On Stochastic Timed Games

TCQV 2016, Mysuru

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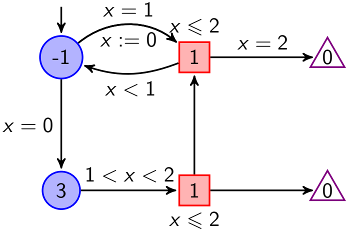
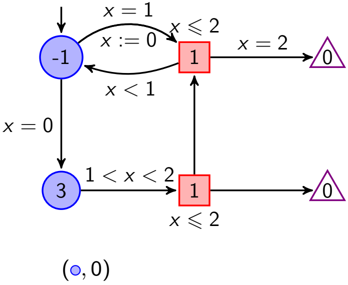
Timed Games



2-player game on TA
Controller, Environment
target locations
reachability objectives

Timed Games



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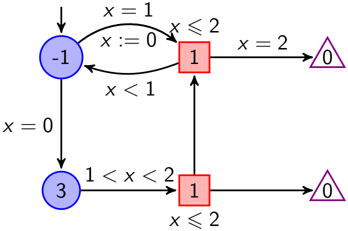

$(\circ, 0) \xrightarrow{0,\downarrow} (\circ, 0)$

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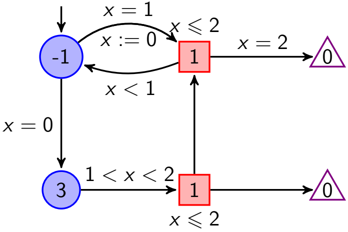

$(\circ, 0) \xrightarrow{0,\downarrow} (\circ, 0) \xrightarrow{1.6,\rightarrow} (\square, 1.6)$

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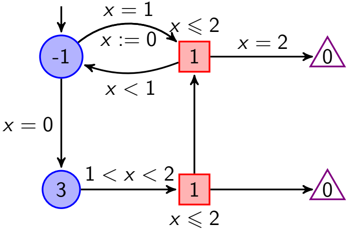

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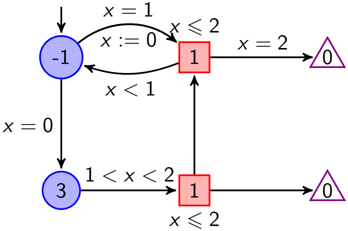
$$(\circ, 0) \xrightarrow{0,\downarrow} (\circ, 0) \xrightarrow{1.6,\rightarrow} (\square, 1.6) \xrightarrow{.4,\rightarrow} (\triangle, 2)$$

Timed Games



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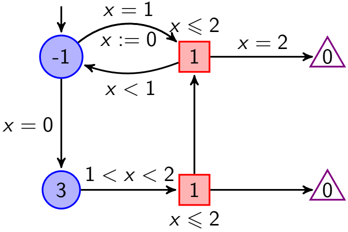
$$0 \quad + \quad 3 \times 1.6 \quad + \quad 1 \times 0.4 \quad + \quad 0 \quad = 5.2$$

Timed Games



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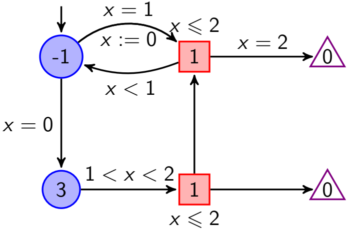
$0 \quad + \quad 3 \times 1.6 \quad + \quad 1 \times 0.4 \quad + \quad 0 \quad = 5.2$

$(\circ, 0) \xrightarrow{1,\rightarrow} (\square, 0) \xrightarrow{0.9,\leftarrow} (\circ, 0.9) \xrightarrow{0.1,\rightarrow} (\square, 0) \xrightarrow{0.9,\leftarrow} (\circ, 0.9) \quad \cdots$

$-1 \quad +0.9 \quad -0.1 \quad +0.9 \quad \cdots \quad = +\infty(\triangle \text{ not reached})$

Timed Games



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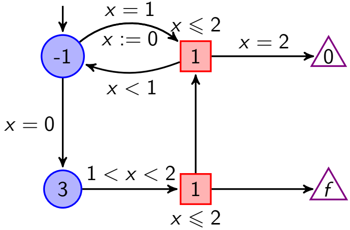
$$-1 \quad +0.9 \quad -0.1 \quad +0.9 \quad \cdots \quad = +\infty (\triangle \text{ not reached})$$

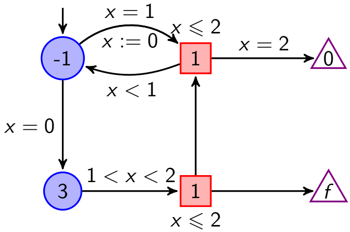
Cost of a play: $\begin{cases} +\infty & \text{if } \triangle \text{ not reached} \\ \text{accumulated cost up to } \triangle & \text{otherwise} \end{cases}$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

Strategies and objectives

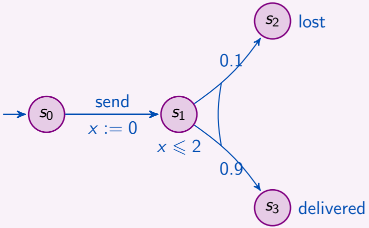


Strategy for each player: mapping of finite runs to a delay and an action

Goal of player ○: reach △ **and** minimize accumulated cost
Goal of player □: avoid △ **or, if not possible,** maximize accumulated cost

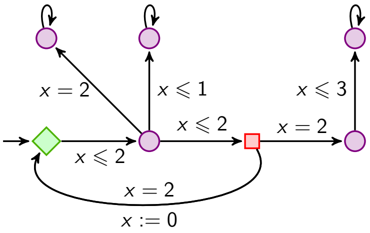
Adding stochastic features

- to model probabilistic behaviours



- Probabilistic timed automata model (PRISM, UPPAAL−PRO)

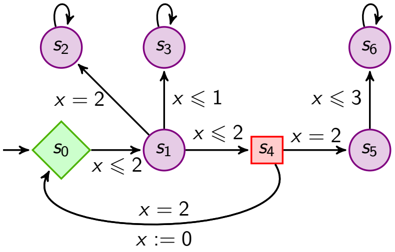
Stochastic Timed Games (STGs)



Stochastic player ○
Classical players ◇, □
Prescribed probability distributions from ○

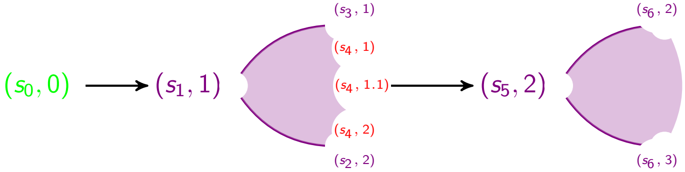
- Players ◇, □ play according to standard strategies
- Player ○ plays according to fixed probability distributions
  - choose a delay according to some distribution
  - choose an action according to some discrete distribution

A Play



◇ : goto s_1 when $x = 1$
□ : goto s_5 when $x = 2$

- From the game and strategies, we obtain a Markov chain



Attaching probabilities to delays

- The exponential distribution, as in continuous time Markov chains, with delays in $[0, \infty)$

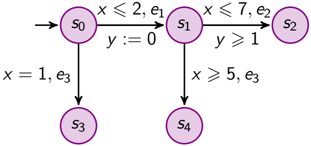
$$\text{density function t} = \begin{cases} \lambda.\exp(-\lambda t), & \text{if } t \geqslant 0, \\ 0, & \text{otherwise.} \end{cases}$$

- For bounded intervals, the uniform distribution,

$$\text{density function t} = \begin{cases} \frac{1}{|I|} & \text{if } t \geqslant 0, \\ 0, & \text{otherwise.} \end{cases}$$

Semantics of STGs

- STGs having only the stochastic player ◯ : $\frac{1}{2}$ player games.



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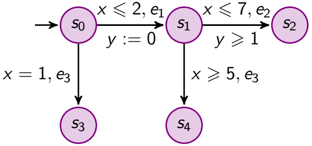
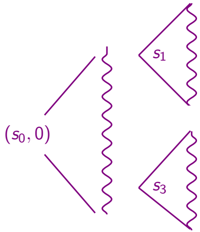


- Path $\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2})$
- $\{s_0 \xrightarrow{\tau_1, e_1} s_1 \xrightarrow{\tau_2, e_2} s_2 \mid$
  $\tau_1 \leqslant 2, \tau_1 + \tau_2 \leqslant 7, \tau_2 \geqslant 1\}$
- Compute $\mathbb{P}(\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2}))$



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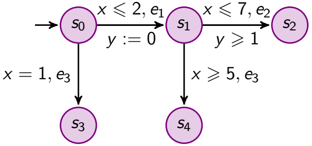

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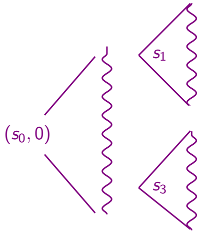
- Compute $\mathbb{P}(\pi(s_0 \xrightarrow{e_1}\xrightarrow{e_2}))$

- $\int_{t \in I(s_0, e_1)} \alpha \ \mathbb{P}(\pi(s_t \xrightarrow{e_2})) d\mu_{s_0}(t)$
- $\alpha = p_{s_0+t}(e_1)$
- α discrete distribution over transitions enabled at $s_0 + t$, given by weights on transitions



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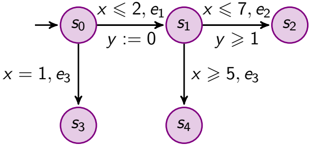

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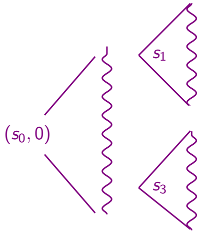


- $\int_{t \in I(s_0, e_1)} \alpha\ \mathbb{P}(\pi(s_t \xrightarrow{e_2}))d\mu_{s_0}(t)$
- $\alpha = p_{s_0+t}(e_1)$
- α discrete distribution over transitions enabled at $s_0 + t$, given by weights on transitions
- $I(s_0, e_1) = \{\tau \mid s_0 \xrightarrow{\tau, e_1}\}$
- μ_{s_0} distribution over $I(s_0)$
- $I(s_0) = \bigcup_e I(s_0, e)$
- $s_0 \xrightarrow{t} s_0 + t \xrightarrow{e_1} s_t$

The $\frac{1}{2}$ player model

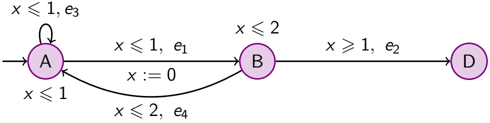
$$\mathbb{P}(\pi(s_0 \xrightarrow{e_1} \cdots \xrightarrow{e_n})) = \int_{t \in I(s_0,e_1)} p_{s_0+t}(e_1)\mathbb{P}(\pi(s_t \xrightarrow{e_2} \cdots \xrightarrow{e_n}))d\mu_{s_0} t$$

- n-dimensional integral
- For infinite runs:

$$\mathsf{Cyl}(\pi(s_0 \xrightarrow{e_1} \cdots \xrightarrow{e_n})) = \{\rho.\rho' \mid \rho \in \pi(s_0 \xrightarrow{e_1} \cdots \xrightarrow{e_n})\}$$

- $\mathbb{P}$ is extended in a standard and unique way to the σ-algebra Ω generated by the cylinders.
- For every state s, $\mathbb{P}$ is a probability measure over $(\mathsf{Runs}(s), \Omega(s))$

An Example



$$\mathcal{P}(\pi((A,0), e_1 e_2)) = \int_0^1 \frac{\mathcal{P}(\pi((B,0), e_2))}{2} d\mu_{(A,0)}(t)$$

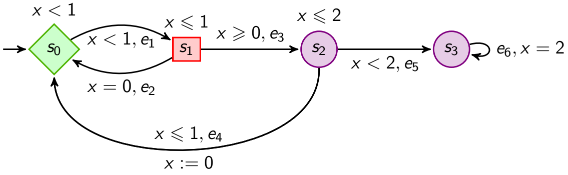
$$= \int_0^1 \frac{1}{2}(\int_1^2 \frac{1}{2} d\mu_{(B,0)}(u))\ d\mu_{(A,0)}(t)$$

$$= \frac{1}{2}\int_0^1 (\int_1^2 \frac{1}{2}\frac{1}{2} du))\ dt) = \frac{1}{8}$$

$d\mu_{(A,0)}$ uniform distribution over [0,1], $d\mu_{(B,0)}$ uniform distribution over [0,2].

$1\frac{1}{2}$ player and $2\frac{1}{2}$ player models

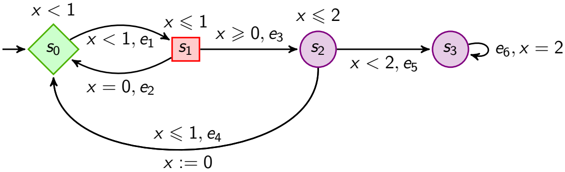
- Extend using standard strategies for other players ◇ and □



- Strategy profile $\Lambda = (\lambda_{\diamond}, \lambda_{\square})$ with $\lambda_{\square} = (0, e_3)$ and

$$\lambda_{\diamond} = \begin{cases} (0.5, e_1) \text{ if}(s_0, \nu) \text{ is such that } \nu \leqslant 0.5, \\ (0, e_1) \text{ otherwise.} \end{cases}$$

$2\frac{1}{2}$ player Example



If $\rho = (s_0, 0) \stackrel{0.5, e_1}{\to} (s_1, 0.5) \stackrel{0, e_3}{\to} (s_2, 0.5)$, then

$$\begin{aligned}
\mathbb{P}_\Lambda(\rho, e_4 e_1 e_3 e_4) &= \frac{1}{1.5}\int_{t=0}^{0.5} \frac{1}{2}\mathbb{P}_\Lambda(\rho \stackrel{t, e_4}{\to} (s_0, 0), e_1 e_3 e_4))dt \\
&= \frac{1}{1.5}\int_{t=0}^{0.5} \frac{1}{2}\mathbb{P}_\Lambda(\rho \stackrel{t, e_4}{\to} (s_0, 0) \stackrel{0.5, e_1}{\to} (s_1, 0.5), e_3 e_4))dt \\
&= \frac{1}{1.5}\int_{t=0}^{0.5} \frac{1}{2}\mathbb{P}_\Lambda(\rho \stackrel{t, e_4}{\to} (s_0, 0) \stackrel{0.5, e_1}{\to} (s_1, 0.5) \stackrel{0, e_3}{\to} (s_2, 0.5), e_4))dt \\
&= \frac{1}{1.5}\int_{t=0}^{0.5} \frac{1}{2}(\frac{1}{1.5}\int_{t'=0}^{0.5} \frac{1}{2}dt')dt = \frac{1}{36}
\end{aligned}$$

Models

- $\frac{1}{2}$ player game = pure stochastic process
- CTMC = timed automata with one clock, reset on all transitions. Exponential distributions, with a rate per location.
- PTA = subclass of $1\frac{1}{2}$ player games, where no time elapse happens in stochastic nodes. So, only discrete probabilities based on weights of outgoing edges.

Synthesis and Reachability Problems

Games

- ○ — $\frac{1}{2}$ player game
- ○, ◇ — $1\frac{1}{2}$ player game
- ○, ◇, □ — $2\frac{1}{2}$ player game

Reachability

- Qualitative (reach with probability $\bowtie r, r \in \{0, 1\}$)
- Quantitative (reach with probability $\bowtie r, r \in [0, 1]$)

Synthesis

Given a game $\mathcal{G}$, an untimed safety property φ, and a rational threshold r, does ◇ have a strategy $\lambda_{◇}$ against all possible strategies $\lambda_{□}$ of □ such that $\mathbb{P}(\mathcal{G}_{\lambda_{◇}, \lambda_{□}} \models \varphi) \bowtie r$?

The STG Landscape

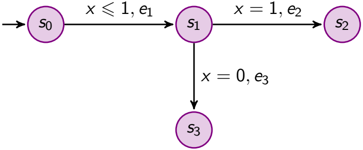
- Safety : Decidability for $\frac{1}{2}, 1\frac{1}{2}$ as well as $2\frac{1}{2}$ player games
- Reachability :

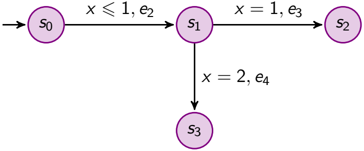
Model		Qual.Reach	Quant.Reach
$\frac{1}{2}$ player	1 clock	D [1]	D [2]
	n clocks	D[1] (reactive)	Open
$1\frac{1}{2}$ player	1 clock	D [3]	D (Initialized)
	n clocks	D (reactive)	U
$2\frac{1}{2}$ player	1 clock	Open	D (Initialized)
	n clocks	Open	U[3], U(Time bounded)

[1] [Bertrand, Bouyer, Brihaye, Menet, Baier, Größer, and Jurdzinski, 2014]
[2] Initialized, [Bertrand, Bouyer, Brihaye, and Markey, 2008]
[3] [Bouyer and Forejt, 2009]

Thick and Thin Paths

Thick and Thin:



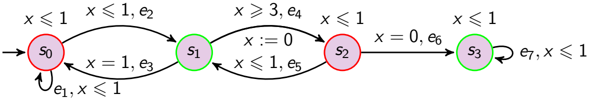


- $e_1 e_3$ thin, $e_1 e_2$ thick
- Both $e_1 e_2, e_1 e_3$ thick

Qualitative Synthesis [Bertrand, Bouyer, Brihaye, Menet, Baier, Größer, and Jurdzinski, 2014]

Given state s, $s \models \varphi$ iff $\mathbb{P}(\{\rho \in \text{Runs}(s) \mid \rho \models \varphi\}) = 1$

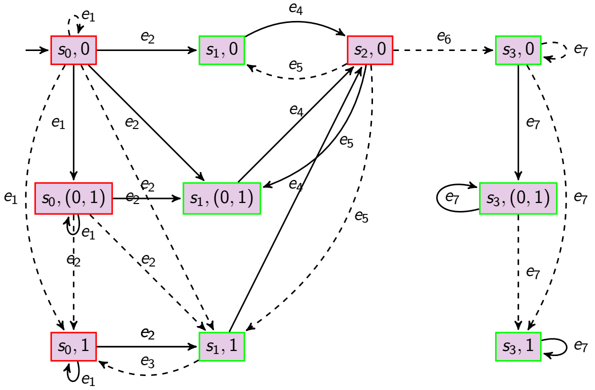
For ease of notations, we use colors in place of propositions.



$\mathcal{A} \not\models \square(\textit{green} \rightarrow \diamond(\textit{red}))$, but $\mathbb{P}(\mathcal{A} \models \square(\textit{green} \rightarrow \diamond(\textit{red}))) = 1$

- Probability of thick paths > 0, while probability of thin paths is 0.
- For safety, if all thick paths satisfy the property, that is good!

Timed Region Graph : Thick and Thin



Almost sure satisfaction : Safety

Work on the thick graph Thick($\mathcal{A}$), obtained by removing the thin edges.

Every infinite path in Thick($\mathcal{A}$) satisfies safety property φ from state $\imath(s)$ iff $\mathcal{A}, s \models_{\mathbb{P}} \varphi$.

Large Satisfaction

- Connecting probabilistic semantics and topological semantics
- Algorithmic solutions to almost sure satisfaction using the thick graph

Topology on Infinite Runs of $\mathcal{A}$

Let s be a state of $\mathcal{A}$. $\mathcal{T}_s^{\mathcal{A}}$ topology on $\mathsf{Runs}(\mathcal{A}, s)$.

- Basic open sets : $\emptyset$, $\mathsf{Runs}(\mathcal{A}, s)$, as well as $\mathsf{Cyl}(\pi)$ for thick paths π

- Meagre sets : countable union of nowhere dense ($\mathring{\overline{A}} = \emptyset$) sets
- Large : Complement is meagre
- Topological games characterizing largeness

Banach-Mazur Games

Let $(A, \mathcal{T})$ be a toplological space and $\mathcal{B}$ a family of subsets of A satisfying

- For all $B \in \mathcal{B}$, $\mathring{B} \neq \emptyset$
- For every open set O of A, $B \subseteq O$ for some $B \in \mathcal{B}$.

Game: Pick some set $C \subseteq A$.

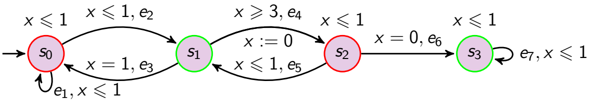
- Player 1 picks some $B_1 \in \mathcal{B}$
- Player 2 responds with some $B_2 \in \mathcal{B}, B_2 \subseteq B_1$
- Repeat

Player 1 wins iff $\bigcap_{i=1}^{\infty} B_i \cap C \neq \emptyset$. Else, player 2 wins.

[Oxtoby, 1957]

Player 2 wins a Banach-Mazur game with target set C iff C is meagre

Large Satisfaction



Is $C = \bigcup_i (e_1^i e_2 (e_4 e_5)^\omega)$ large?

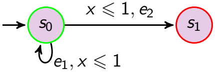
- Player 1 starts with $\mathsf{Cyl}(e_1^n)$ or $\mathsf{Cyl}(e_1^n e_2)$ for some $n \in \mathbb{N}$.
- Player 2 responds with $\mathsf{Cyl}(e_1^n e_2)$ or $\mathsf{Cyl}(e_1^n e_2 e_4)$
- Game continues picking only thick edges, and converges into a run of C and player 1 wins.
- C is hence large

Large and Almost sure : Safety Properties

- Given a safety property φ, if the set of paths satisfying φ is large, then φ is true almost surely.
- If $\neg\varphi$ is large, then there is a thick prefix violating φ
- Large and almost sure satisfaction coincide; result extends to $2\frac{1}{2}$ player case.

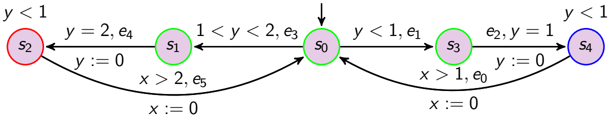
Large and Almost sure : Safety Properties

- Given a safety property φ, if the set of paths satisfying φ is large, then φ is true almost surely.
- If $\neg\varphi$ is large, then there is a thick prefix violating φ
- Large and almost sure satisfaction coincide; result extends to $2\frac{1}{2}$ player case.
- However, this does not help for general properties.



$\Diamond(\textit{red})$ is violated by a thick path, but it is true almost surely!

Qualitative Reachability : $\frac{1}{2}$ player



- $\varphi = \Diamond$ blue is satisfied by every fair and thick path.

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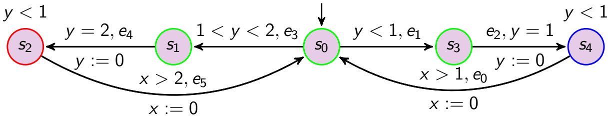
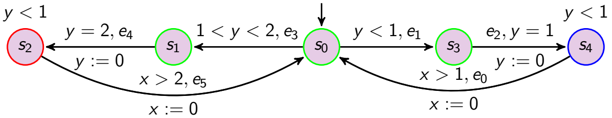


- $\varphi = \Diamond$ blue is satisfied by every fair and thick path.
- However, $\mathbb{P}_{\mathcal{A}}(\pi(s_0, (e_3 e_4 e_5)^\omega)) > 0$. Hence, $\mathbb{P}_{\mathcal{A}}(s_0 \models \text{fair}) < 1$.
- If the underlying system is such that $\mathbb{P}_{\mathcal{A}}(s_0 \models \text{fair}) = 1$, then checking all BSCCs in Thick($\mathcal{A}$) suffice!

Qualitative Reachability : $\frac{1}{2}$ player

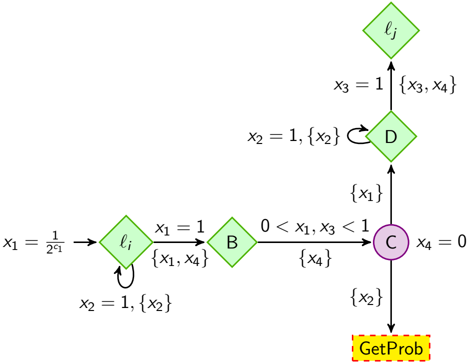


- $\varphi = \Diamond$ blue is satisfied by every fair and thick path.
- However, $\mathbb{P}_{\mathcal{A}}(\pi(s_0, (e_3 e_4 e_5)^\omega)) > 0$. Hence, $\mathbb{P}_{\mathcal{A}}(s_0 \models \text{fair}) < 1$.
- If the underlying system is such that $\mathbb{P}_{\mathcal{A}}(s_0 \models \text{fair}) = 1$, then checking all BSCCs in Thick($\mathcal{A}$) suffice!
- 1-clock $\frac{1}{2}$ player automata has this nice property.
- Result extends to $1\frac{1}{2}$ player case, but $2\frac{1}{2}$ case is open!

Quant. Reachability : $1\frac{1}{2}$ player (> 1 cl.)

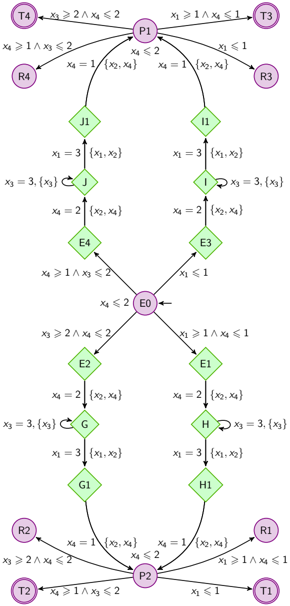
- Suprisingly undecidable with 4 clocks and uniform distributions
- Non-halting of 2-counter machine iff reachability with probability $= \frac{1}{2}$. Extends to all objectives $\bowtie \frac{1}{2}$.
- ◊ simulates a computation of the two-counter machine, encodes counter values in clocks $x_1 = \frac{1}{2^{c_1}}, x_2 = \frac{1}{2^{c_2}}$
- ○ checks for cheating, using probabilities

Increment c_1



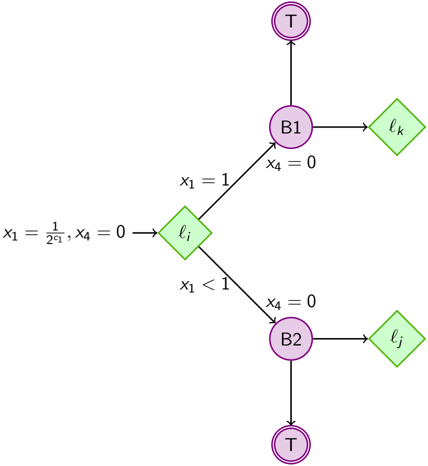
- Start with $x_1 = \frac{1}{2^{c_1}}, x_2 = \frac{1}{2^{c_2}}$
- $x_3 = x_4 = 0$
- Time $0 < t < 1$ spent at B
- $x_1 = t$ at C
- $x_1 = \frac{1}{2^{c_1}} - t$ at ℓ_j
- Is $t = \frac{1}{2^{c_1+1}}$?
- Check done by GetProb
- Probability to continue=Probability to check=$\frac{1}{2}$

The Check widget : GetProb



- At E_0, $x_1 = \frac{1}{2^{c_1+1}} \pm \epsilon$
- $\mathbb{P}_{E_0}(\Diamond(P1)) = \frac{1}{2}(1 - 2\epsilon)$
- $\mathbb{P}_{E_0}(\Diamond(P2)) = \frac{1}{2}(1 + 2\epsilon)$
- $\mathbb{P}_{P1}(\Diamond(T3 \text{ or } T4)) = \frac{1}{2}(1 + 2\epsilon)$
- $\mathbb{P}_{P2}(\Diamond(T1 \text{ or } T2)) = \frac{1}{2}(1 - 2\epsilon)$
- $\mathbb{P}_{E_0}(\Diamond(\text{Target})) = \frac{1}{2}(1 - 4\epsilon^2) \leqslant \frac{1}{2}$

Zero Test : c_1



- Probability to reach target=Probability to continue=$\frac{1}{2}$

Putting things together

Player ♢ has a strategy to reach the (set of) target locations in $\mathcal{G}$ with probability $\frac{1}{2}$ iff the two counter machine does not halt.

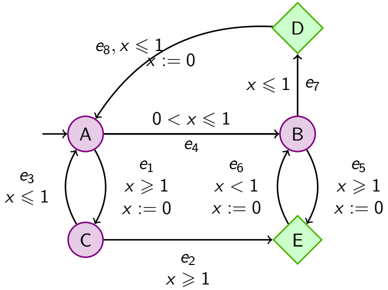
- If the machine does not halt, $\mathbb{P}(T)=\frac{1}{2}.\eta_1 + (\frac{1}{2})^2.\eta_2 + (\frac{1}{2})^3.\eta_3 + \cdots + (\frac{1}{2})^k.\eta_k + \ldots$
- $\eta_j = \frac{1}{2}(1 - 4\epsilon_j^2) \leqslant \frac{1}{2}$
- $\epsilon_j = 0$ iff ♢ faithful

Quantitative Reachability: $1\frac{1}{2}$ player, 1 clock

- Initialized 1-clock, $1\frac{1}{2}$ player STG $\mathcal{G}$, with $I(s) = \mathbb{R}^+$, any bounded cycle has a reset, exponential distribution at all locations. Reachability to some $\diamondsuit$ node.

Quantitative Reachability: $1\frac{1}{2}$ player, 1 clock

- Initialized 1-clock, $1\frac{1}{2}$ player STG $\mathcal{G}$, with $I(s) = \mathbb{R}^+$, any bounded cycle has a reset, exponential distribution at all locations. Reachability to some ◇ node.



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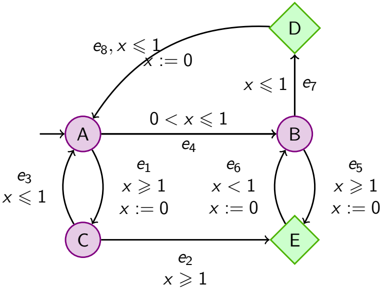
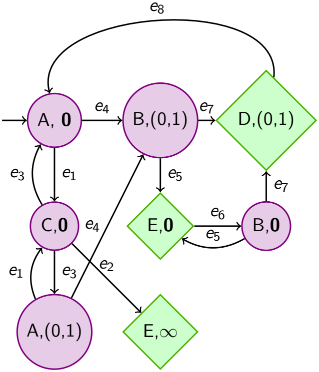
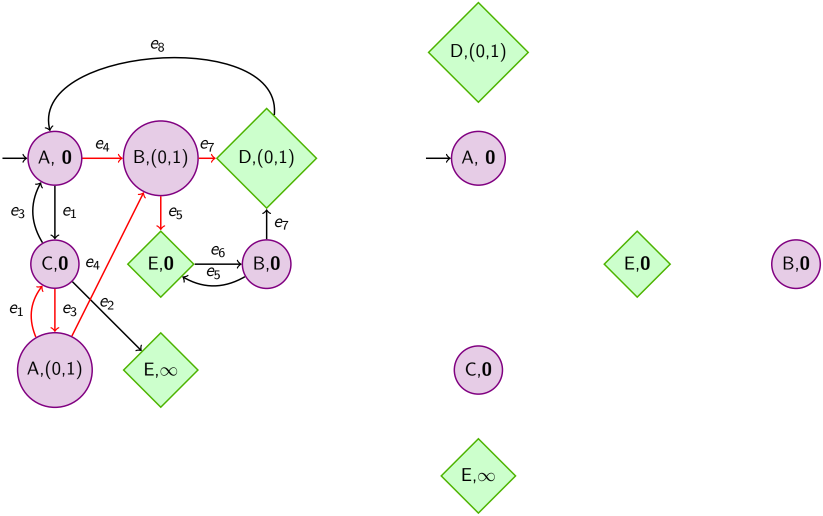
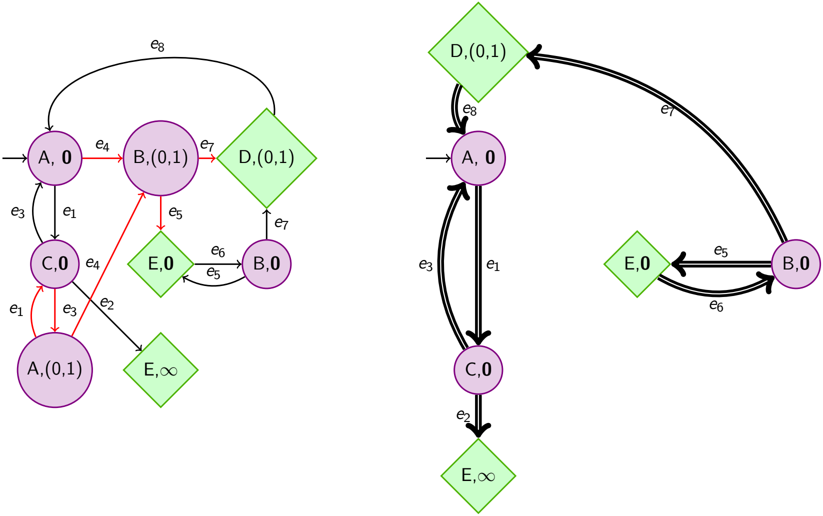




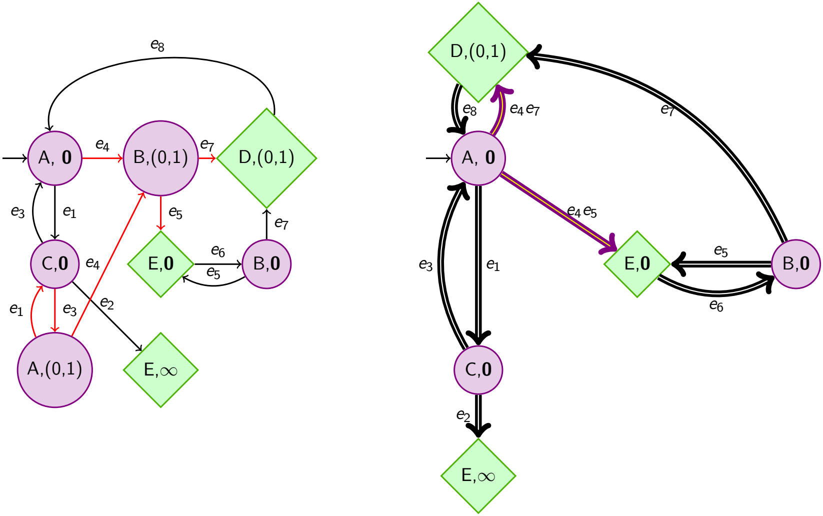
Quantitative Reachability : $1\frac{1}{2}$ player, 1-clock



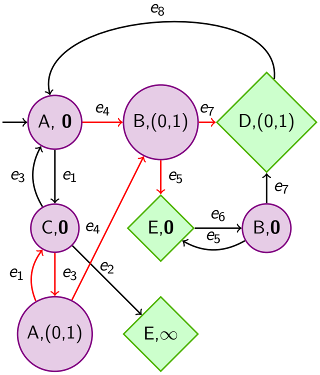
Quantitative Reachability : $1\frac{1}{2}$ player, 1-clock

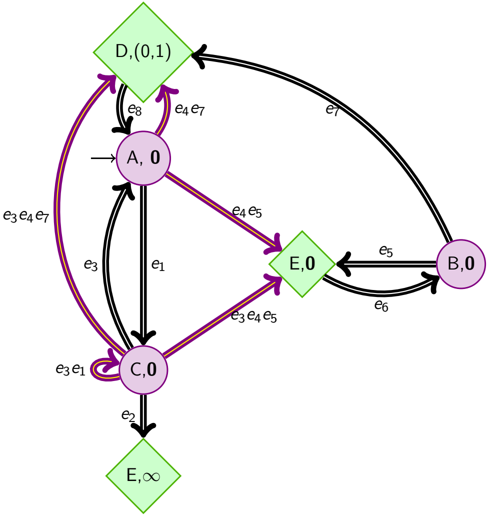


Quantitative Reachability : $1\frac{1}{2}$ player, 1-clock



Quantitative Reachability : $1\frac{1}{2}$ player, 1-clock





Time-bounded Quantitative Reachability

$\mathbb{P}_\sigma(\{\rho \in Run(\mathcal{G}, s_0, \sigma) \mid \rho \text{ visits } T \text{ within } \Delta \text{ time units})\} \bowtie p$

The timed-bounded quantitative reachability problem is undecidable for $2\frac{1}{2}$ STGs with $\geqslant 7$ clocks.

- Simulation of instruction k takes time $\frac{1}{2^k}$
- Reachability with probability $\bowtie \frac{1}{2}$ iff the two counter machine halts, and ◊ simulates faithfully
- □ checks correctness of simulation using the power of probabilities

Many Open Questions....

Model		Qual.Reach	Quant.Reach
$\frac{1}{2}$ player	1 clock	D	D
	n clocks	D(reactive)	Open
$1\frac{1}{2}$ player	1 clock	D	D (Initialized)
	n clocks	D (reactive)	U
$2\frac{1}{2}$ player	1 clock	Open	D (Initialized)
	n clocks	Open	U, U(Time bounded)

Thankyou

References I

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