

# Learning to Use Learning in Verification

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joint work with T. Brázdil (Masaryk University Brno),  
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published at ATVA 2014, CAV 2015, TACAS 2016

Mysore Park Workshop  
Trends and Challenges in Quantitative Verification  
February 3, 2016

विमान के उड़ान पर चलते, उड़ान भरते तथा उड़ाने समय इस स्क्रीन को निचला करने पर लें



स्वागतम्  
Welcome

मनोरंजन कार्यक्रम शीघ्र ही उपलब्ध होगा  
Entertainment system will be available shortly

जैसे ही मनोरंजन सेवाएं उपलब्ध होंगी, आपकी स्क्रीन पर विमान की उड़ान स्थिति दर्शाता मॉपिंग आउटपुट आएगा। आप अपनी सीट के आर्मरेस्ट में लगे कंट्रोल यूनिट द्वारा या स्क्रीन को छूकर अपनी पसंद के अनुसूचित चैनल का चयन, ध्वनि नियंत्रण तथा मोड (ऑडियो/वीडियो/मॉपिंग) में परिवर्तन कर सकते हैं।

Once the entertainment services are activated, a moving map will appear on your screen. You can adjust volume, select channel and change mode (audio/video/map) according to your preferences, through the control unit in the armrest or by touching the screen.

PLEASE STOW MONITOR DURING TAXI, TAKE-OFF AND LANDING





AIR INDIA

सेवा उपलब्ध नहीं है  
Service not available

आपके हवाई यात्रे के दौरान  
use fasten seat belt anytime period

आपके हवाई यात्रे के दौरान  
Life vest use

No Signal (Error Code: 8001)

This could be due to a bad connection or bad weather conditions at your end or at broadcast centre.

1. Check the cable connections and remove any obstruction around the dish.
2. Try to change channels.
3. Restart the set top box by switching power off and then on.

For Installation Setup

☎ 133 / 1333 / 13333 / 133333





TO PREVENT DAMAGE TO  
LAP BELT, ENSURE THE  
BELT IS FASTENED  
BEFORE SEAT PAN IS  
STOWED.

Seat must be occupied  
during take off and landing

```
Address: 00-00-00-00-00-00
Copyright (C) 2000, 2001, 2002, Red Hat, Inc.
RAM: 0x00000000-0x00000000, 0x01000000-0x01000000 available

verifying MBR... OK, MBR:
Partition 0: already exists.
Partition 1: already exists.
Partition 2: already exists.
Partition 3: already exists.

verifying image... OK
-- Booting image in 5 seconds - enter 'c' to abort
load address 0x00000000
image length 0x00000000
loading kernel binary
read image signature... OK
decompressing image...
image loaded from 0x01000000-0x01100000
memory size 0x00000000
checksum sum (kernel): 0000000000000000
OK started at 0x00000000
writing linux image from 0x00000000
```







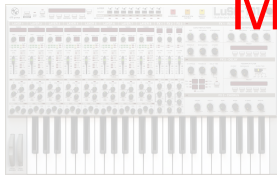
## Formal methods

- ▶ precise
- ▶ scalability issues

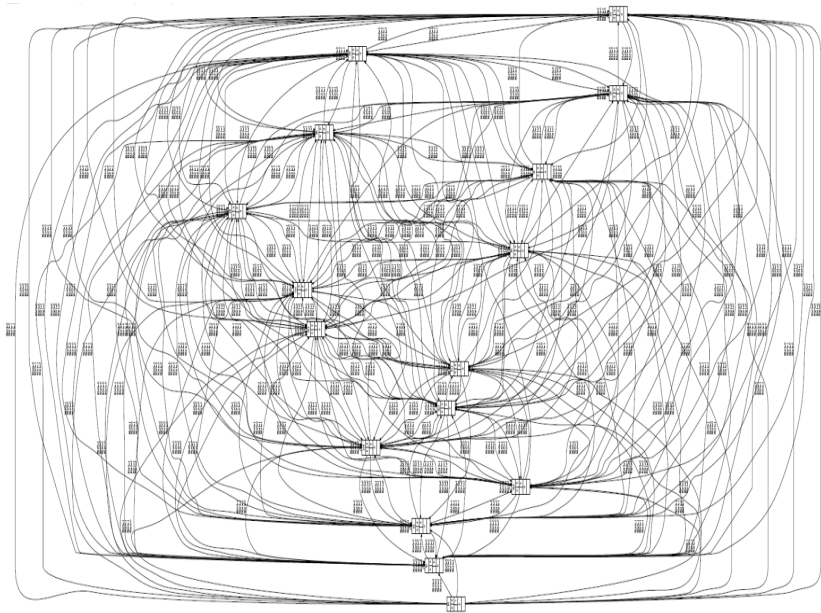


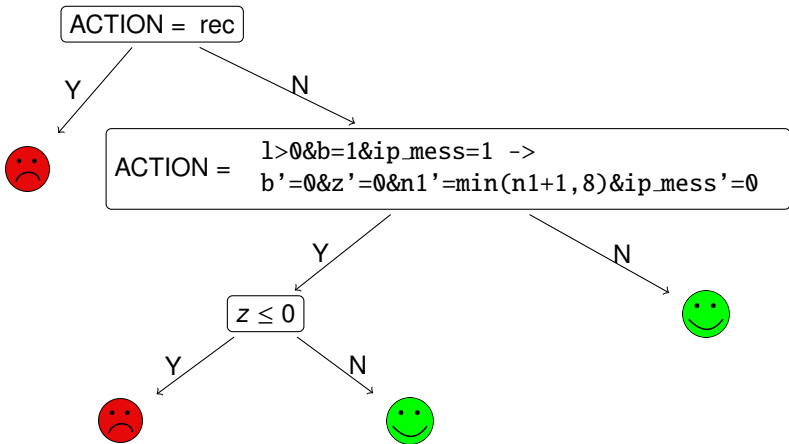
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**MEM-OUT**





## Formal methods

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## Learning

- ▶ weaker guarantees
- ▶ scalable



different objectives

## Formal methods

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- ▶ scalability issues



## Learning

- ▶ weaker guarantees
- ▶ scalable



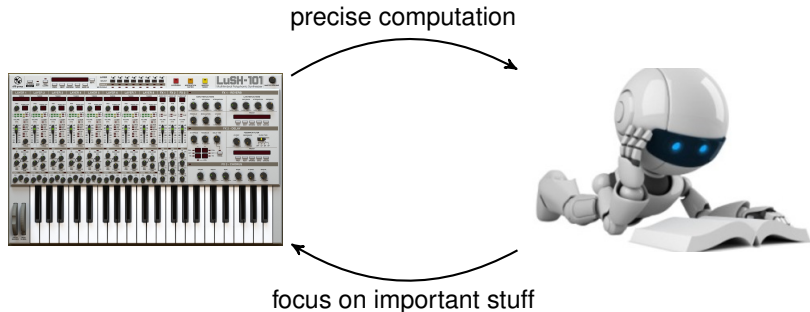


## Formal methods

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- ▶ scalability issues

## Learning

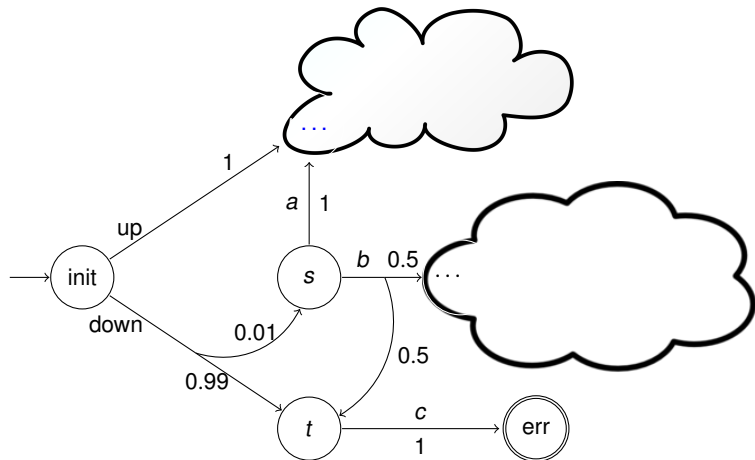
- ▶ weaker guarantees
- ▶ scalable



- ▶ Verification
  - ▶  $(\varepsilon)$ -optimality  $\overset{?}{\rightarrow}$  PAC
  - ▶ hard guarantees  $\overset{?}{\rightarrow}$  probably correct
- ▶ Controller synthesis
  - ▶ convergence is preferable
  - ▶ at least probably correct?
- ▶ Synthesis

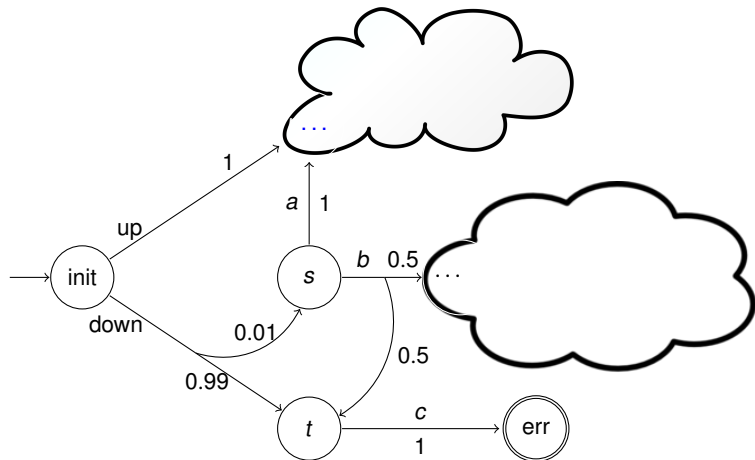
# Markov decision processes

$(S, s_0 \in S, A, \Delta : S \rightarrow A \rightarrow \mathcal{D}(S))$



# Markov decision processes

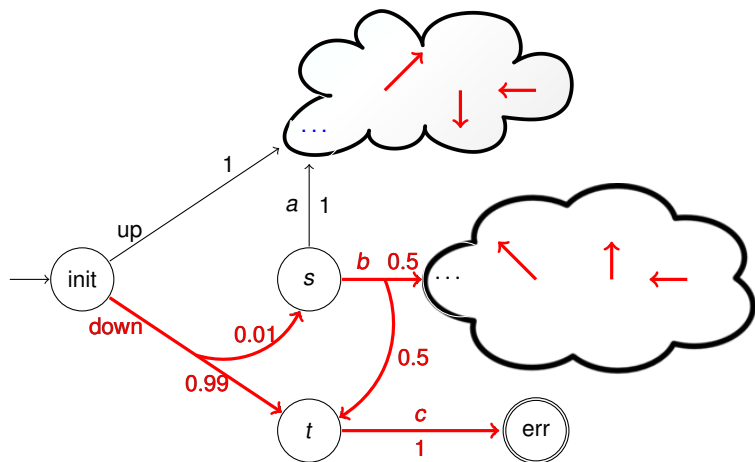
$$(S, s_0 \in S, A, \Delta : S \rightarrow A \rightarrow \mathcal{D}(S))$$



$$\max_{\text{strategy } \sigma} \mathbb{P}^{\sigma}[\text{Reach err}]$$

# Markov decision processes

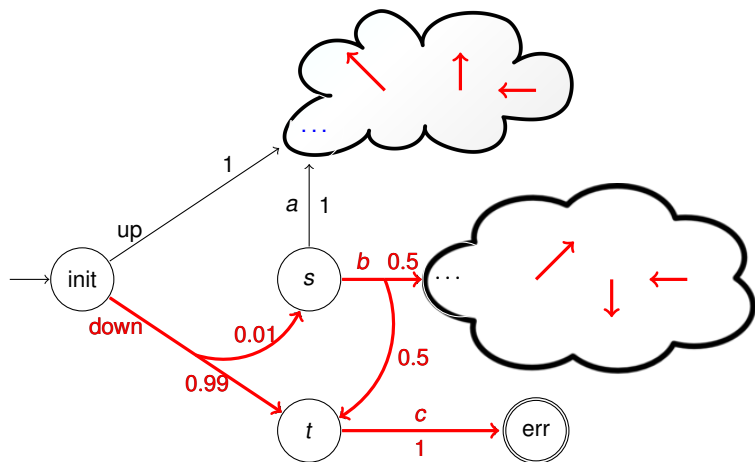
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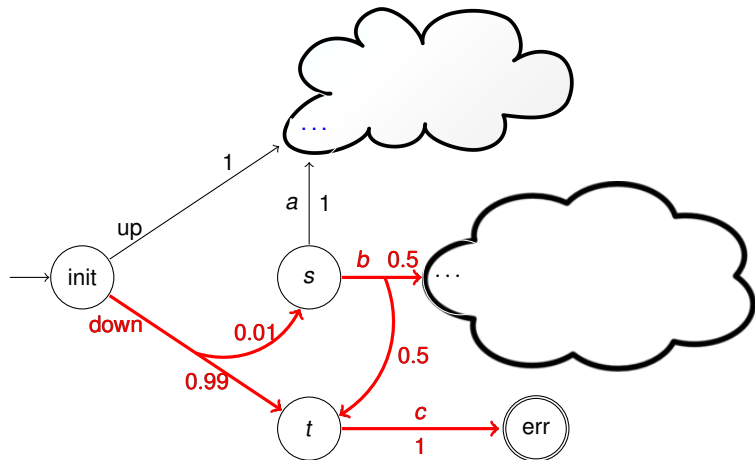
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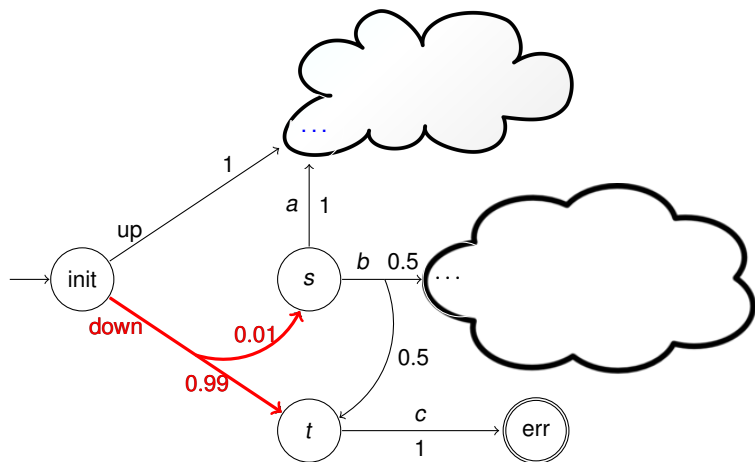
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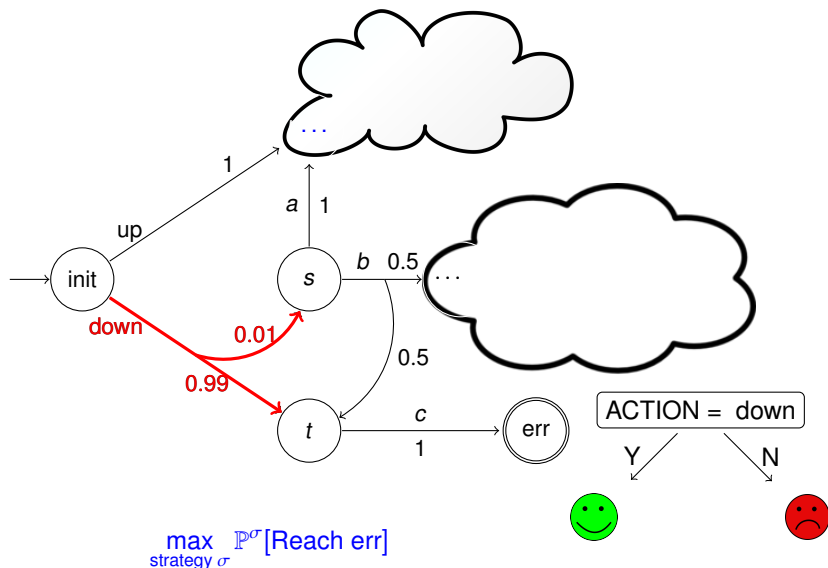
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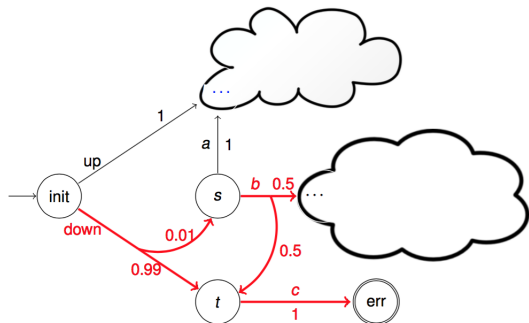
# Markov decision processes

5/16

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# Ex.1: Computing strategies faster: How?

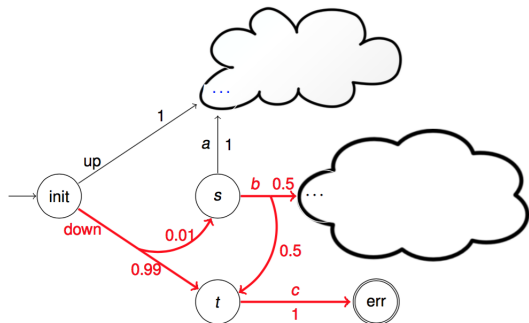


Fixed-point computation

$$V(s) := \max_{a \in \Delta(s)} V(s, a)$$

$$V(s, a) := \sum_{s' \in S} \Delta(s, a, s') \cdot V(s')$$

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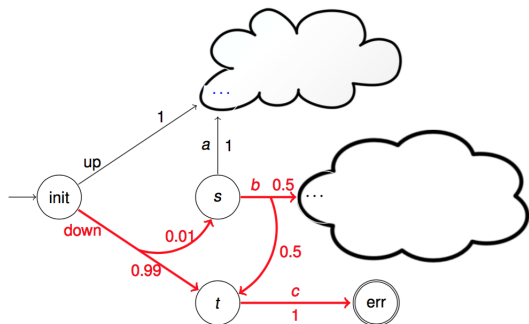
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Order of evaluation?

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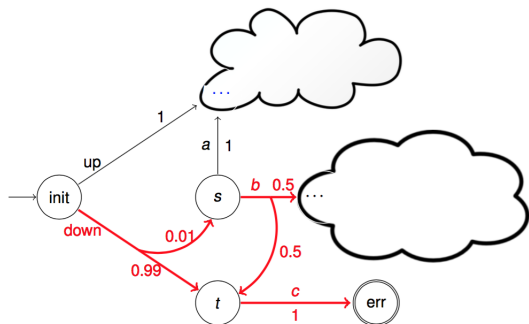
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Order of evaluation? [ATVA'14]

More frequently evaluate those states that are **visited** more frequently

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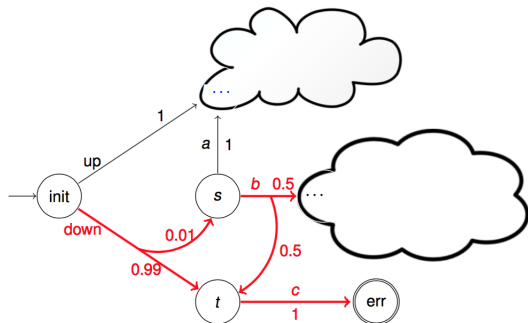
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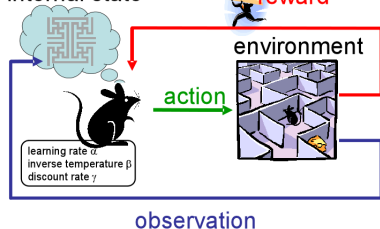
# Ex.1: Computing strategies faster: How?

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## Reinforcement learning

internal state



Fixed-point computation

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Order of evaluation? [ATVA'14]

More frequently evaluate those states that are **visited** more frequently  
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# Ex.1: Computing strategies faster: Algorithm

1:  $U(\cdot, \cdot) \leftarrow 1, L(\cdot, \cdot) \leftarrow 0$

2:  $L(\mathbf{1}, \cdot) \leftarrow 1, U(\mathbf{0}, \cdot) \leftarrow 0$

3: **repeat**

4: **sample a path** from  $s_0$  to  $\{\mathbf{1}, \mathbf{0}\}$

- actions uniformly from  $\arg \max_a U(s, a)$
- states according to  $\Delta(s, a, s')$

7: **until**  $U(s_0) - L(s_0) < \epsilon$

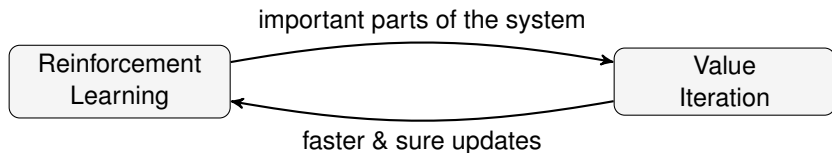


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- 5:     **for all** visited transitions  $(s, a, s')$  **do**
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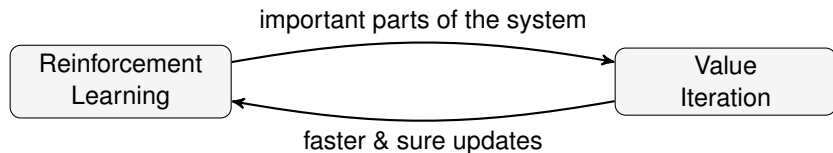
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- 1: **procedure** **UPDATE** $(s, a, \cdot)$
  - 2:      $U(s, a) := \sum_{s' \in \mathcal{S}} \Delta(s, a, s') \cdot U(s')$
  - 3:      $L(s, a) := \sum_{s' \in \mathcal{S}} \Delta(s, a, s') \cdot L(s')$

## Ex.1: Computing strategies faster



Guaranteed upper & lower bounds at all times + practically fast convergence



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Remark:

- ▶ PAC SMC for MDP and (unbounded) LTL [ATVA'14]:  $|S|, \rho_{\min}$
- ▶ practical PAC SMC for MC and (unbounded) LTL + mean payoff [TACAS'16]:  $\rho_{\min}$

# Ex.1: Experimental results

Example	Visited states	
	PRISM	BRTDP
<i>zeroconf</i>	3,001,911	760
	4,427,159	977
	5,477,150	1411
<i>wlan</i>	345,000	2018
	1,295,218	2053
	5,007,548	1995
<i>firewire</i>	6,719,773	26,508
	13,366,666	25,214
	19,213,802	32,214
<i>mer</i>	17,722,564	1950
	17,722,564	2902
	26,583,064	1950
	26,583,064	2900

Sebastian Junges, Nils Jansen, Christian Dehnert, Ufuk Topcu, Joost-Pieter Katoen:

*Safety-Constrained Reinforcement Learning for MDPs*. TACAS 2016.

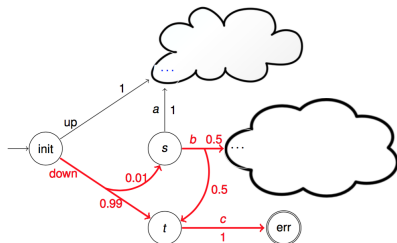
- ▶ safe and cost-optimizing strategies
- ▶ (1) compute safe, permissive strategies
- ▶ (2) learn cost-optimal strategies (convergence) among them

Alexandre David, Peter Gjl Jensen, Kim Guldstrand Larsen, Axel Legay, Didier Lime, Mathias Grund Srensen, Jakob Haahr Taankvist:

*On Time with Minimal Expected Cost!* ATVA 2014.

- ▶ priced timed games → priced timed MDPs
- ▶ time-bounded cost-optimal (convergence) strategies
- ▶ (1) Uppaal TiGa for safe strategies
- ▶ (2) Uppaal SMC and learning for cost-optimal strategies

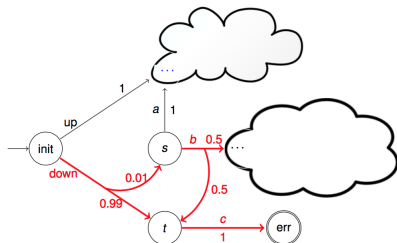
## Ex.2: Computing small strategies: Which decisions?



Importance of a node **s** with respect to  $\diamond target$  and strategy  $\sigma$ :

$$\mathbb{P}^{\sigma}[\diamond s \quad ]$$

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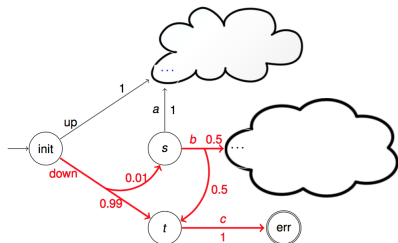


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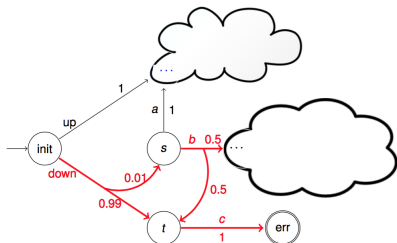


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Cut off states with **zero** importance (unreachable or useless)

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Cut off states with **zero** importance (unreachable or useless)

Cut off states with **low** importance (small error,  $\varepsilon$ -optimal strategy)

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How to make use of the exact *importance*?

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How to make use of the exact importance?

Learn decisions in  $s$  in proportion to importance of  $s$

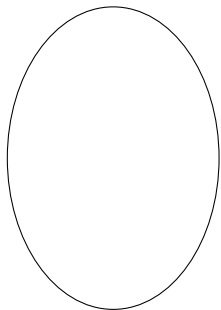
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How to make use of the exact importance?

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Advantages of decision trees over BDDs:

- ▶ more readable: predicates
- ▶ smaller due to good ordering: entropy
- ▶ yet smaller at a cost of an error: pruning



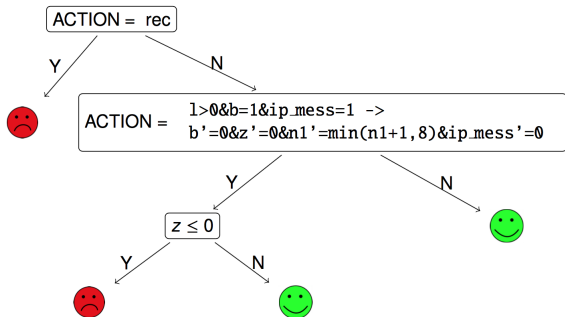
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## Ex.2: Experimental results

<b>Example</b>	#states	Value	Explicit	BDD	DT	Rel.err(DT) %
firewire	481,136	1.0	479,834	4233	1	0.0
investor	35,893	0.958	28,151	783	27	0.886
mer	1,773,664	0.200016	MEM-OUT			*
zeroconf	89,586	0.00863	60,463	409	7	0.106

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\* MEM-OUT in PRISM,  
 whereas BRTDP yields:      1887    619    13    0.00014



Pranav Garg, Daniel Neider, P. Madhusudan, Dan Roth:  
*Learning Invariants using Decision Trees and Implication Counterexamples*. POPL 2016.

- ▶ positive examples from runs of the program
- ▶ negative examples from modifications
- ▶ implication examples

Siddharth Krishna, Christian Puhersch, Thomas Wies:  
*Learning Invariants Using Decision Trees*.

- ▶ positive examples: states reachable when preconditions holds
- ▶ negative examples: states leaving loop and violating a postcondition

## Machine learning in verification

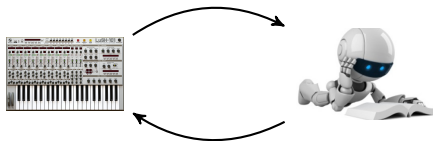
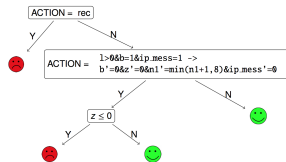
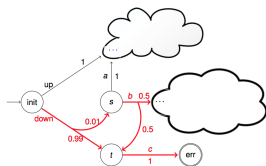
### ▶ Scalable heuristics

#### ▶ Example 1: **Speeding up** value iteration

- ▶ TECHNIQUE: **reinforcement learning**, BRTDP
- ▶ IDEA: focus on updating “**most important parts**”  
= most often visited by good strategies

#### ▶ Example 2: **Small and readable strategies**

- ▶ TECHNIQUE: **decision tree learning**
- ▶ IDEA: based on the **importance of states**,  
feed the decisions to the learning algorithm



## Verification using machine learning



- ▶ How far do we want to compromise?
- ▶ Do we have to compromise?
  - ▶ BRTDP, invariant generation, strategy representation don't
- ▶ Don't we want more than ML?
  - ▶ ( $\epsilon$ -)optimal controllers?
  - ▶ arbitrary controllers – is it still verification?
- ▶ What do we actually want?
  - ▶ scalability shouldnt overrule guarantees?
    - ▶ SMC should be PAC; when is it enough?
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