CS344: Introduction to Artificial Intelligence

(associated lab: CS386)

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Lecture–3: Fuzzy Inferencing: Inverted Pendulum

Inferencing

- Two methods of inferencing in classical logic
  - Modus Ponens
    - Given p and $p \rightarrow q$, infer q
  - Modus Tolens
    - Given $\sim q$ and $p \rightarrow q$, infer $\sim p$
- How is fuzzy inferencing done?

A look at reasoning

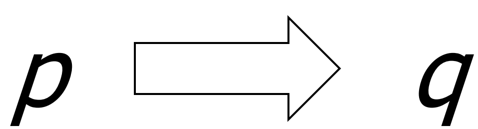
- Deduction: *p, p→q |- q*
- Induction: $p_1, p_2, p_3, \ldots$ *|- for_all p*
- Abduction: *q, p→q |- p*
- Default reasoning: Non-monotonic reasoning: Negation by failure
  - If something cannot be proven, its negation is asserted to be true
  - E.g., in Prolog

Completeness and Soundness

- Completeness question
  - Provability - Is the machine powerful enough to establish a fact?
- Soundness – Anything that is proved to be true is indeed true
  - Truth - Is the fact true?

Fuzzy Modus Ponens in terms of truth values

- Given $t(p)=1$ and $t(p \rightarrow q)=1$, infer $t(q)=1$
- In fuzzy logic,
  - given $t(p)>=a$, $0<=a<=1$
  - *and* $t(p \rightarrow >q)=c$, $0<=c<=1$
  - What is $t(q)$
- How much of truth is transferred over the channel



Lukasiewitz formula for Fuzzy Implication

- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- t($P \rightarrow Q$) = min[1,1 -t(P)+t(Q)]

$\underbrace{\hspace{12cm}}$

Lukasiewitz definition of implication

Use Lukasiewitz definition

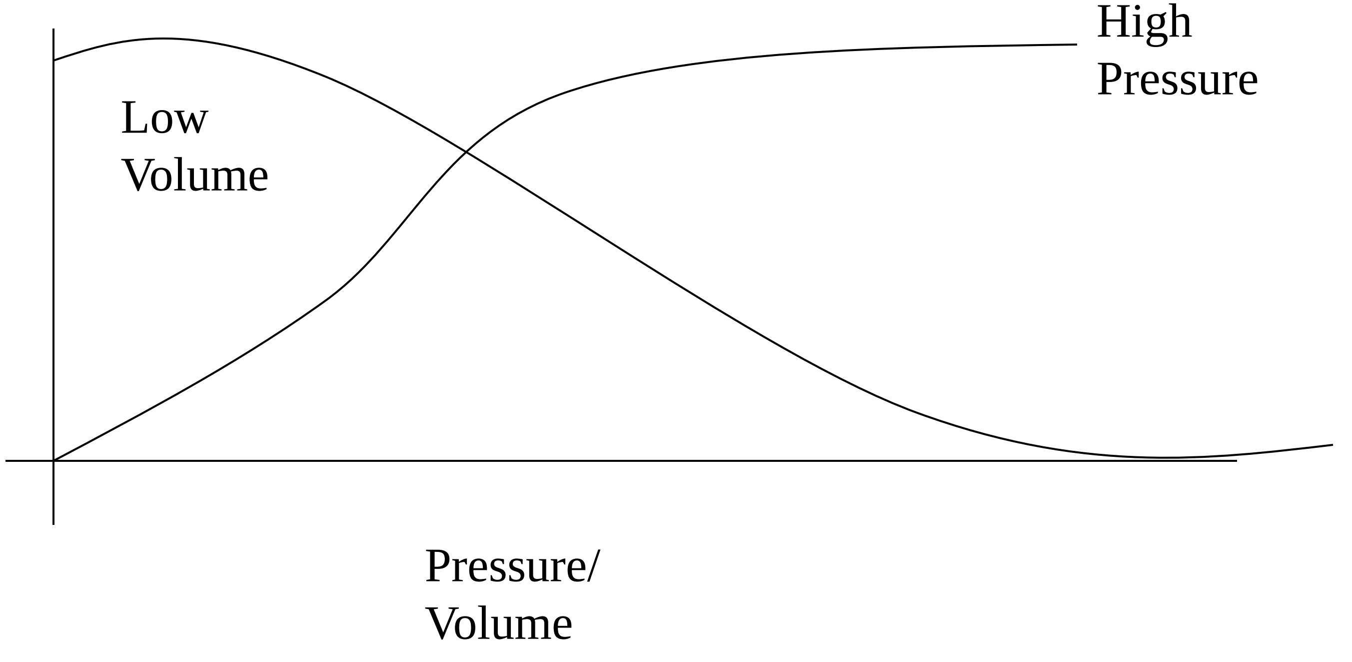
- $t(p \rightarrow q) = min[1,1 - t(p)+t(q)]$
- We have $t(p\text{-}>q)=c$, *i.e.*, $min[1,1 - t(p)+t(q)]=c$
- Case 1:
- $c=1$ gives $1 - t(p)+t(q)>=1$, *i.e.*, $t(q)>=a$
- Otherwise, $1 - t(p)+t(q)=c$, *i.e.*, $t(q)>=c+a-1$
- Combining, $t(q)=max(0,a+c-1)$
- This is the amount of truth transferred over the channel $p \rightarrow q$

ANDING of Clauses on the LHS of implication

$$t(P \wedge Q) = \min(t(P), t(Q))$$

Eg: If pressure is high then Volume is low

$$t(high(pressure) \rightarrow low(volume))$$



Fuzzy Inferencing

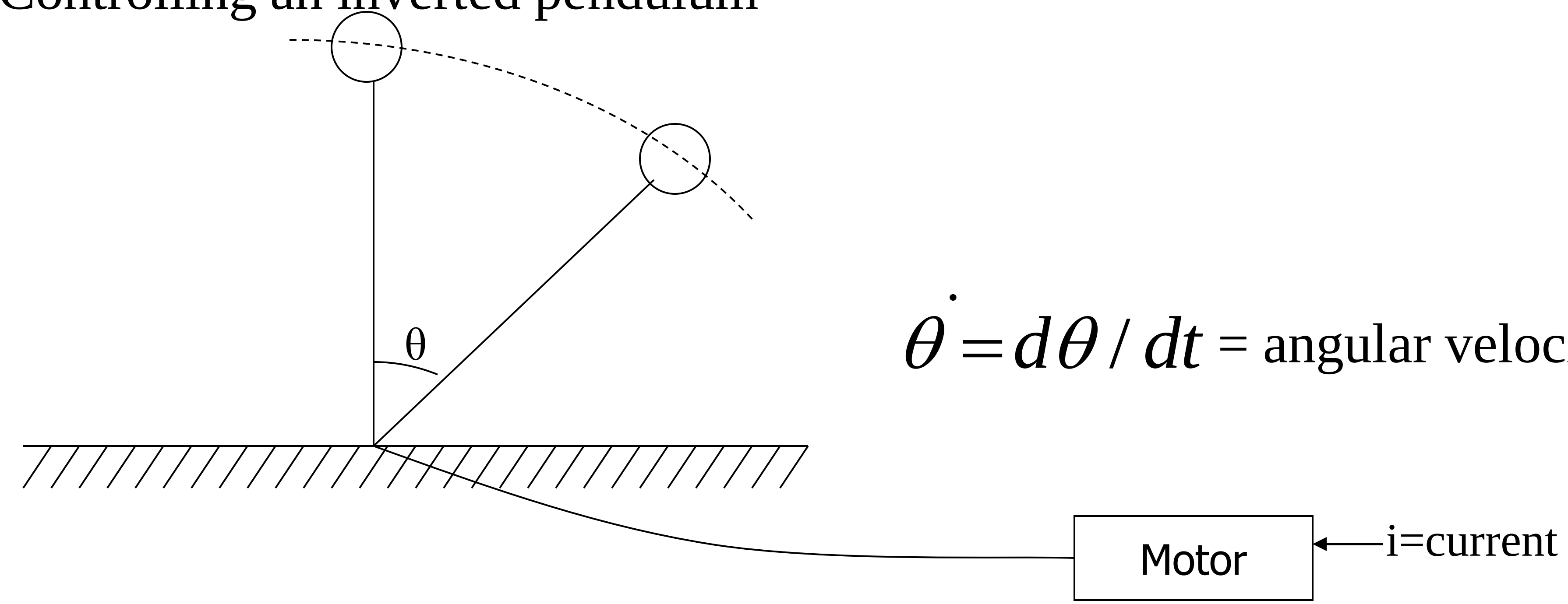
Core

The Lukasiewitz rule

$$t(P \rightarrow Q) = \min[1, 1 + t(P) - t(Q)]$$

An example

Controlling an inverted pendulum



$$\dot{\theta} = d\theta / dt \text{ = angular velocity}$$

The goal: To keep the pendulum in vertical position (θ=0) in dynamic equilibrium. Whenever the pendulum departs from vertical, a torque is produced by sending a current ‘i’

Controlling factors for appropriate current

Angle θ, Angular velocity $\dot{\theta}$

Some intuitive rules

If θ is +ve small and $\dot{\theta}$ is –ve small

then current is zero

If θ is +ve small and $\dot{\theta}$ is +ve small

then current is –ve medium

Control Matrix

$\dot{\theta}$ \ θ	-ve med	-ve small	Zero	+ve small	+ve med
-ve med					
-ve small		+ve med	+ve small	Zero	
Zero		+ve small	Zero	-ve small	
+ve small		Zero	-ve small	-ve med	
+ve med					

Region of interest

Each cell is a rule of the form

If θ is <> and $\dot{\theta}$ is <>

then i is <>

<u>4 “Centre rules”</u>

1. if θ = = Zero and $\dot{\theta}$ = = Zero then i = Zero

2. if θ is +ve small and $\dot{\theta}$ = = Zero then i is –ve small

3. if θ is –ve small and $\dot{\theta}$ = = Zero then i is +ve small

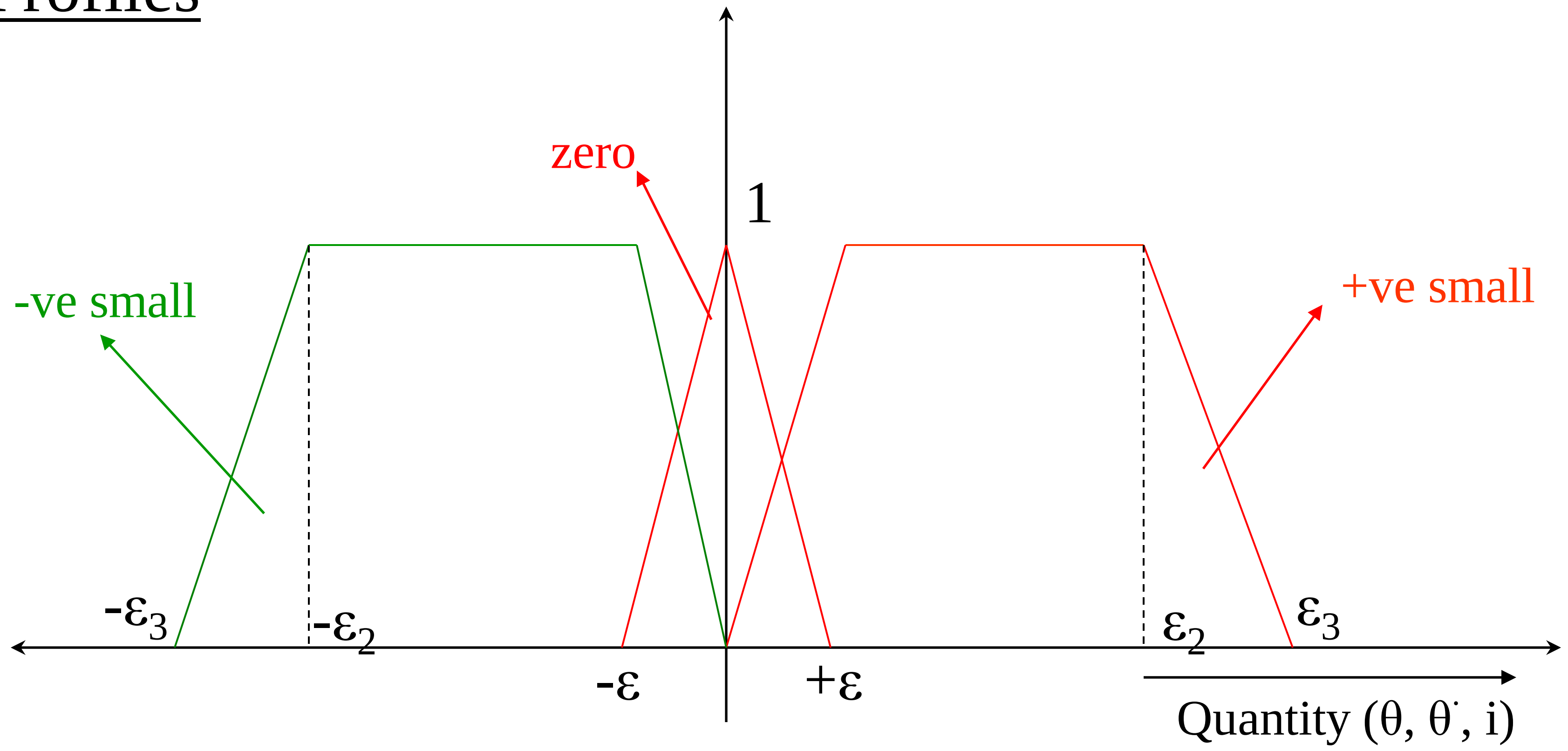
4. if θ = = Zero and $\dot{\theta}$ is +ve small then i is –ve small

5. if θ = = Zero and $\dot{\theta}$ is –ve small then i is +ve small

Linguistic variables

1. Zero
2. +ve small
3. -ve small

Profiles

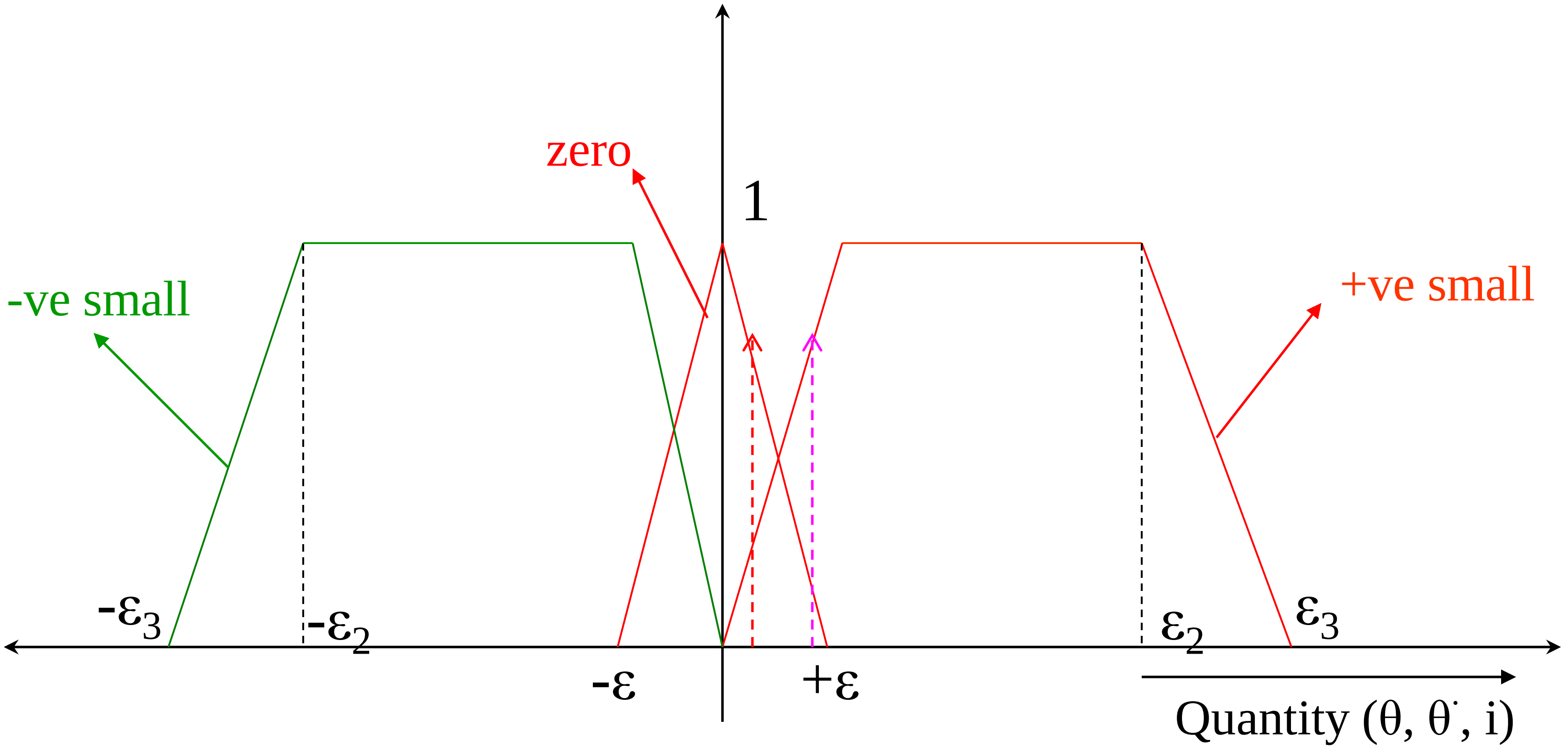


Inference procedure

1. Read actual numerical values of θ and $\dot{\theta}$
2. Get the corresponding μ values μ_{Zero}, $\mu_{(+ve\ small)}$, $\mu_{(-ve\ small)}$. This is called FUZZIFICATION
3. For different rules, get the fuzzy *i* values from the R.H.S of the rules.
4. "Collate" by some method and get ONE current value. This is called DEFUZZIFICATION
5. Result is one numerical value of *i*.

Rules Involved

if θ is Zero and dθ/dt is Zero then i is Zero
if θ is Zero and dθ/dt is +ve small then i is –ve small
if θ is +ve small and dθ/dt is Zero then i is –ve small
if θ +ve small and dθ/dt is +ve small then i is -ve medium



Fuzzification

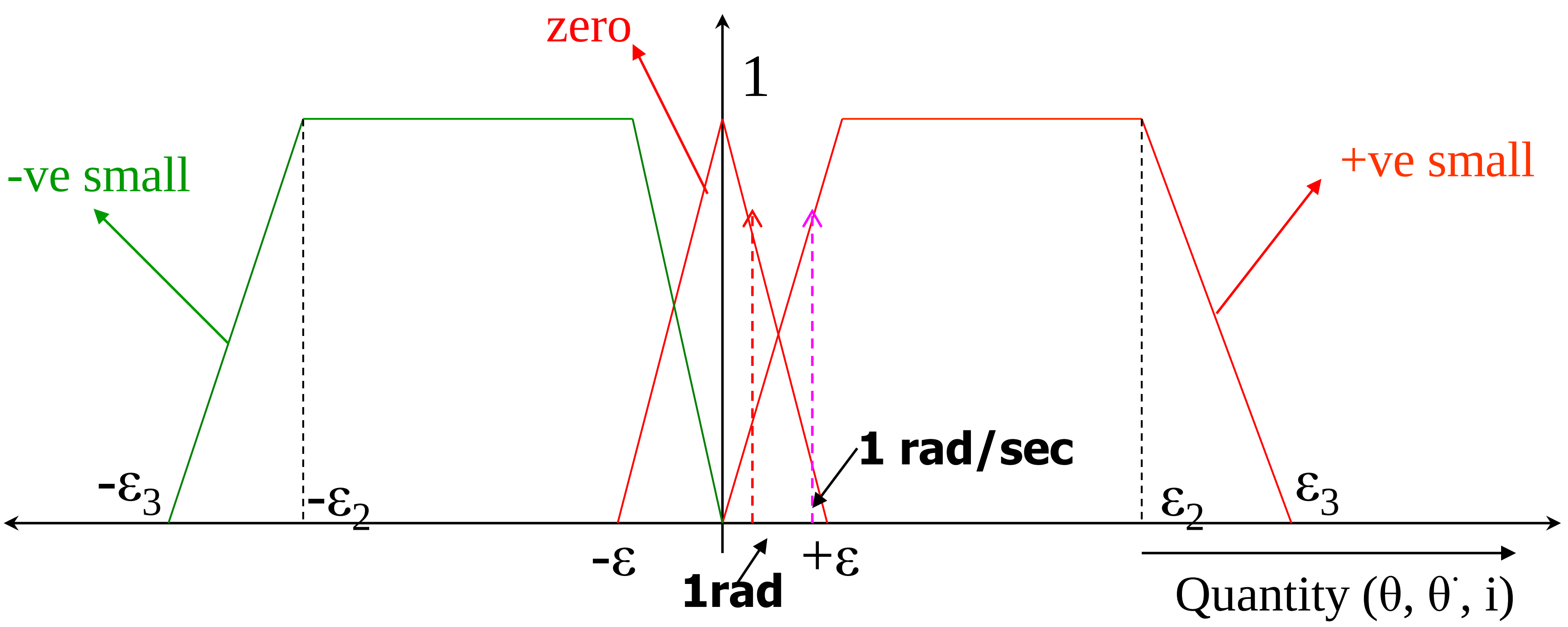
Suppose θ is 1 radian and dθ/dt is 1 rad/sec

$\mu_{zero}(\theta = 1) = 0.8$ (say)

$\mu_{+ve\text{-}small}(\theta = 1) = 0.4$ (say)

$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)

$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)



Fuzzification

Suppose θ is 1 radian and dθ/dt is 1 rad/sec
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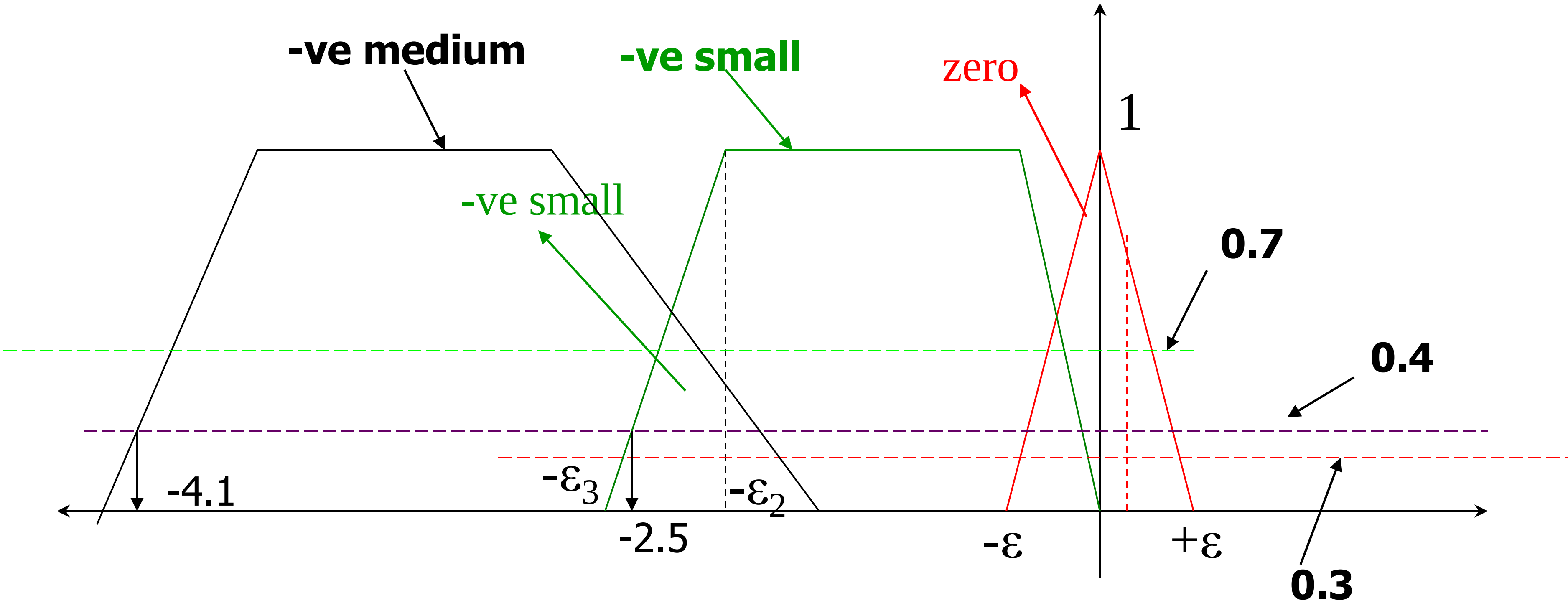
$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)
$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)

if θ is Zero and dθ/dt is Zero then i is Zero
min(0.8, 0.3)=0.3
hence $\mu_{zero}(i)=0.3$
if θ is Zero and dθ/dt is +ve small then i is –ve small
min(0.8, 0.7)=0.7
hence $\mu_{-ve\text{-}small}(i)=0.7$
if θ is +ve small and dθ/dt is Zero then i is –ve small
min(0.4, 0.3)=0.3
hence *μ-ve-small(i)=0.3*
if θ +ve small and dθ/dt is +ve small then i is -ve medium
min(0.4, 0.7)=0.4
hence $\mu_{-ve\text{-}medium}(i)=0.4$

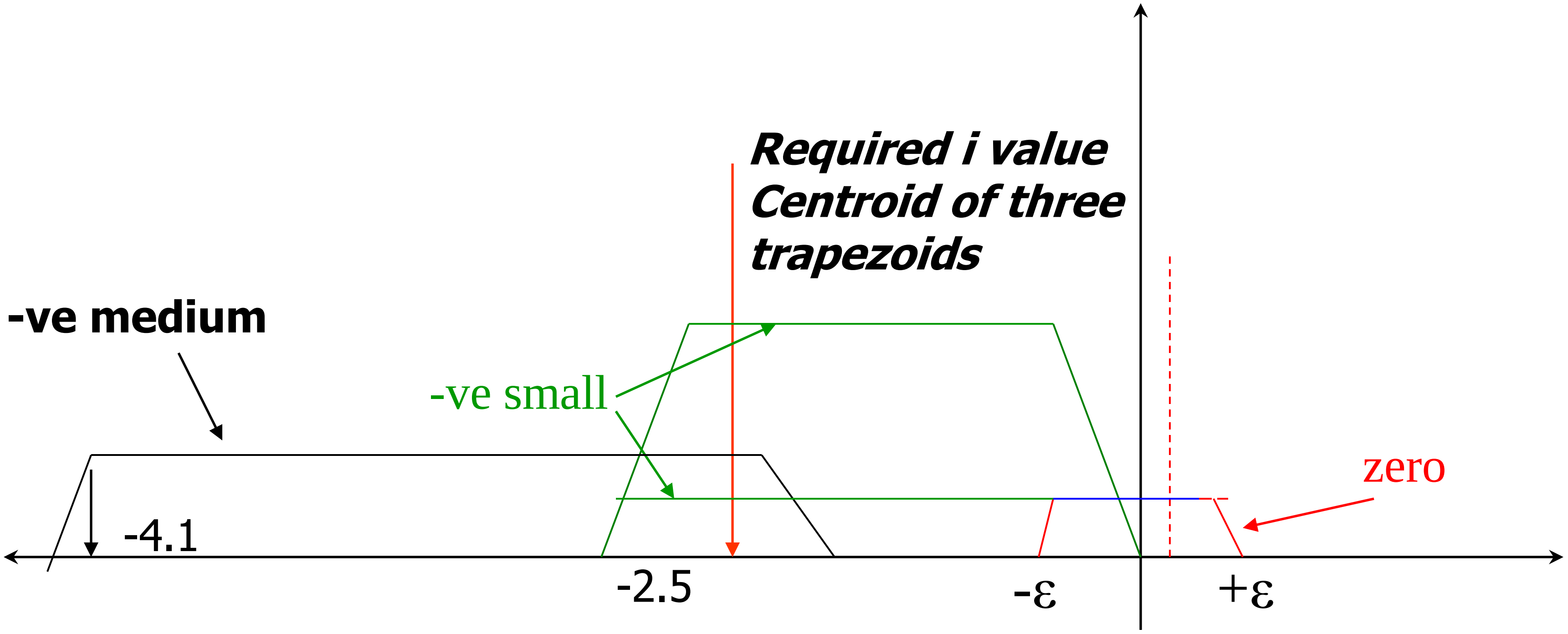
Finding *i*



Possible candidates:

i=0.5 and -0.5 from the "zero" profile and μ=0.3
i=-0.1 and -2.5 from the "-ve-small" profile and μ=0.3
i=-1.7 and -4.1 from the "-ve-small" profile and μ=0.3

Defuzzification: Finding *i* by the *centroid* method



Possible candidates:

i is the x-coord of the centroid of the areas given by the blue trapezium, the green trapeziums and the black trapezium