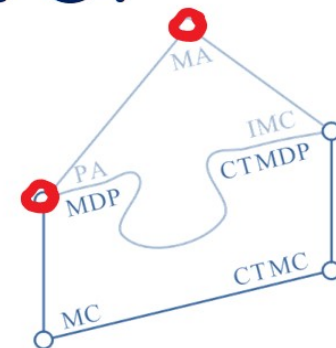
- explained on PA -

How can this work?

There is no state on the left that matches state \circ .

Well, there is a matching distribution.

Bisimulations on distributions!

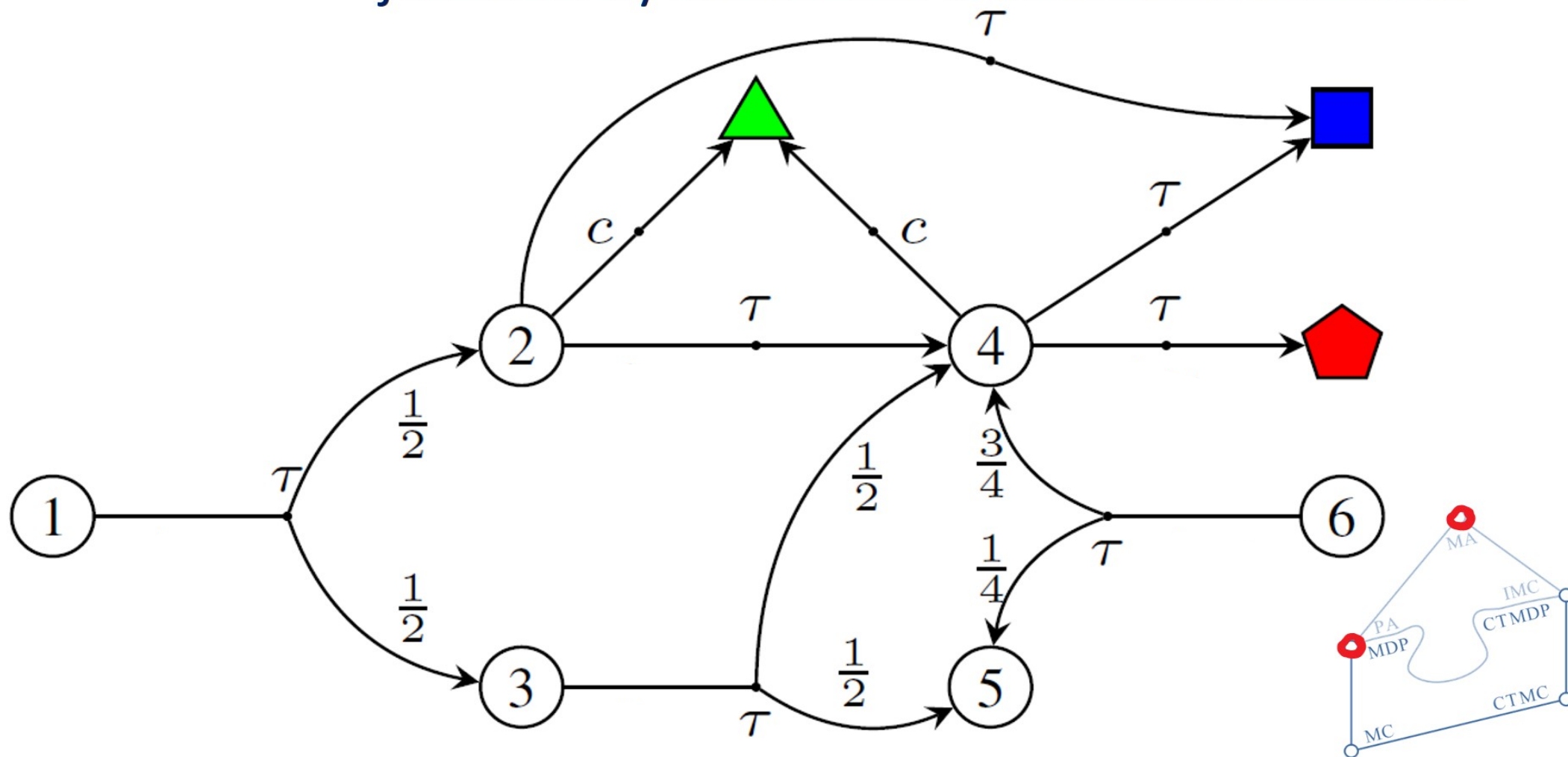




Weak Distribution Bisimulation in Action

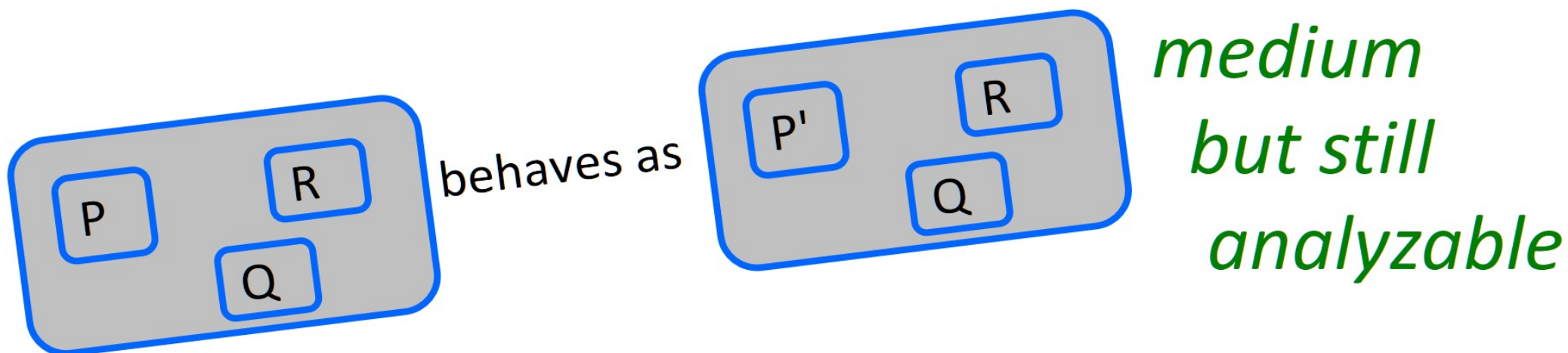
State ① exhibits the same observable behaviour as ⑥.

justified by weak distribution bisimulation.





Compositional Equivalences



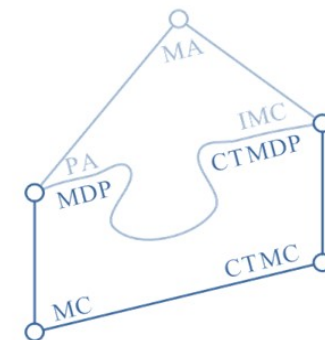
*very
large*

if P behaves as P'

small

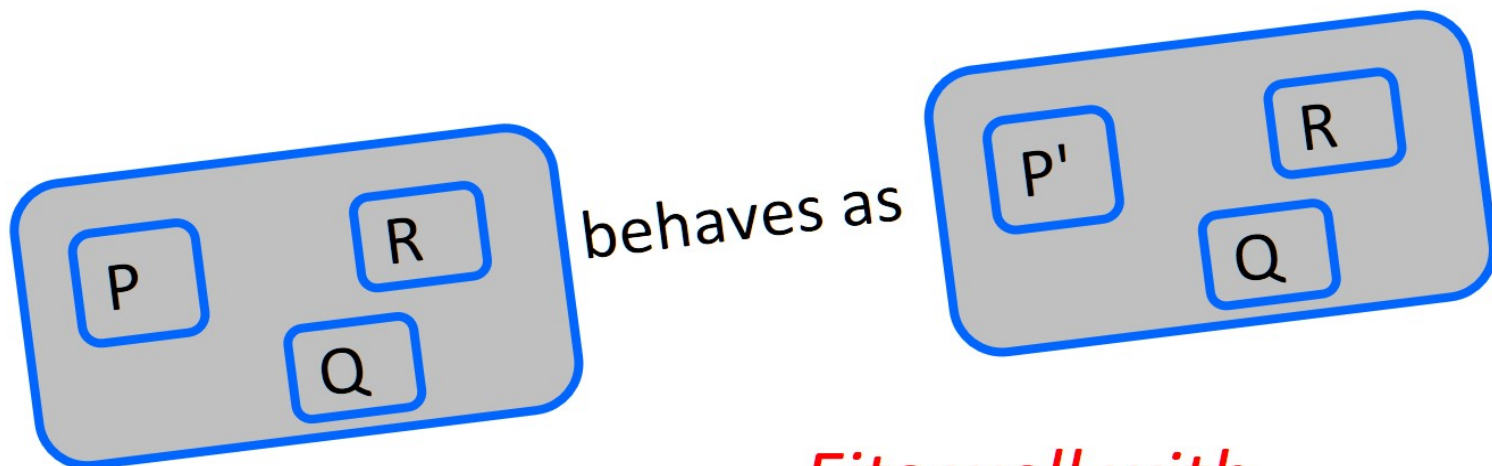
large

“Applied” Process Algebra





Compositional Equivalences

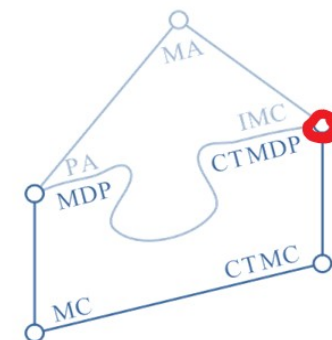


*medium
but still
analyzable*

*very
large*

*Fits well with
dependability
evaluation*

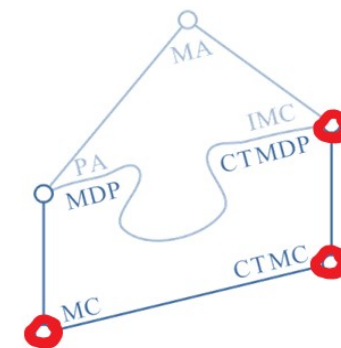
> 10²⁰⁰ reduced to 10⁶





Compositional Minimization

- Based on (strong, or better) weak bisimulations;
- Congruence property wrt parallel composition and hiding operators;
- Requires an efficient decision algorithm which can be turned into a minimization algorithm;
- Cubic algorithms for the base models.

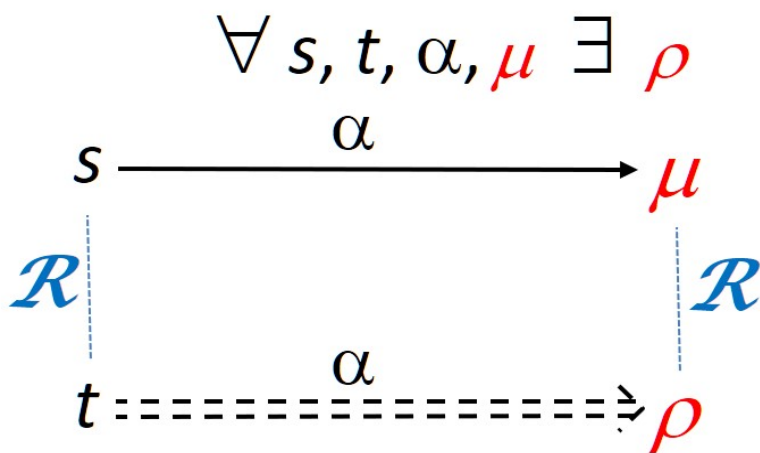




PA Weak Bisimulation Decision Algorithm

Core Problem:

matching a challenger transition



with a weak defender transition

can be coded as an LP-problem

and be embedded in

partition refinement strategy

$\max \sum f_i$
under constraints

$$0 \leq f_i \leq c_i (< \infty)$$

$$f_0 + f_8 = f_1 + f_4$$

$$f_1 = f_2 + f_3$$

$$f_2 = f_9$$

$$f_3 = f_{10}$$

$$f_4 = f_5 + f_6 + f_7$$

$$f_5 = f_8 + f_{11}$$

$$f_6 = f_{12}$$

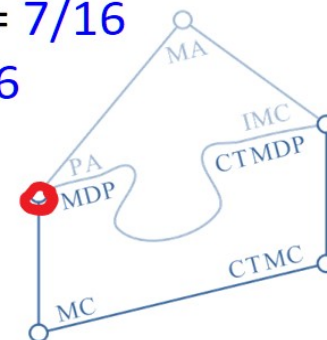
$$f_7 = f_{13}$$

$$f_0 = 1$$

$$f_9 + f_{12} = 5/16$$

$$f_{10} + f_{11} = 7/16$$

$$f_{13} = 4/16$$





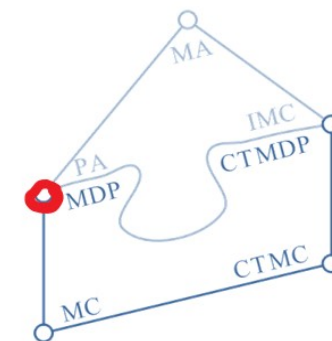
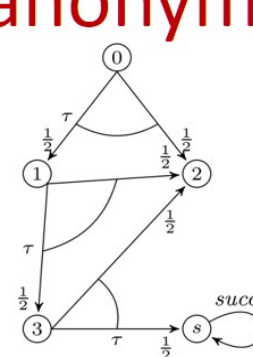
Weak Bisimulation in Action

Sweep
over the
PRISM PA
collection

| Problem | $ S $ | $ D $ | $ S_{\bowtie} $ | $ D_{\bowtie} $ | $T_{\mathcal{A}_{\bowtie}}$ | $ [S]_{\approx} $ | $ [D]_{\approx} $ | $T_{[\mathcal{A}_{\bowtie}]_{\approx}}$ |
|--------------|-------|-------|-----------------|-----------------|-----------------------------|---------------------|---------------------|---|
| csma2 | 1038 | 1054 | 835 | 849 | 1s | 449 | 459 | 1s |
| csma2-sa | 1038 | 1054 | 621 | 630 | 7s | 233 | 237 | < 1s |
| csma2-sa-nt | 1038 | 1054 | 91 | 98 | < 1s | 87 | 90 | < 1s |
| dining4 | 2165 | 4540 | 161 | 300 | < 1s | 1 | 1 | < 1s |
| firewire3 | 611 | 694 | 425 | 469 | 5s | 425 | 469 | 5s |
| firewire3-nt | 611 | 694 | 29 | 62 | < 1s | 4 | 4 | < 1s |
| wlan_d10dl6 | 97 | 148 | 63 | 94 | < 1s | 59 | 86 | 1s |
| wlan0col0 | 2954 | 3972 | 1097 | 1591 | 14s | 798 | 1092 | 120s |
| zeroconf | 670 | 827 | 341 | 433 | < 1s | 334 | 420 | 14s |
| zeroconf-nt | 670 | 827 | 52 | 75 | < 1s | 41 | 52 | < 1s |

| Components | $ S_{\circ} $ | $ D_{\circ} $ | $ [S]_{\approx} $ | $ [D]_{\approx} $ | T_{\approx} |
|---|---------------|---------------|---------------------|---------------------|---------------|
| $i_1 = d_1 \parallel d_2$ | 41 | 92 | 20 | 41 | 4s |
| $i_2 = [i_1]_{\approx} \parallel d_3$ | 105 | 247 | 33 | 75 | 33s |
| $i_3 = [i_2]_{\approx} \parallel d_4$ | 180 | 482 | 45 | 107 | 330s |
| $i_4 = [i_3]_{\approx} \parallel d_5$ | 248 | 706 | 57 | 139 | 20m |
| $[i_4]_{\approx} \parallel d_6$ | 178 | 372 | 7 | 6 | 22m |
| $d_1 \parallel d_2 \parallel d_3 \parallel d_4$ | 2242 | 4708 | 5 | 4 | 39s |
| $d_1 \parallel d_2 \parallel \dots \parallel d_5$ | 12042 | 31184 | 6 | 5 | 335s |
| $d_1 \parallel d_2 \parallel \dots \parallel d_6$ | 63511 | 196642 | 7 | 6 | 22m |
| $d_1 \parallel d_2 \parallel \dots \parallel d_7$ | 329784 | 1189626 | 8 | 7 | 59m |
| $d_1 \parallel d_2 \parallel \dots \parallel d_8$ | 1689417 | 6961480 | 9 | 8 | 161m |

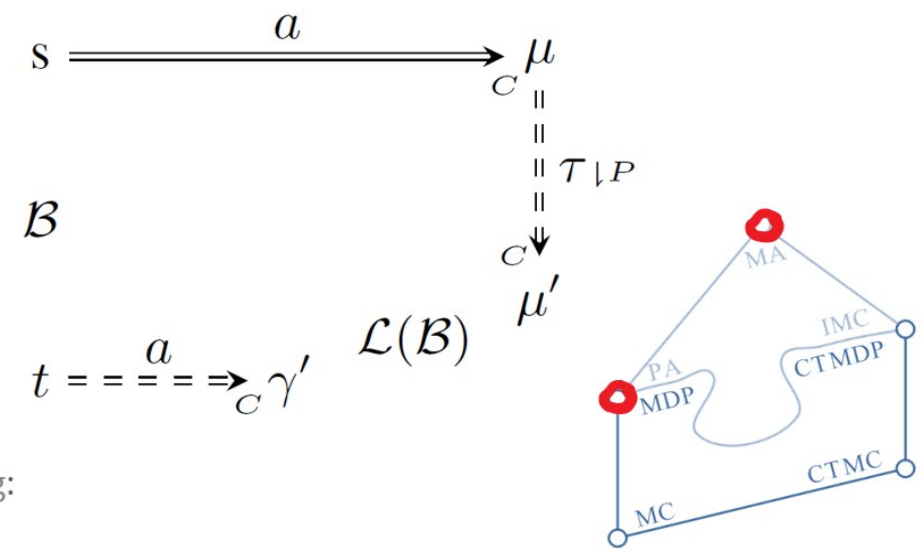
Compositional
minimization
applied to dining
cryptographer
anonymity





Weak Distribution Bisimulation Decision Algorithm

- Exploits a state-based characterisation.
- Preprocessing: Eliminate maximal τ -end components.
- Brute force guess of “preserving transition” set P .
- Overall strategy: standard partition refinement approach.
- Step conditions are encoded as LP-problems.
- Challenger transitions range over weak transitions induced by Dirac determinate schedulers.





Complexity

Polynomial-time decision algorithms
except for Markov automata.

There: Two sources of exponentiality.

- **Weak transitions appear as challengers.**
We do not know if strong challengers suffice.

- **Set of preserving transitions is guessed.**

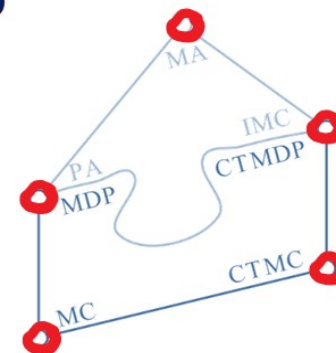
Possibility to intertwine with computation of \mathcal{B} ?

Johann Schuster, Markus Siegle:

Markov Automata: Deciding weak bisimulation by means of non-naïvely vanishing states. Inf. Comput. 237: 151-173 (2014)

Christian Eisentraut, Holger Hermanns, Julia Krämer, Andrea Turrini, Lijun Zhang:

Deciding Bisimilarities on Distributions. QEST 2013: 72-88

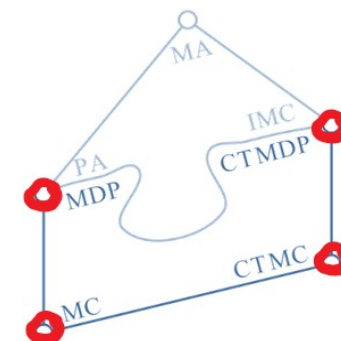




Minimality

Quotient Construction:

- Take one representative per equivalence class.
 - Then: Number of states is minimal.
 - Then: Number of transitions can be made minimal.
 - Then: Sum of transition fanout can be made minimal.
-
- This works for weak bisimulation (on PA etc).
 - But not for weak distribution bisimulation.





Joost-Pieter Katoen:

Concurrency Meets Probability: Theory and Practice - (Abstract). CONCUR 2013: 44-45

Mark Timmer, Jaco van de Pol, Mariëlle Stoelinga:

Confluence Reduction for Markov Automata. FORMATS 2013: 243-257

Mark Timmer, Joost-Pieter Katoen, Jaco van de Pol, Mariëlle Stoelinga:

Efficient Modelling and Generation of Markov Automata. CONCUR 2012: 364-379

Markov Automata

Construction & Compression

Marco Bozzano, Alessandro Cimatti, Joost-Pieter Katoen, Viet Yen Nguyen, Thomas Noll, Marco Roveri:

Safety, Dependability and Performance Analysis of Extended AADL Models. Comput. J. 54(5): 754-775 (2011)

Hichem Boudali, Pepijn Crouzen, Mariëlle Stoelinga:

A Rigorous, Compositional, and Extensible Framework for Dynamic Fault Tree Analysis.

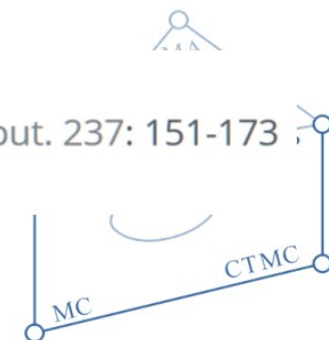
128-143 (2010)

Johann Schuster, Markus Siegle:

Markov Automata: Deciding weak bisimulation by means of non-naïvely vanishing states. Inf. Comput. 237: 151-173 (2014)

Christian Eisentraut, Holger Hermanns, Julia Krämer, Andrea Turrini, Lijun Zhang:

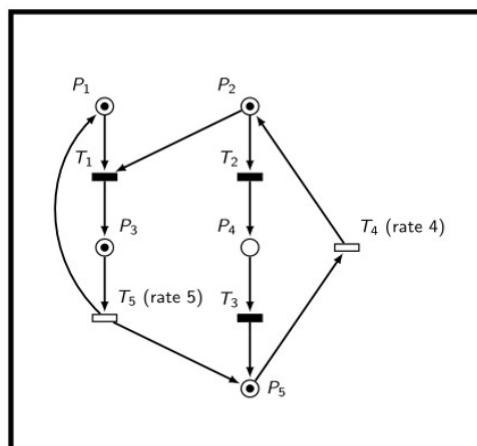
Deciding Bisimilarities on Distributions. QEST 2013: 72-88





MAMA Tool Chain

GSPN
(PNML)



reach P1 = 1 & P5 = 2

GEMMA

MAPA

```
GSPN(P1:N,P2:N,P3:N,
P4:N,P5:N) =
P2 >= 1 => T2 .
  GSPN[P2--, P4++]
+ P5 >= 1 => (4.0) .
  GSPN[P2++, P5--]
+ ...
init GSPN(1,1,1,0,1)
```

reach P1 = 1 & P5 = 2

SCOOP

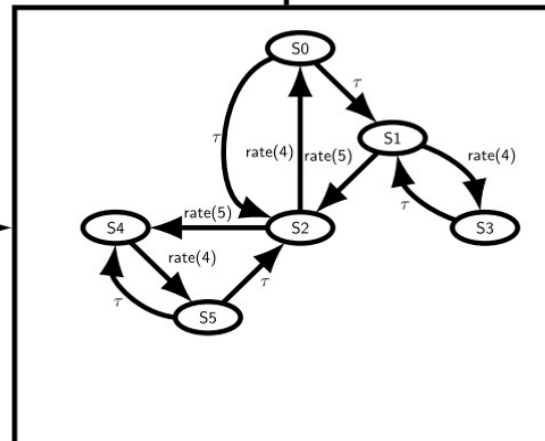
Min. reach. [1, 1.5]: 0.007
Max. reach. [1, 1.5]: 0.930

Min. expected time: 0.0
Max. expected time: 0.2

Min. LRA: 0.0
Max. LRA: 0.4

#GOALS S2

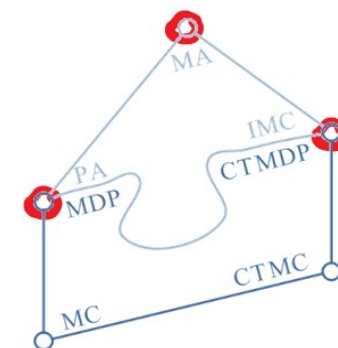
IMCA



#GOALS S2

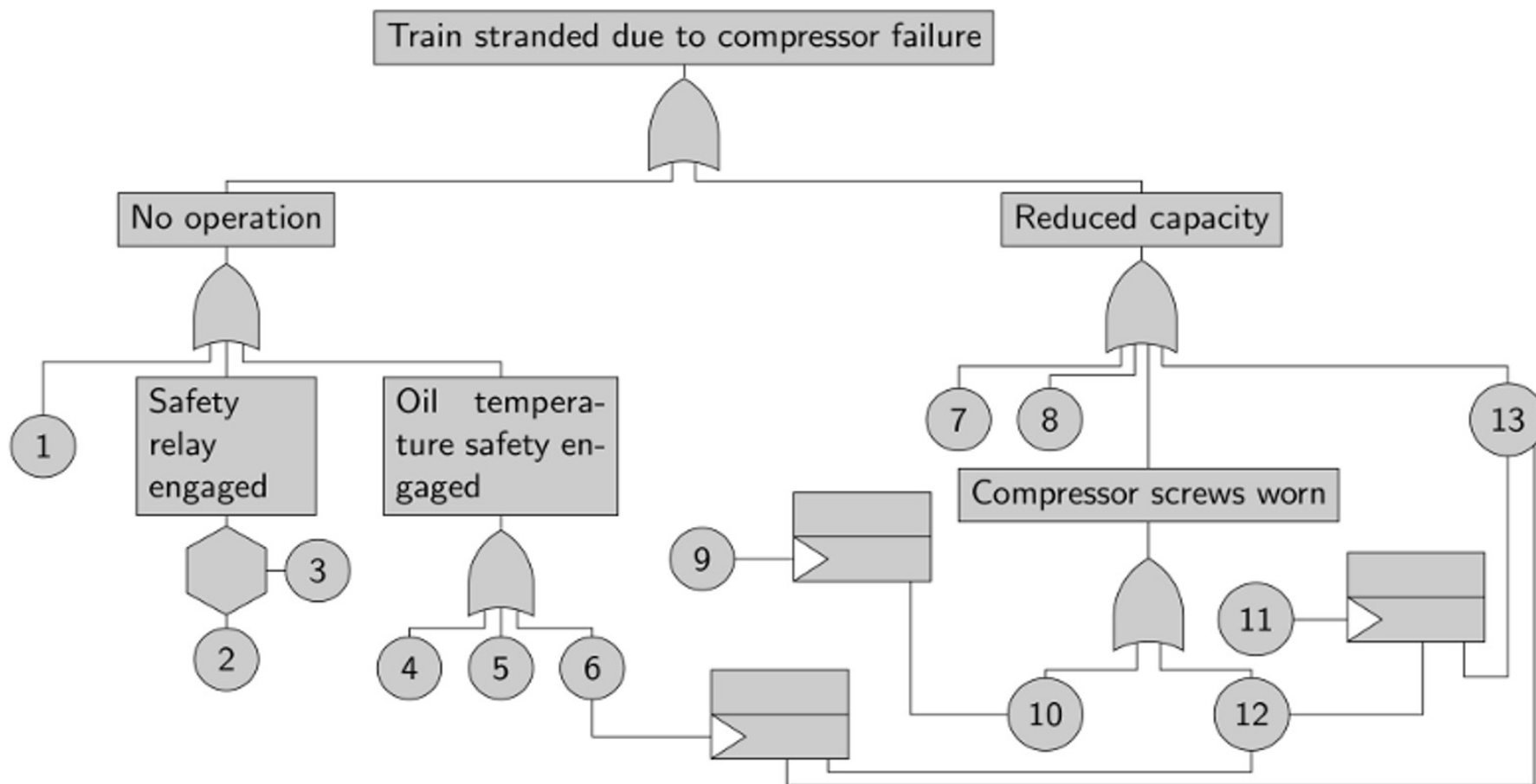
Results

MRA





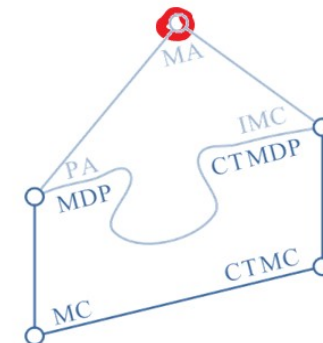
More of this: Failure-Prone Systems



Can all be coded up into MA components.

Marco Bozzano, Alessandro Cimatti, Joost-Pieter Katoen, Viet Yen Nguyen, Thomas Noll, Marco Roveri:
Safety, Dependability and Performance Analysis of Extended AADL Models. Comput. J. 54(5): 754-775 (2011)

Hichem Boudali, Pepijn Crouzen, Mariëlle Stoelinga:
A Rigorous, Compositional, and Extensible Framework for Dynamic Fault Tree Analysis.
IEEE Trans. Dependable Sec. Comput. 7(2): 128-143 (2010)



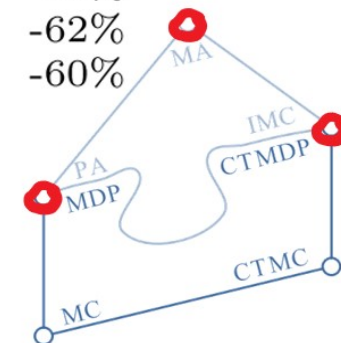


More of this: Confluence Reduction

Rewrites the state space representation

so as to eliminate τ -confluent transition.

| Specification | Original state space | | | | Reduced state space | | | | Impact | |
|---------------|----------------------|---------|-------|---------|---------------------|--------|-------|---------|--------|------|
| | States | Trans. | SCOOP | IMCA | States | Trans. | SCOOP | IMCA | States | Time |
| leader-3-7 | 25,505 | 34,257 | 4.7 | 102.5 | 5,564 | 6,819 | 5.1 | 9.3 | -78% | -87% |
| leader-3-9 | 52,465 | 71,034 | 9.7 | 212.0 | 11,058 | 13,661 | 10.4 | 17.8 | -79% | -87% |
| leader-3-11 | 93,801 | 127,683 | 18.0 | 429.3 | 19,344 | 24,043 | 19.2 | 31.9 | -79% | -89% |
| leader-4-2 | 8,467 | 11,600 | 2.1 | 74.0 | 2,204 | 2,859 | 2.5 | 6.8 | -74% | -88% |
| leader-4-3 | 35,468 | 50,612 | 9.0 | 363.8 | 7,876 | 10,352 | 8.7 | 33.3 | -78% | -89% |
| leader-4-4 | 101,261 | 148,024 | 25.8 | 1,309.8 | 20,857 | 28,023 | 24.3 | 94.4 | -79% | -91% |
| polling-2-2-4 | 4,811 | 8,578 | 0.7 | 3.7 | 3,047 | 6,814 | 0.7 | 2.3 | -37% | -32% |
| polling-2-2-6 | 27,651 | 51,098 | 12.7 | 91.0 | 16,557 | 40,004 | 5.4 | 49.0 | -40% | -48% |
| polling-2-4-2 | 6,667 | 11,290 | 0.9 | 39.9 | 4,745 | 9,368 | 0.9 | 26.6 | -29% | -33% |
| polling-2-5-2 | 27,659 | 47,130 | 4.0 | 1,571.7 | 19,721 | 39,192 | 4.0 | 1,054.6 | -29% | -33% |
| polling-3-2-2 | 2,600 | 4,909 | 0.4 | 7.1 | 1,914 | 4,223 | 0.5 | 4.8 | -26% | -29% |
| polling-4-6-1 | 15,439 | 29,506 | 3.1 | 330.4 | 4,802 | 18,869 | 3.0 | 109.4 | -69% | -66% |
| polling-5-4-1 | 21,880 | 43,760 | 5.1 | 815.9 | 6,250 | 28,130 | 5.1 | 318.3 | -71% | -61% |
| processor-2 | 2,508 | 4,608 | 0.7 | 2.8 | 1,514 | 3,043 | 0.8 | 1.2 | -44% | -43% |
| processor-3 | 10,852 | 20,872 | 3.1 | 66.3 | 6,509 | 13,738 | 3.3 | 23.0 | -45% | -62% |
| processor-4 | 31,832 | 62,356 | 10.8 | 924.5 | 19,025 | 41,018 | 10.3 | 365.6 | -45% | -60% |





Hassan Hatefi, Holger Hermanns:

Model Checking Algorithms for Markov Automata. ECEASST 53 (2012)

Dennis Guck, Hassan Hatefi, Holger Hermanns, Joost-Pieter Katoen, Mark Timmer:

Analysis of Timed and Long-Run Objectives for Markov Automata. Logical Methods in Computer Science 10(3) (2014)

Bettina Braitting, Luis María Ferrer Fioriti, Hassan Hatefi, Ralf Wimmer, Bernd Becker, Holger Hermanns:

MeGARA: Menu-based Game Abstraction and Abstraction Refinement of Markov Automata. QAPL 2014: 48-63

Markov Automata

Model Checking

Dennis Guck, Mark Timmer, Hassan Hatefi, Enno Ruijters, Mariëlle Stoelinga:

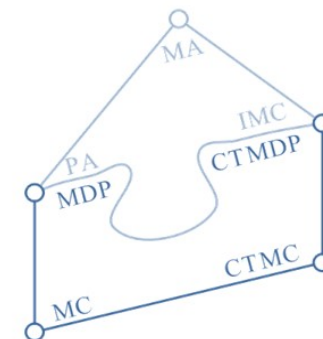
Modelling and Analysis of Markov Reward Automata. ATVA 2014: 168-184

Bettina Braitting, Luis María Ferrer Fioriti, Hassan Hatefi, Ralf Wimmer, Bernd Becker, Holger Hermanns:

Abstraction-Based Computation of Reward Measures for Markov Automata. VMCAI 2015: 172-189

Hassan Hatefi, Bettina Braitting, Ralf Wimmer, Luis María Ferrer Fioriti, Holger Hermanns, Bernd Becker:

Cost vs. Time in Stochastic Games and Markov Automata. SETTA 2015: 19-34



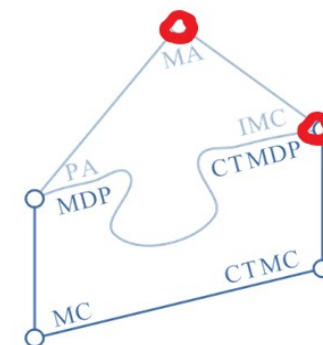
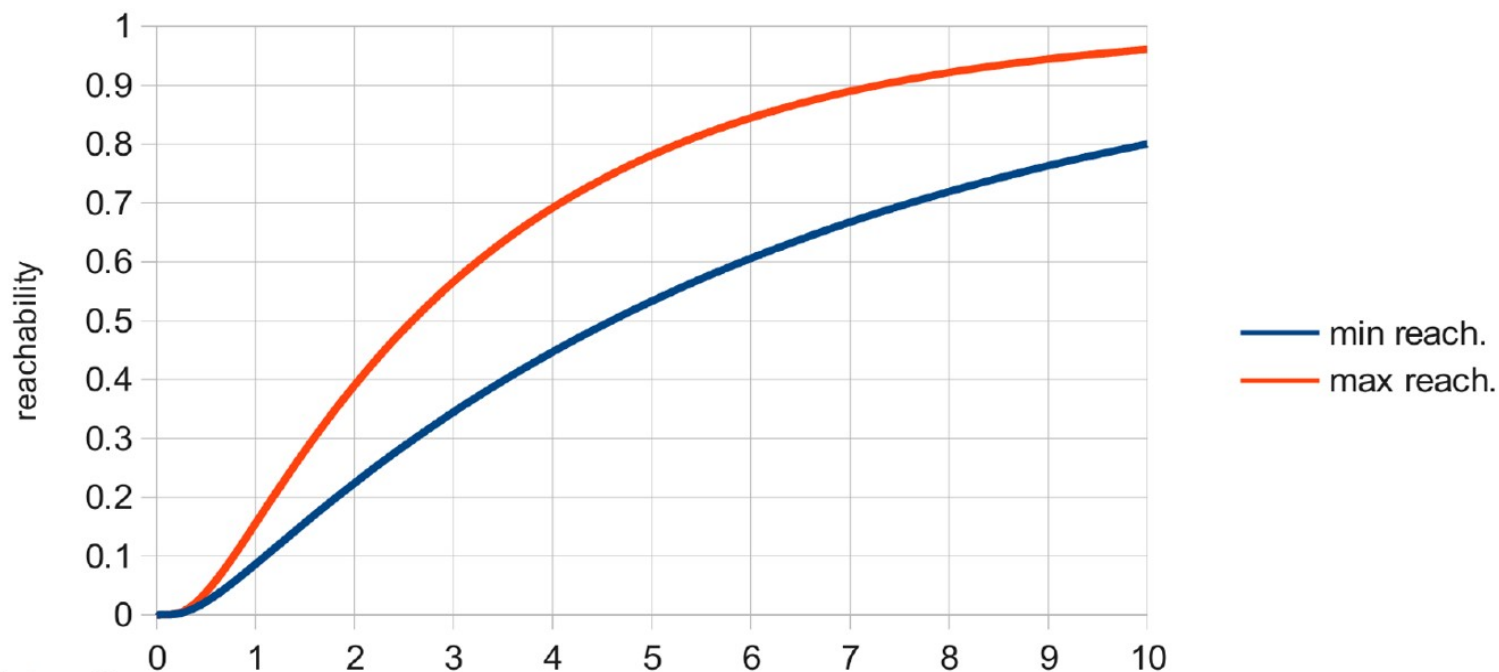


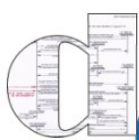
Model Checking CSL/PCTL

$$\Phi ::= a \mid \neg\Phi \mid \Phi \wedge \Phi \mid \mathcal{P}_{\leq p}(\phi)$$

$$\phi ::= \mathcal{X}'\Phi \mid \Phi \mathcal{U} \Phi \mid \Phi \mathcal{U}' \Phi$$

- Mostly reduces to MDP setting.
- Main Challenge: Time bounded reachability.

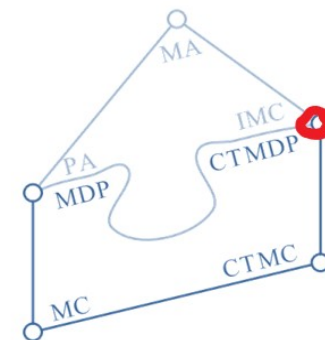
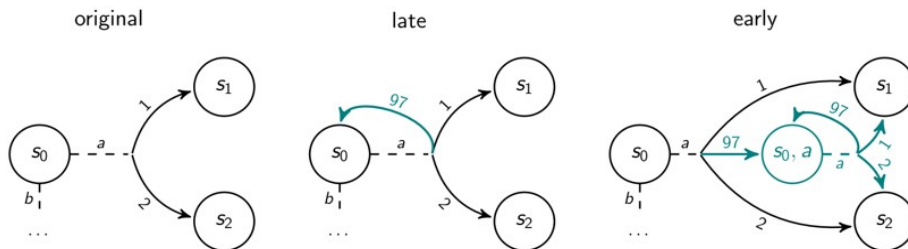


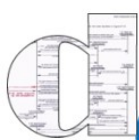


CTMDP Time Bounded Reachability

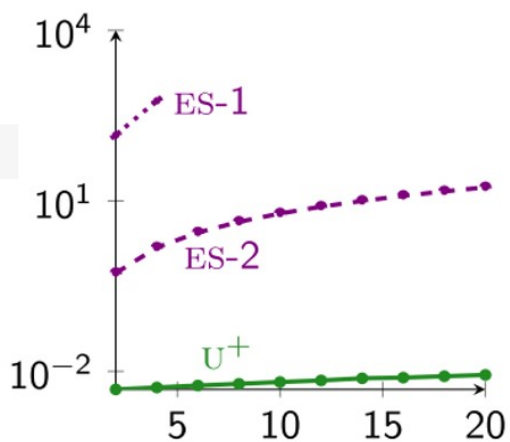
- Various algorithm have been proposed, in different setting.
- None of them achieve the efficiency known from CTMC.
- Exception:
 - Greedy Algorithm for Uniform Models.
 - Assumes time-abstract policies only.
- U^+ lifts both restrictions at once, and is very simple.
- It uniformises the model, doubling the uniformisation rate until sufficient.

Christel Baier, Holger Hermanns, Joost-Pieter Katoen, Boudewijn R. Haverkort: Efficient computation of time-bounded reachability probabilities in uniform continuous-time Markov decision processes. Theor. Comput. Sci. 345(1): 2-26 (2005)

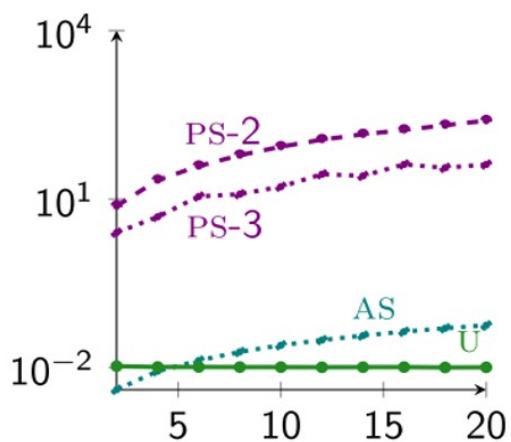




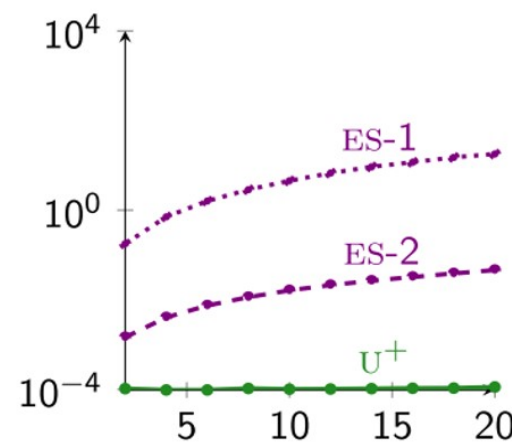
CTMDP Time Bounded Reachability



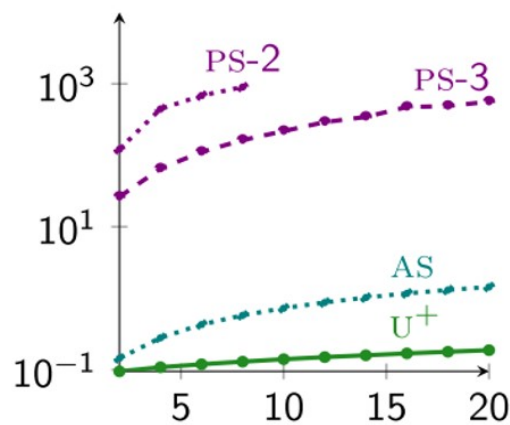
ES-1000-10, early, $\Delta = 2$, $|S| = 1004$, $\lambda = 10$, $\epsilon = 10^{-4}$



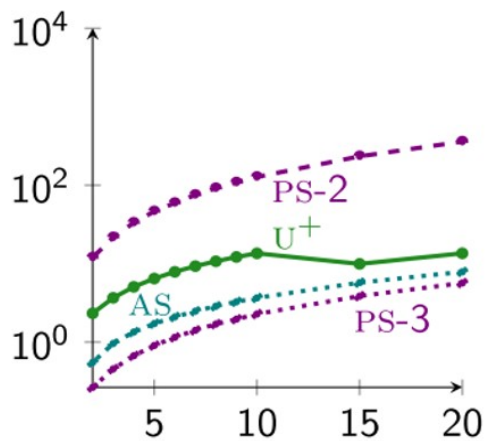
ES-1000-10, late, $\Delta = 2$, $|S| = 1004$, $\lambda = 10$, $\epsilon = 10^{-6}$



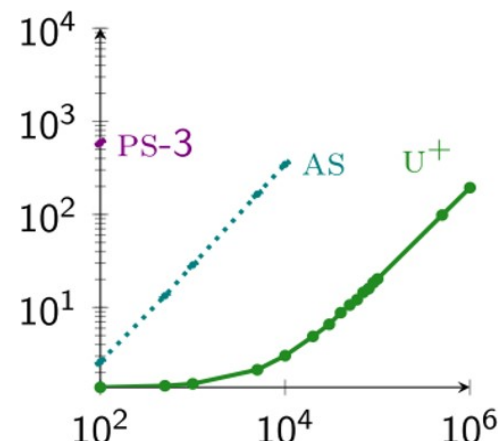
PS-1-1, early, $\Delta = 2$, $|S| = 17$, $\lambda = 2.8$, $\epsilon = 10^{-4}$



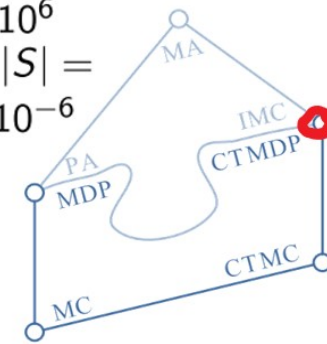
PS-4-2, late, $\Delta = 4$, $|S| = 10593$, $\lambda = 5.6$, $\epsilon = 10^{-6}$



QS-2-2, late, $\Delta = 8$, $|S| = 796$, $\lambda = 11.3$, $\epsilon = 10^{-6}$



FTWC-16, late, $\Delta = 5$, $|S| = 10130$, $\lambda = 2.06$, $\epsilon = 10^{-6}$





Long-Run Averages & Expectations

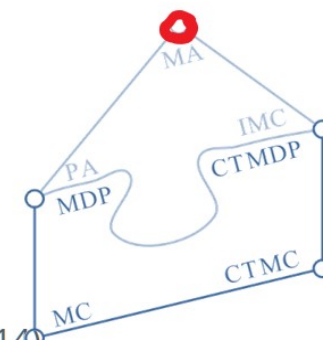
Given a set G of states of interest, one aims for

- Minimal and maximal expected time to reach G

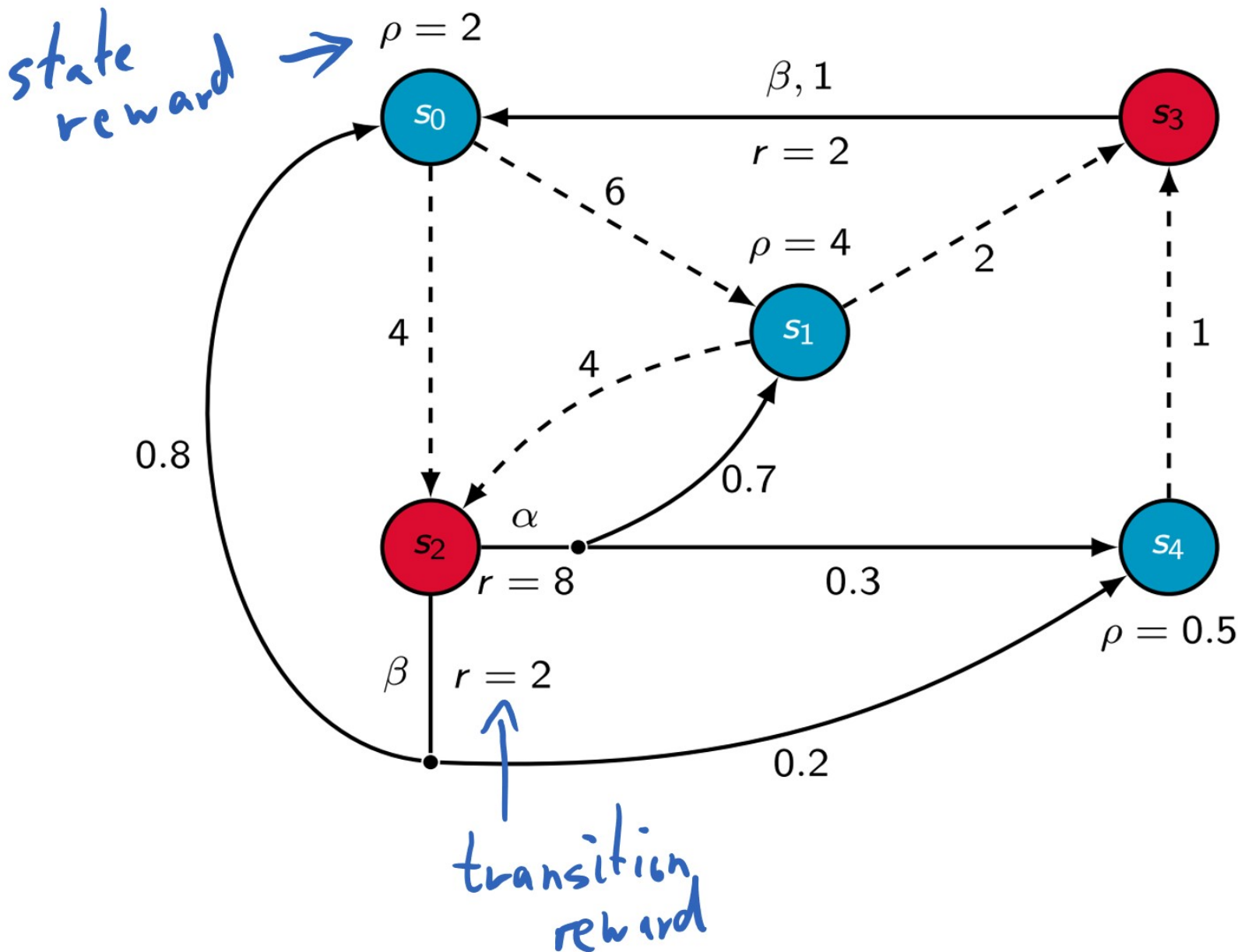
can be solved via stochastic shortest path computations

- Minimal and maximal long run fraction of time spent in G

reduces to MDP setting with residence times as costs



Adding Costs: Markov Reward Automata

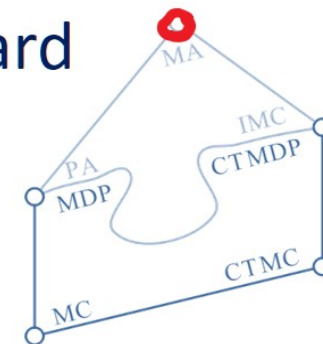


Algorithms extend smoothly

- Expected reward prior to goal
- Long-run reward rate

[de Alfaro] helps

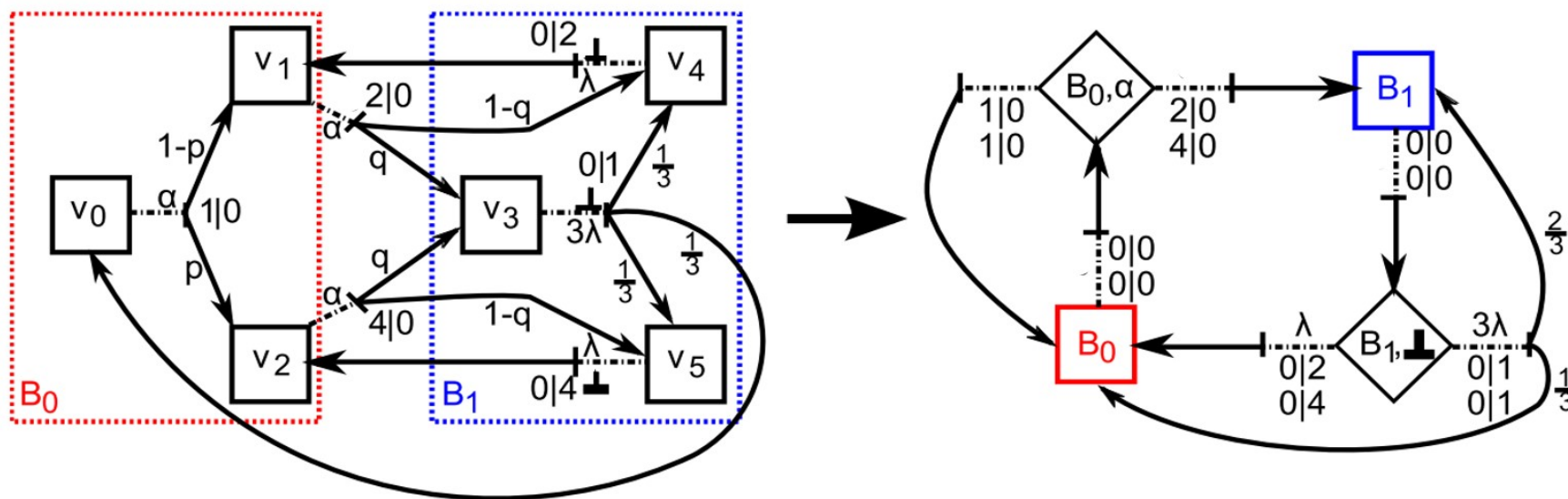
- Time bounded reward



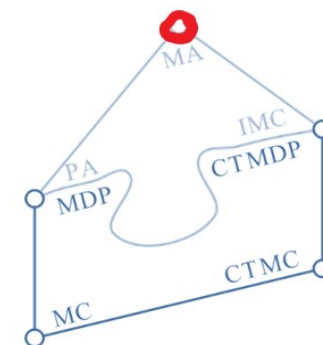


Attacking Size

- Abstraction refinement applied to Markov Reward Automata.



Game-based abstractions

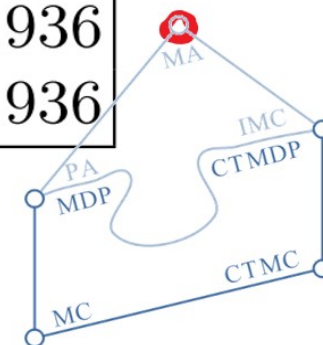




Attacking Size

- Abstraction refinement applied to Markov Reward Automata.

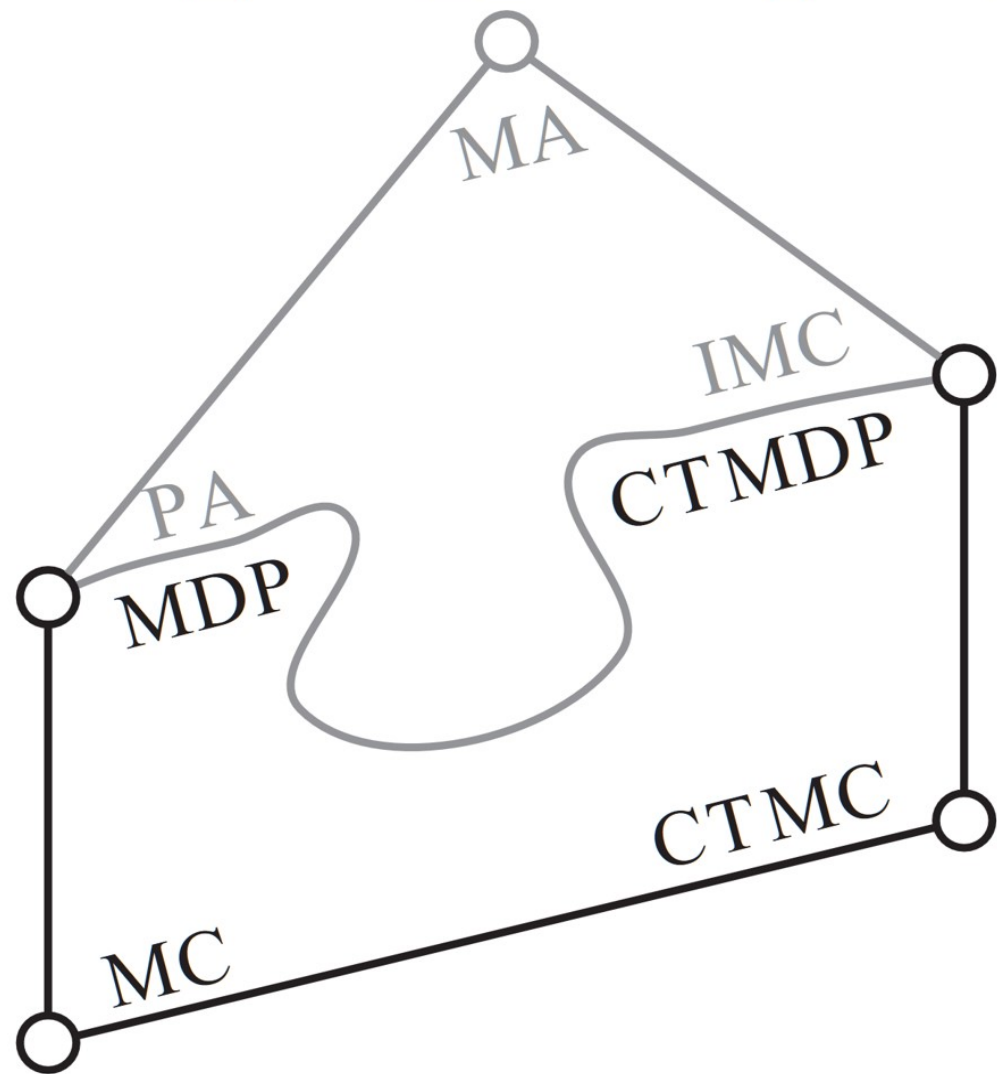
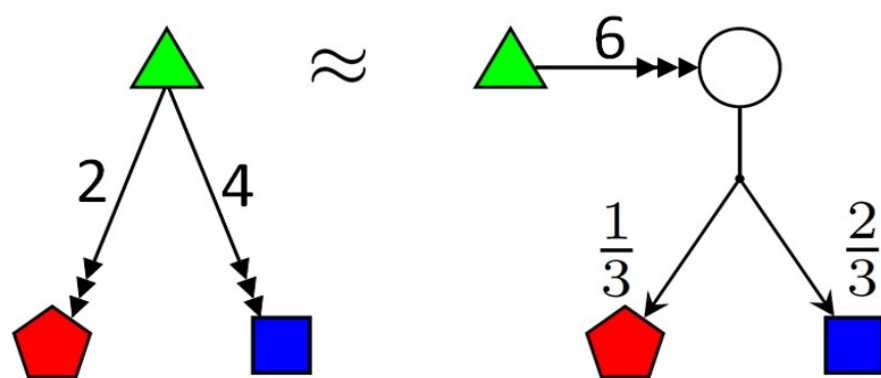
| name | #states | budget = 10 | | budget = 20 | | budget = 50 | |
|-----------|---------|-------------|-------|-------------|-------|-------------|-------|
| | | min | max | min | max | min | max |
| DPMS-2-5 | 508 | 0.759 | 0.859 | 1.557 | 1.924 | 3.910 | 5.150 |
| DPMS-2-10 | 1,588 | 0.759 | 0.859 | 1.557 | 1.924 | 3.910 | 5.150 |
| DPMS-2-20 | 5,548 | 0.759 | 0.859 | 1.557 | 1.924 | 3.910 | 5.150 |
| DPMS-3-5 | 5,190 | 0.785 | 0.883 | 1.617 | 1.930 | 4.129 | 5.088 |
| DPMS-3-10 | 29,530 | 0.785 | 0.883 | 1.617 | 1.930 | 4.129 | 5.088 |
| DPMS-3-20 | 195,810 | 0.785 | 0.883 | 1.617 | 1.930 | 4.129 | 5.088 |
| DPMS-4-5 | 47,528 | 0.784 | 0.877 | 1.617 | 1.889 | 4.143 | 4.936 |
| DPMS-4-10 | 492,478 | 0.784 | 0.877 | 1.617 | 1.889 | 4.143 | 4.936 |





Summary

- What?
- Why?
- How?
 - Construction
 - Compression
 - Verification
 - Extension
- Open Challenges

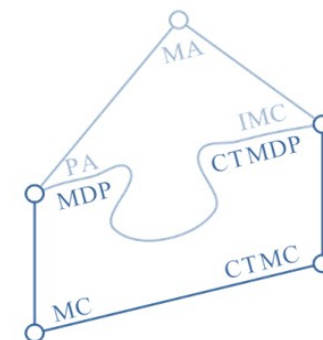




Outlook

Solve the open challenges

- Polynomial-time algorithm
for weak distribution bisimulation?
- Lifting U^+ to Markov Reward Automata.
- ...





Not Covered

Open Interpretation of CT Models

Tomás Brázdil, Holger Hermanns, Jan Krcál, Jan Kretínský, Vojtech Rehák:
Verification of Open Interactive Markov Chains. FSTTCS 2012: 474-485

Holger Hermanns, Jan Krcál, Jan Kretínský:
Compositional Verification and Optimization of Interactive Markov Chains. CONCUR 2013: 364-379

Distributed Synthesis in Continuous Time

justifies absence of interleaving scheduler
provided distributions have
continuous support.

Holger Hermanns, Jan Krcál, Steen Veester:
Distributed Synthesis in Continuous Time. FOSSACS 2016. To appear

