Metric Temporal Logic With Counting

S.N.Krishna, Khushraj Madnani, Paritosh.K.Pandya

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- Counting within a given time slot is a very natural and useful property in real time systems.
- Thus it becomes interesting to study satisfiability checking for its fragments and their extensions with ability to count.

Presentation Flow

- Model : Timed Words
- Timed Logic with Counting : Syntax and Semantics
- Temporal Projections : Simple and Oversampled
- Expressiveness Relations with Counting Extensions
- Satisfiability Checking: Decidability
- Conclusion
- Future Work

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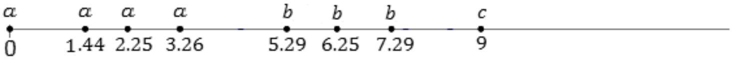

Figure: A finite timed word over $\Sigma = \{a, b, c\}$. A strictly monotonic timed word can be seen as a real line annotated with symbols from Σ

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Metric Temporal Logic : Semantics

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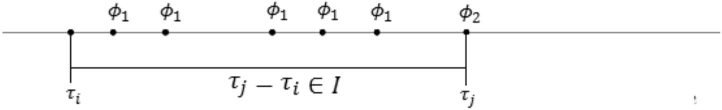

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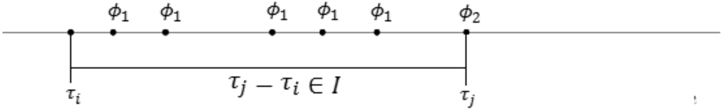

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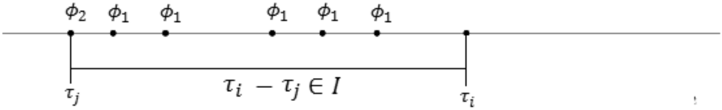

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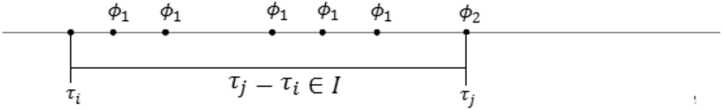

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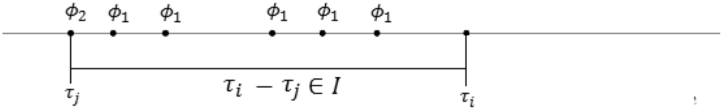

Ex: work-hard $\mathsf{U}_{[5,10]}$ giving-up

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Time Logic : MTL Fragments

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- We will restrict to future only fragment of MTL.

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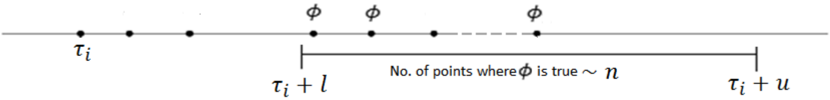

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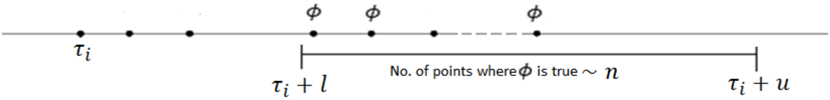

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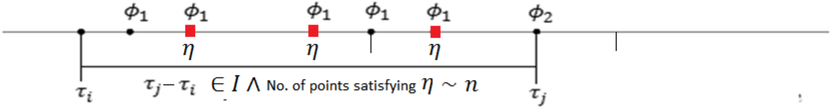

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- Scheduling of HVAC to limit peak power demand below threshold.
- HVAC are more flexible as compared to devices like microwave oven.
- Constant mode switching (OFF-¿ON) causes wear and tear and more power consumption due to transient currents.
- No. of Switch yet another important parameter to grade such scheduling algorithms.

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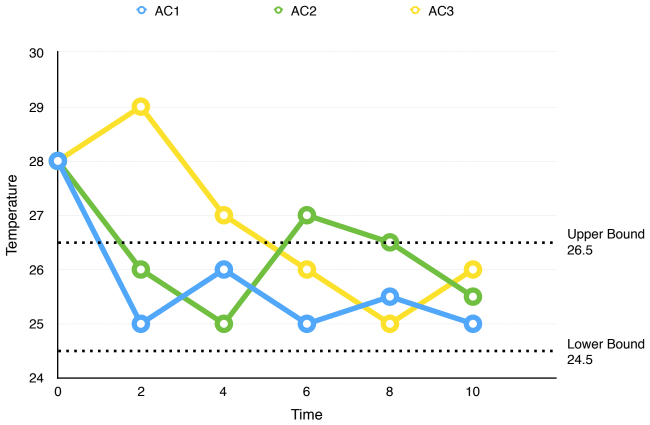

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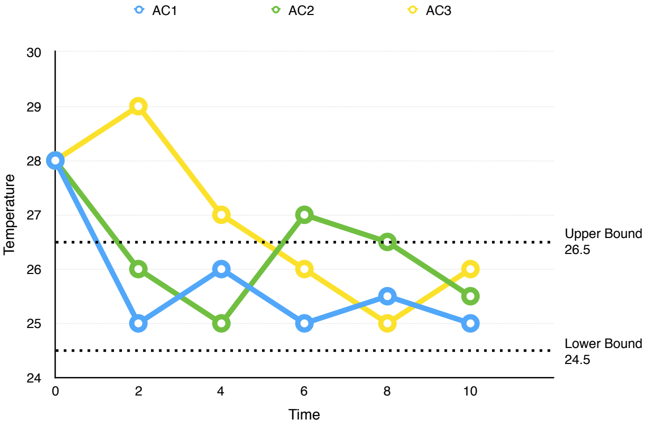
$\Diamond_{(0,3),\#Switch-ON-AC\leq 3}(Comfort - AC_1 \wedge Comfort - AC_2 \wedge Comfort - AC_3)$



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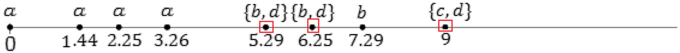

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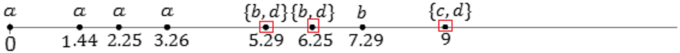

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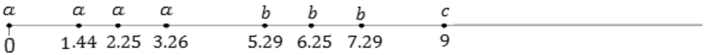

The simple projection of the above Extension over Σ

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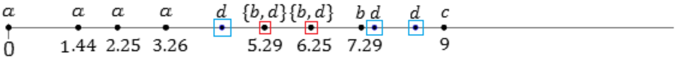

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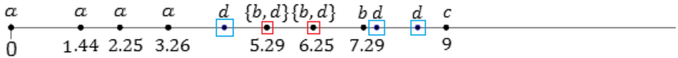

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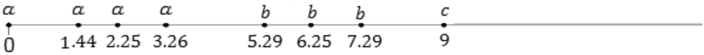

The oversampled projection of the above oversampled behaviour over Σ

We say that φ over Σ is equisatisfiable modulo temporal projection ψ over $\Sigma \cup 2^X$ iff:

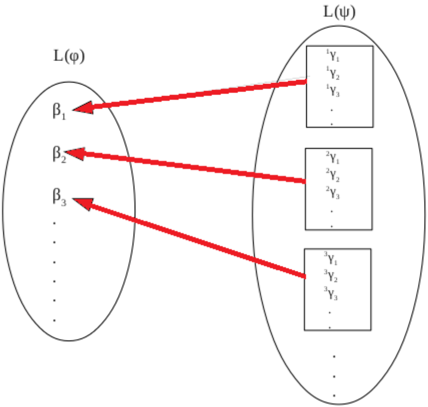

Figure: Figure Illustrating φ is equisatisfiable to ψ. Arrow represents the temporal(simple or oversampled) projection function

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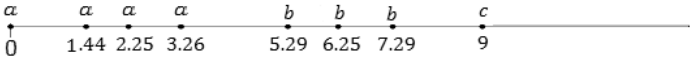

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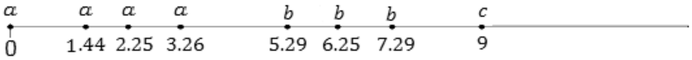

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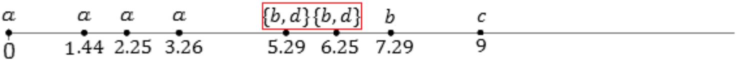

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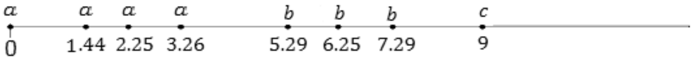

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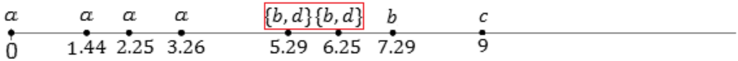

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Thus flattening is an example of a reduction preserving satisfiability modulo simple projections.

Related Work

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- *MTL* with counting over signals is expressively complete with $FO[<, +1]$ over reals [Hunter *CSL* 2013].

Related Work

- Satisfiability Checking of *MITL* is decidable with *EXPSPACE* complexity. [Alur *et al*. *J.ACM* 1996]
- Satisfiability problem for MTL[U_I] is decidable by reducing it to reachability problem for 1-clock Timed Alternating Automata.[Ouaknine *et al*. *LICS* 2005]
- Satisfiability Checking of QTL with counting is decidable with *EXPSPACE* complexity. [Rabinovich *et.al*. *FORMATS* 2008]
- Counting adds expressiveness to MITL over signals [Rabinovich *FORMATS* 2008].
- *MTL* with counting over signals is expressively complete with $FO[<, +1]$ over reals [Hunter *CSL* 2013].
- Counting LTL is equivalent to LTL and has $EXP-SPACE$ complete satisfiability checking.[Laroussinie *et. al*. *TIME* 2010].

Our Results

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- Satisfiability Checking for CTMTL is decidable.

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- Satisfiability Checking for CTMTL is decidable.
- Exploring Expressiveness relations amongst fragments of MTL with counting over timed words(Pointwise Semantics).

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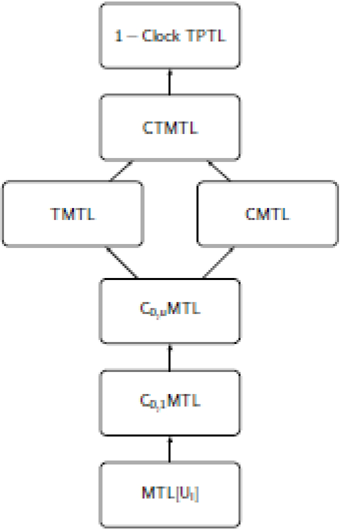
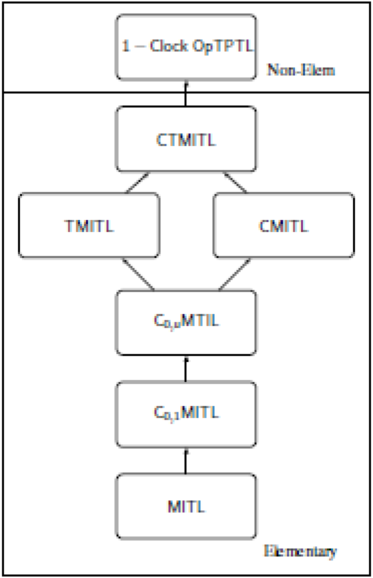
Expressiveness Heirarchy : Logic with counting

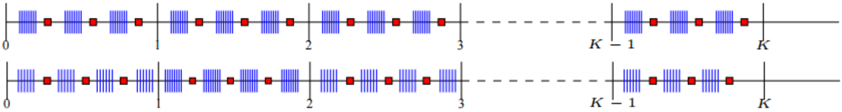


Expressiveness Heirarchy : Non-Punctual Fragments



TMTL − CMTL $\neq \emptyset$

$$\varphi = \Diamond_{(0,1),\#a \geq 3} b$$



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Satisfiability Checking : Decidability

- **Flatten the formula**

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- All the counting modalities are of the form $\Box(w \leftrightarrow \mathsf{C}_I^{\sim n} a)$ and $\Box(w \leftrightarrow a \mathsf{U}_{I,\#x \sim n} b)$

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- All the counting modalities are of the form $\Box(w \leftrightarrow \mathsf{C}_I^{\sim n} a)$ and $\Box(w \leftrightarrow a\mathsf{U}_{I,\#x\sim n} b)$
- Next we eliminate counting modalities from the above flattened formula preserving satisfiability to show decidability.

Eliminating $C_I^{\geq n}b$ modality

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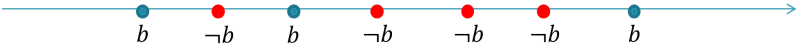
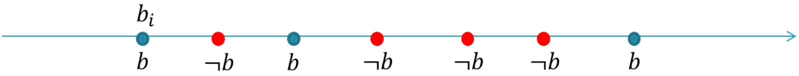
- Given a word ρ over Σ we construct a simple extension ρ' over $\Sigma \cup \{b_0, b_1, \ldots, b_{n-1}\}$

Eliminating $\mathsf{C}_I^{\geq n}\mathsf{b}$ modality

- Given a word ρ over Σ we construct a simple extension ρ' over $\Sigma \cup \{b_0, b_1, \ldots, b_{n-1}\}$
- $\{b_0, b_1, \ldots, b_{n-1}\}$ works as a counter. Using their behaviour we precisely mark a as the witness for $\mathsf{C}_I^{\geq n}\mathsf{b}$.

Construction of ρ'





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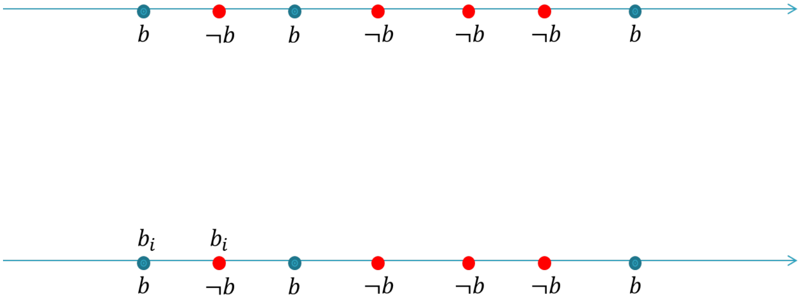

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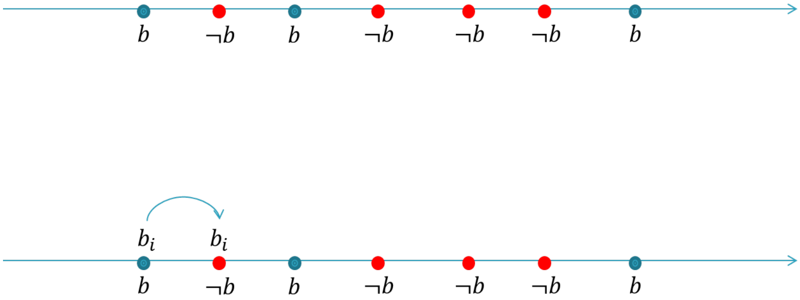

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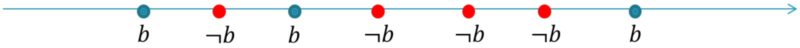

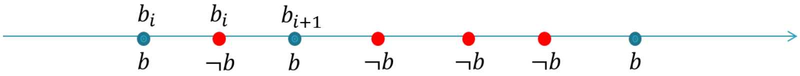

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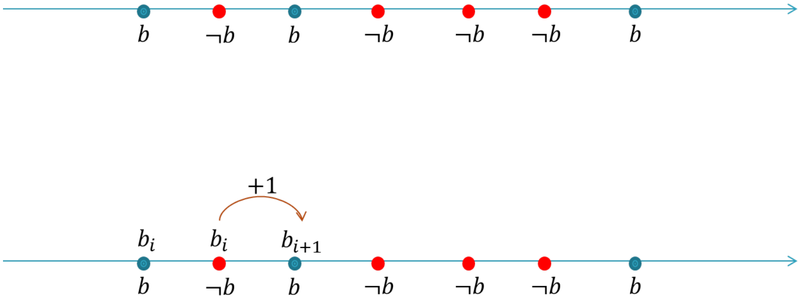

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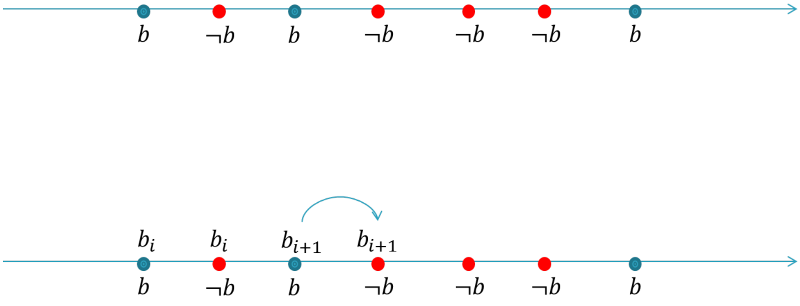

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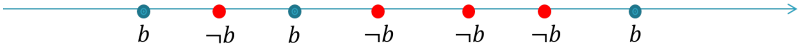

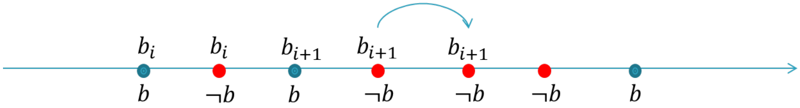

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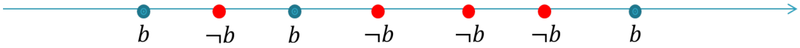

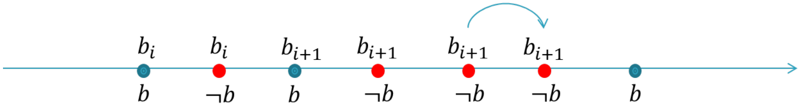

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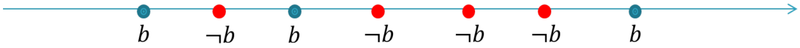
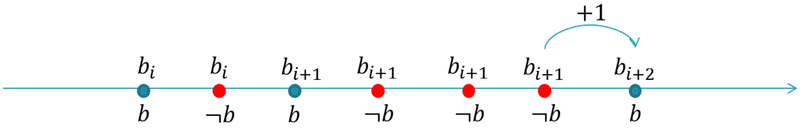
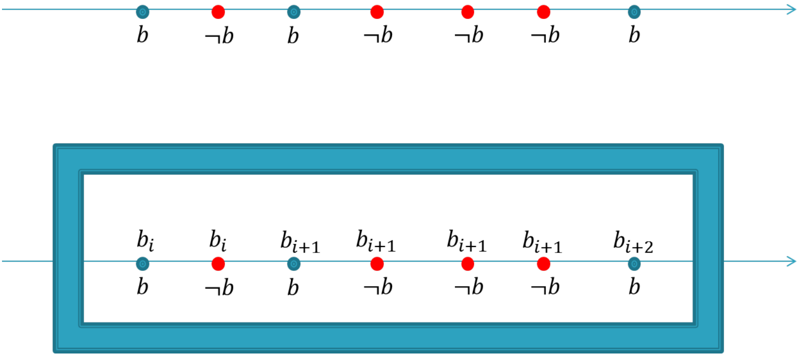




Construction of ρ'

Note how the behaviour of b_i helps in finding the occurrences of b



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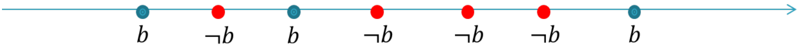
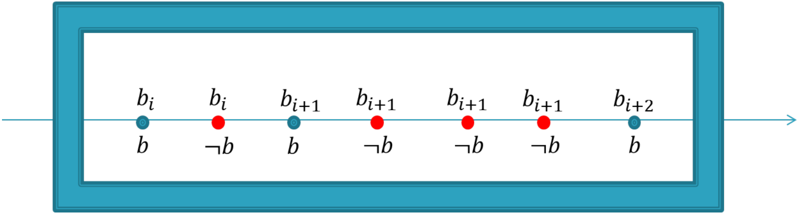


$No. of\ points\ where\ b\ holds\ = No. of\ different\ indices\ along\ with\ b$



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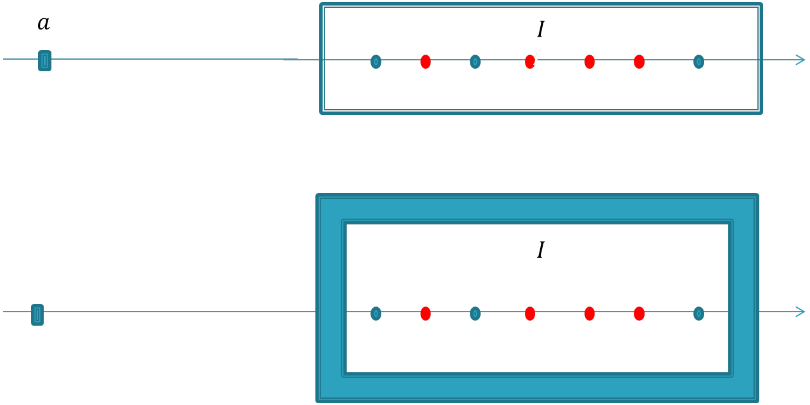

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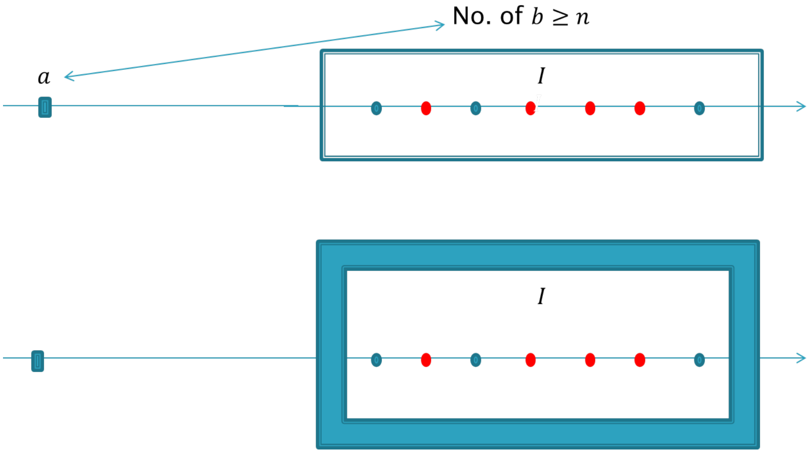

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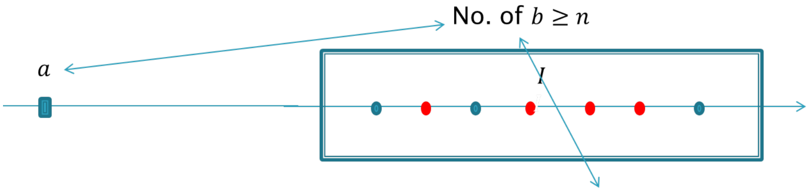
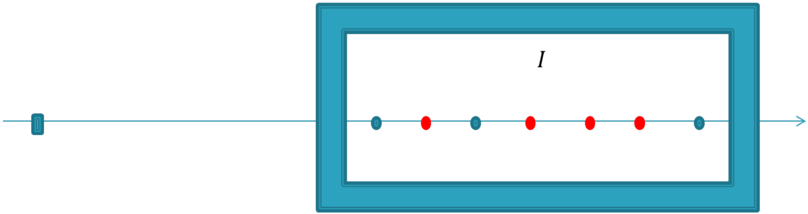


All the symbols $\{b \wedge b_0, \ldots, b \wedge b_n\}$ occur at least once



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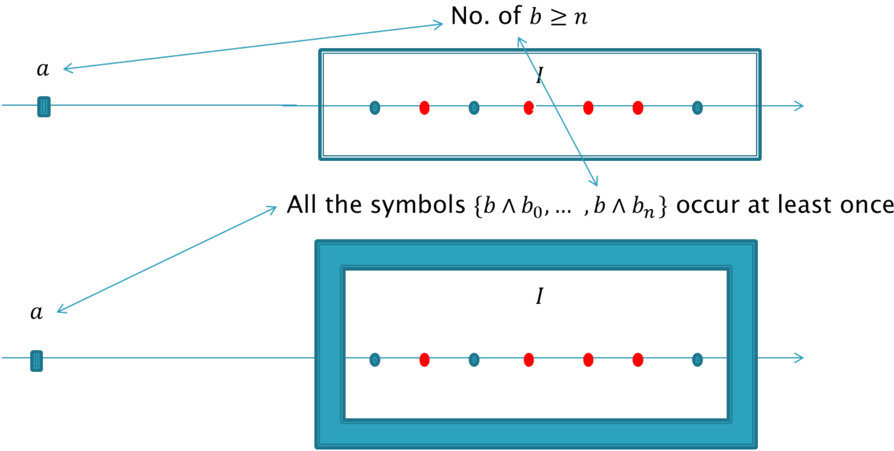

Eliminating UT modality

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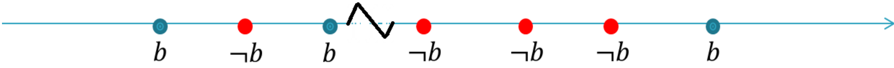
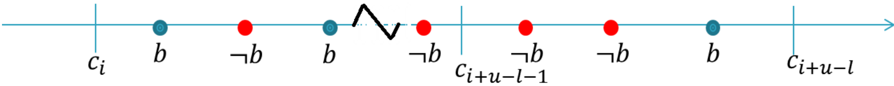
Eliminating UT modality

- Given a word ρ over Σ we construct a **oversampling** ρ' over $\Sigma \cup C \cup B$
  - $C = \{c_0, \ldots, c_u\}$:These propositions oversample the model at integer time stamps.
  - $B = \bigcup_{i=0}^{u} B^i$ where $B^i = \{b_0^i, \ldots b_n^i\}$: These propositions are used as counters for b. Counter B^i resets at integer point marked c_i and saturates once the value reaches n till the next reset.

Oversample the word





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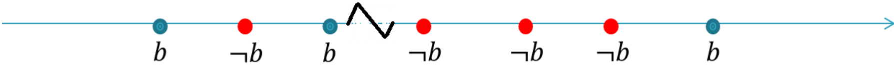
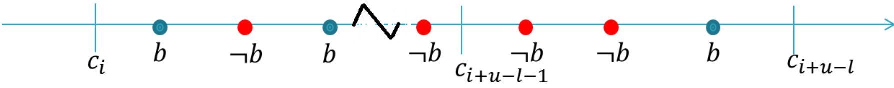
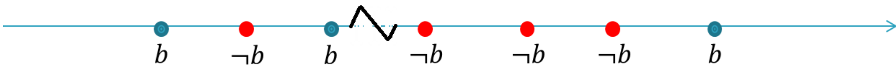
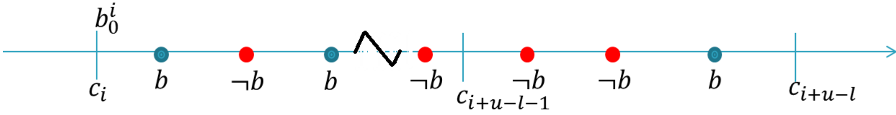




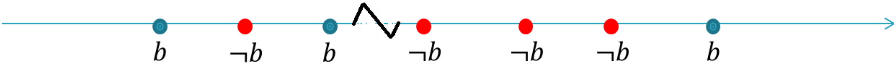
Construction of ρ'

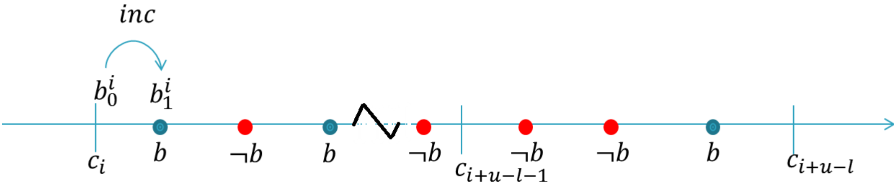
Initiate the counter





Propagate the counter





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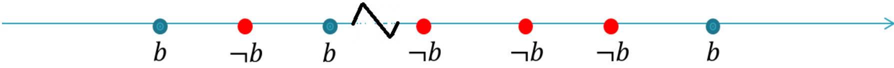

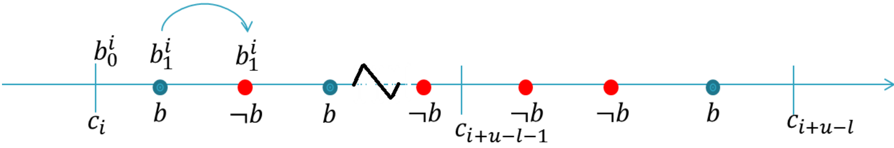

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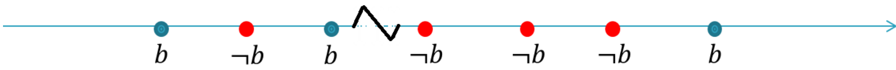

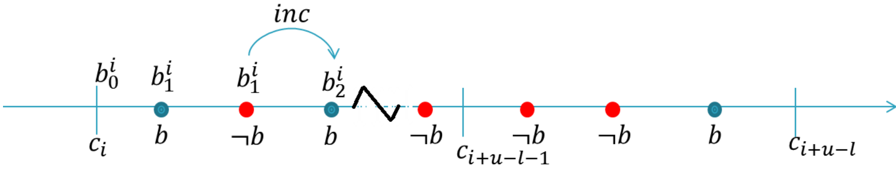

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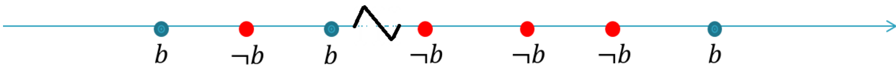
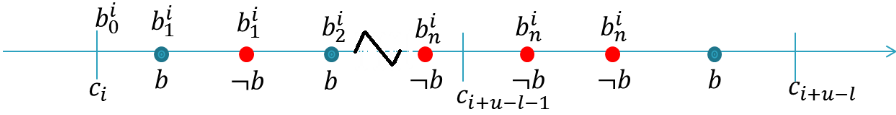
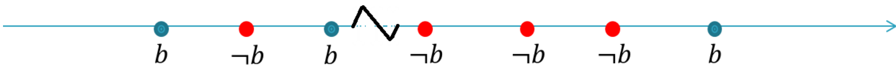
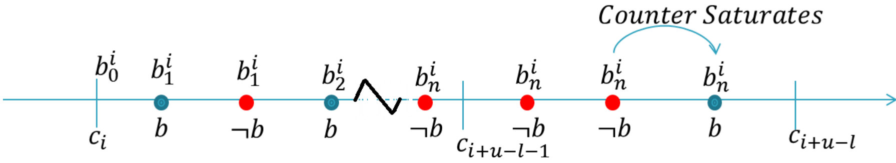




Construction of ρ'

Propagate the counter and stop incrementing after highest val





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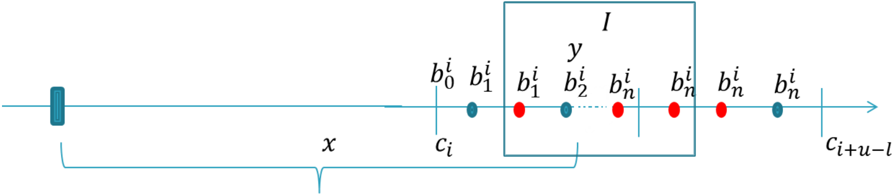
We count the number of occurrence of b in two stages:



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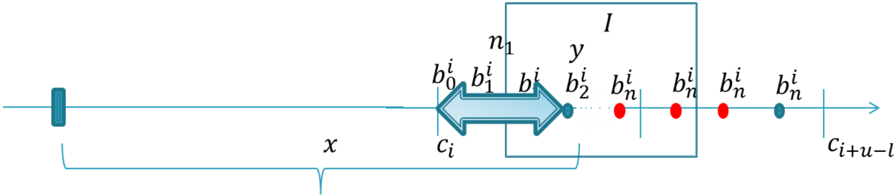

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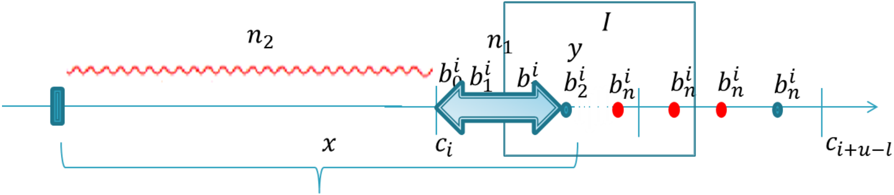

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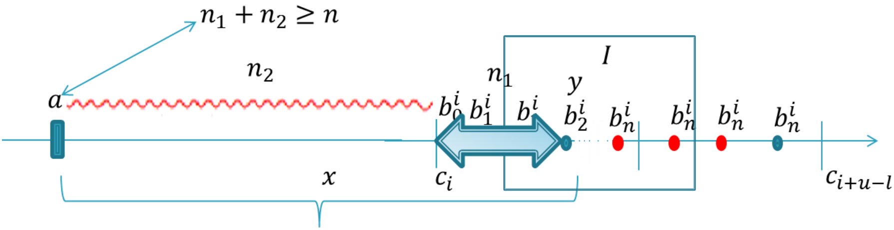

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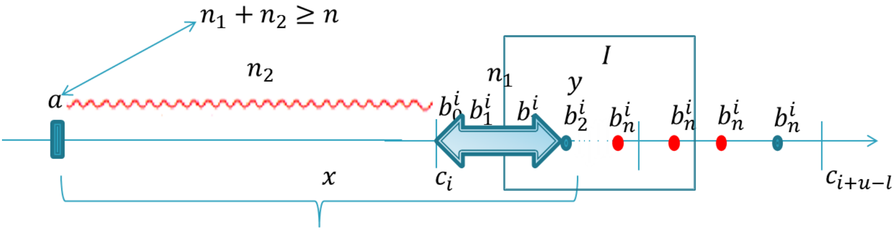
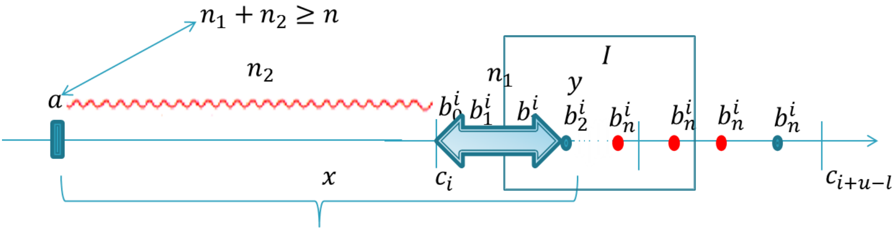


$n_1 \geq n \vee n_2 \geq n$



$n_1 \geq n \vee n_2 \geq n$ Or, $n_1 < n \wedge n_2 < n$ and thus bounded number of cases (disjunctions).

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Conclusion

- Two ways of extending MTL with counting threshold constraints is studied.
- Both ways add expressiveness to MTL orthogonally.
- Satisfiability checking for the logic CTMTL is decidable.
- Both the extensions enjoy benefits of relaxing punctuality.
- Unlike continuous semantics, pointwise semantics creates a zoo of sub-logics in the expressiveness hierarchy.

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Future Work

- Exploring complexity results for satisfiability checking of CTMTL.
- Extending logics with modulo counting and study the expressiveness and satisfiability checking for those extensions.
- Complete picture of expressiveness of these counting extensions with different versions of past operators.
- Study model checking and synthesis problems for these extensions.

Thank You

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