CS344: Introduction to Artificial Intelligence

(associated lab: CS386)

Pushpak Bhattacharyya
CSE Dept.,
IIT Bombay

Lecture–4: Fuzzy Control of Inverted Pendulum + Propositional Calculus based puzzles

Lukasiewitz formula for Fuzzy Implication

- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- t($P \to Q$) = min[1,1 -t(P)+t(Q)]

Lukasiewitz definition of implication

Use Lukasiewitz definition

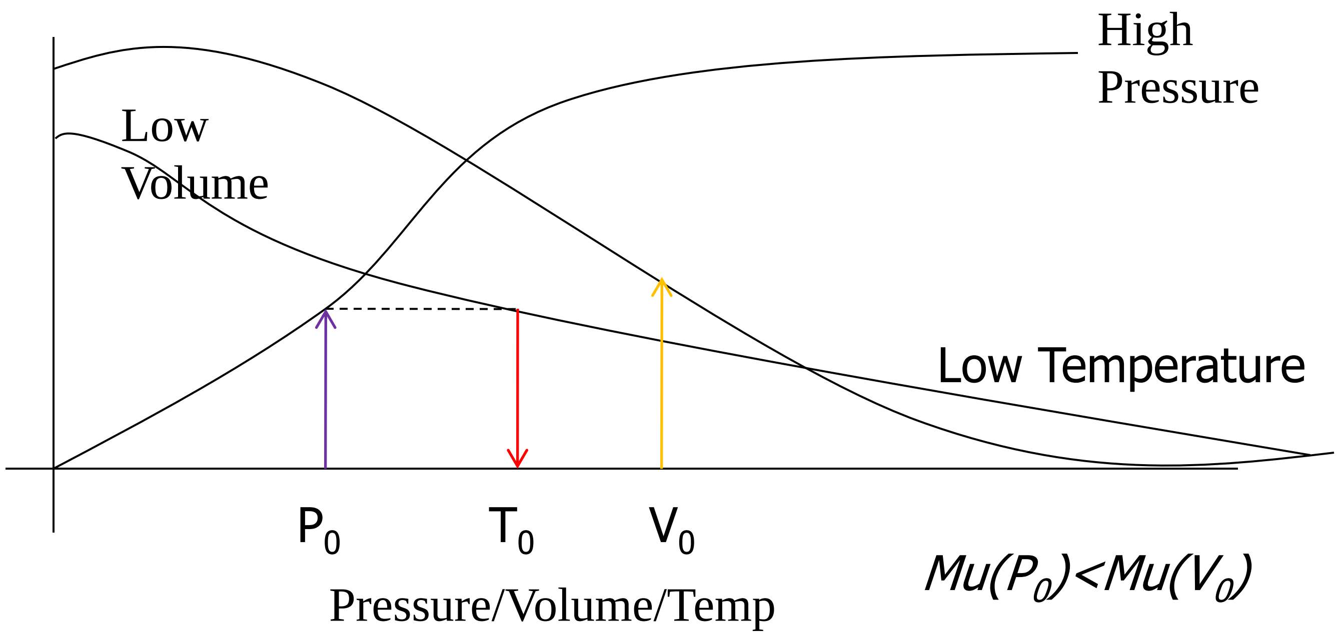
- $t(p \rightarrow q) = min[1, 1 - t(p) + t(q)]$
- We have $t(p\text{->}q)=c$, *i.e.*, $min[1, 1 - t(p) + t(q)] = c$
- Case 1:
- $c=1$ gives $1 - t(p) + t(q) >= 1$, *i.e.*, $t(q) >= a$
- Otherwise, $1 - t(p) + t(q) = c$, *i.e.*, $t(q) >= c + a - 1$
- Combining, $t(q) = max(0, a + c - 1)$
- This is the amount of truth transferred over the channel $p \rightarrow q$

Fuzzification and Defuzzification

ANDING of Clauses on the LHS of implication

$$t(P \wedge Q) = \min(t(P), t(Q))$$

Eg: If Pressure is high AND Volume is low then make Temperature Low



$Mu(P_0) < Mu(V_0)$

Hence *$Mu(T_0) = Mu(P_0)$*

Fuzzy Inferencing

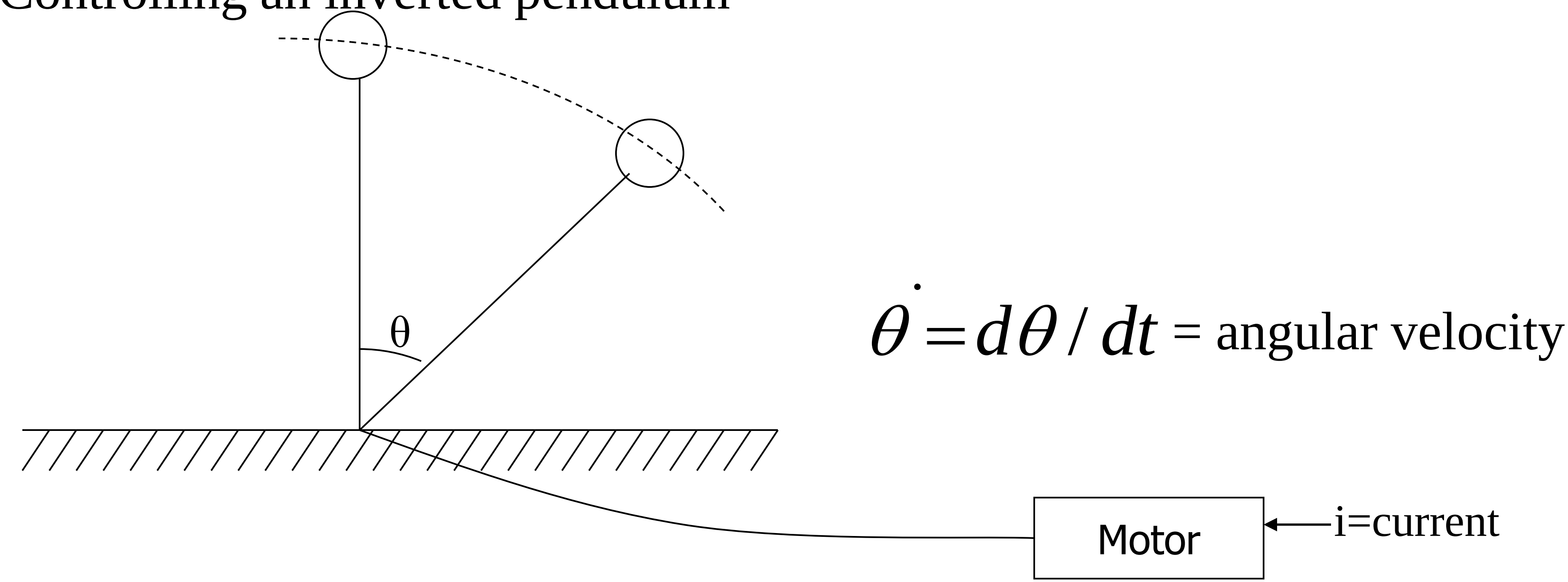
Core

The Lukasiewitz rule

$$t(P \rightarrow Q) = \min[1, 1 + t(P) - t(Q)]$$

An example

Controlling an inverted pendulum



$$\dot{\theta} = d\theta / dt = \text{angular velocity}$$

The goal: To keep the pendulum in vertical position ($\theta=0$) in dynamic equilibrium. Whenever the pendulum departs from vertical, a torque is produced by sending a current ‘i’

Controlling factors for appropriate current

Angle θ, Angular velocity $\dot{\theta}$

<u>Some intuitive rules</u>

If θ is +ve small and $\dot{\theta}$ is –ve small

then current is zero

If θ is +ve small and $\dot{\theta}$ is +ve small

then current is –ve medium

Control Matrix

$\dot{\theta}$ \ θ	-ve med	-ve small	Zero	+ve small	+ve med	
-ve med						
-ve small		+ve med	+ve small	Zero		
Zero		+ve small	Zero	-ve small		
+ve small		Zero	-ve small	-ve med		
+ve med						

Region of interest

Each cell is a rule of the form

If θ is <> and $\dot{\theta}$ is <>

then i is <>

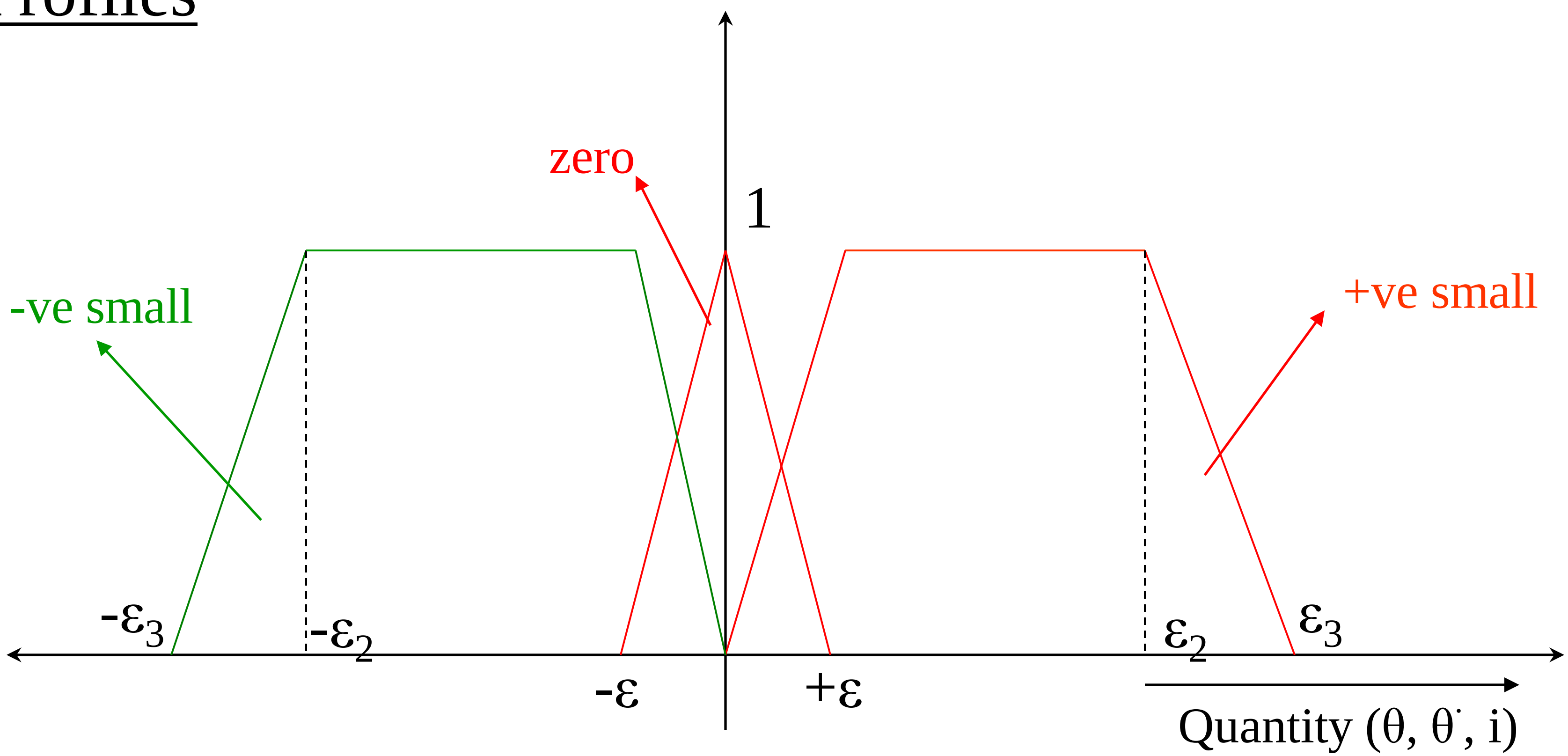
<u>4 “Centre rules”</u>

1. if θ = = Zero and $\dot{\theta}$ = = Zero then i = Zero
2. if θ is +ve small and $\dot{\theta}$ = = Zero then i is –ve small
3. if θ is –ve small and $\dot{\theta}$ = = Zero then i is +ve small
4. if θ = = Zero and $\dot{\theta}$ is +ve small then i is –ve small
5. if θ = = Zero and $\dot{\theta}$ is –ve small then i is +ve small

Linguistic variables

1. Zero
2. +ve small
3. -ve small

Profiles

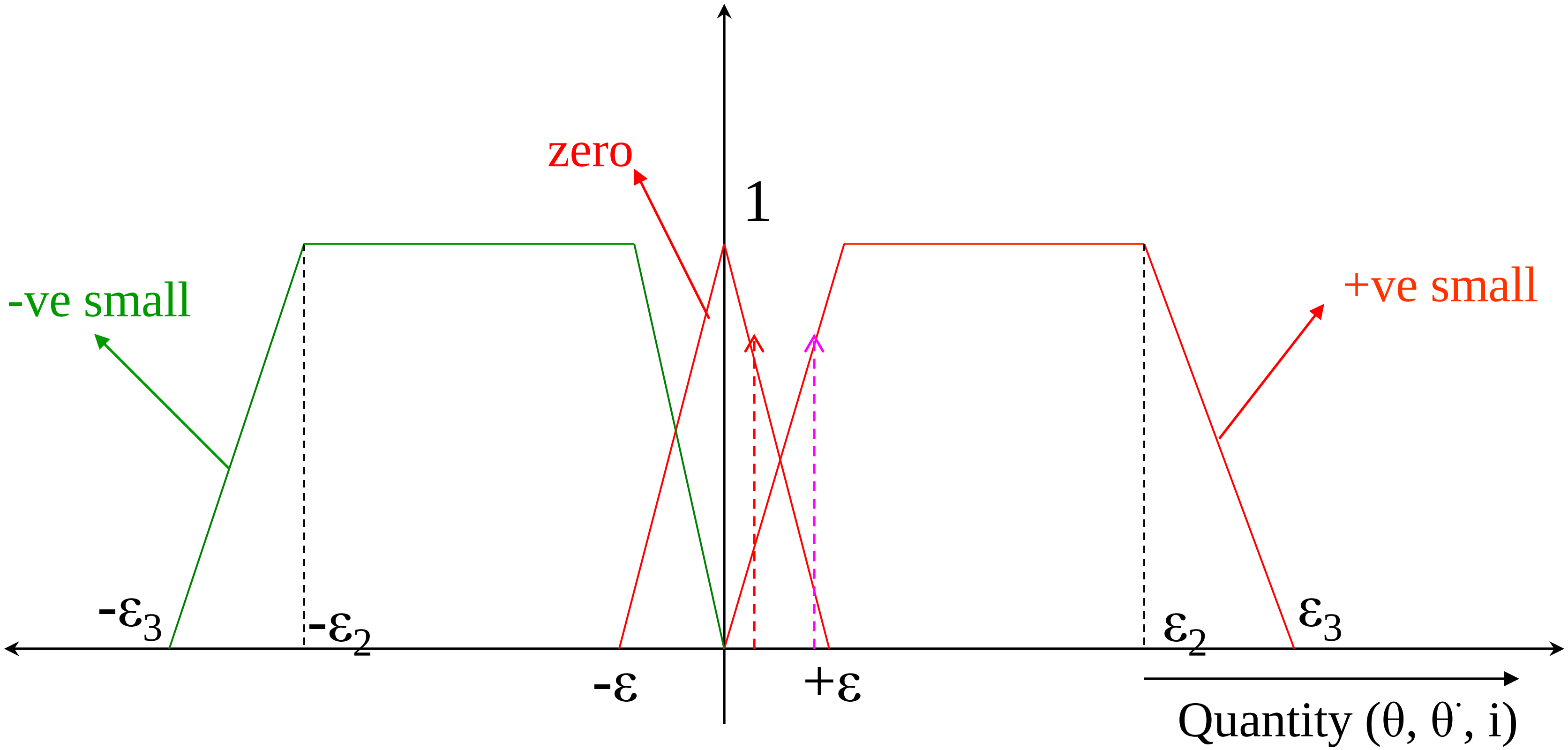


Inference procedure

1. Read actual numerical values of θ and $\dot{\theta}$
2. Get the corresponding μ values μ_{Zero}, $\mu_{(+ve\ small)}$, $\mu_{(-ve\ small)}$. This is called FUZZIFICATION
3. For different rules, get the fuzzy *i* values from the R.H.S of the rules.
4. "Collate" by some method and get ONE current value. This is called DEFUZZIFICATION
5. Result is one numerical value of *i*.

Rules Involved

if θ is Zero and dθ/dt is Zero then i is Zero
if θ is Zero and dθ/dt is +ve small then i is –ve small
if θ is +ve small and dθ/dt is Zero then i is –ve small
if θ +ve small and dθ/dt is +ve small then i is -ve medium



Fuzzification

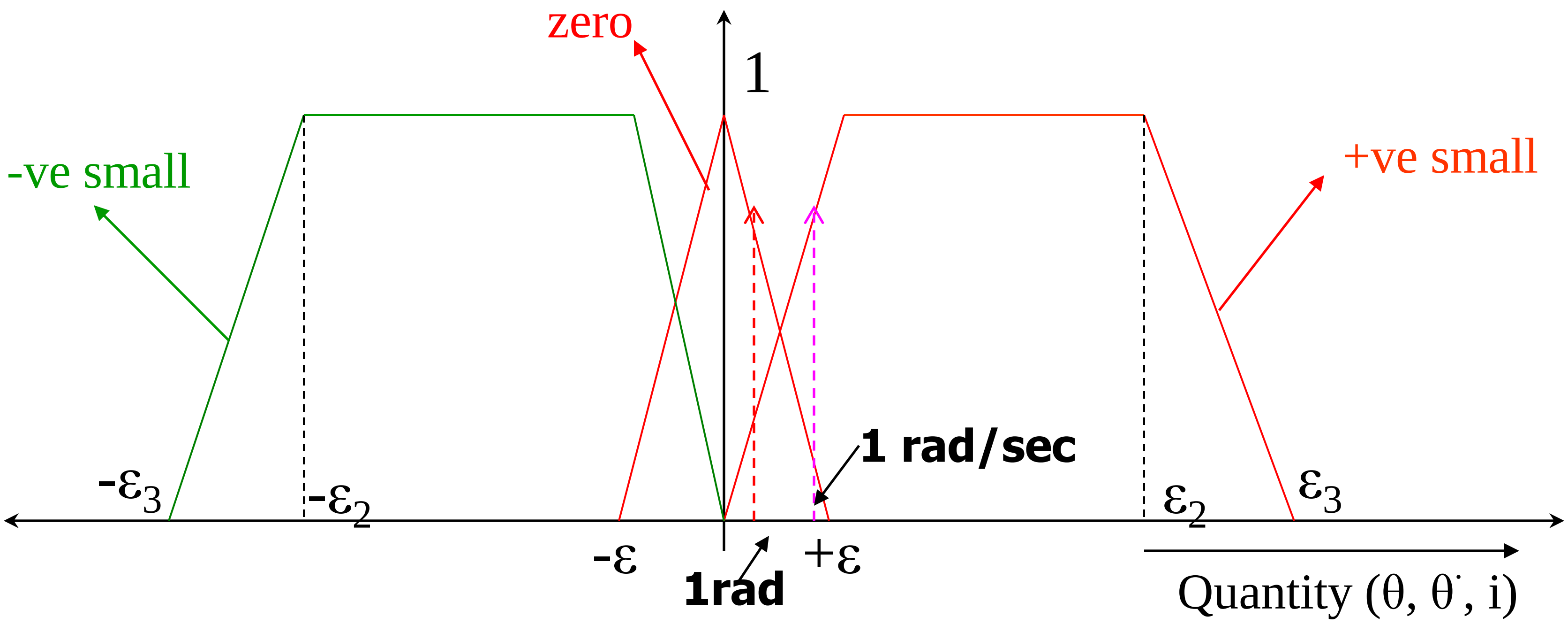
Suppose θ is 1 radian and dθ/dt is 1 rad/sec

$\mu_{zero}(\theta = 1) = 0.8$ (say)

$\mu_{+ve\text{-}small}(\theta = 1) = 0.4$ (say)

$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)

$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)

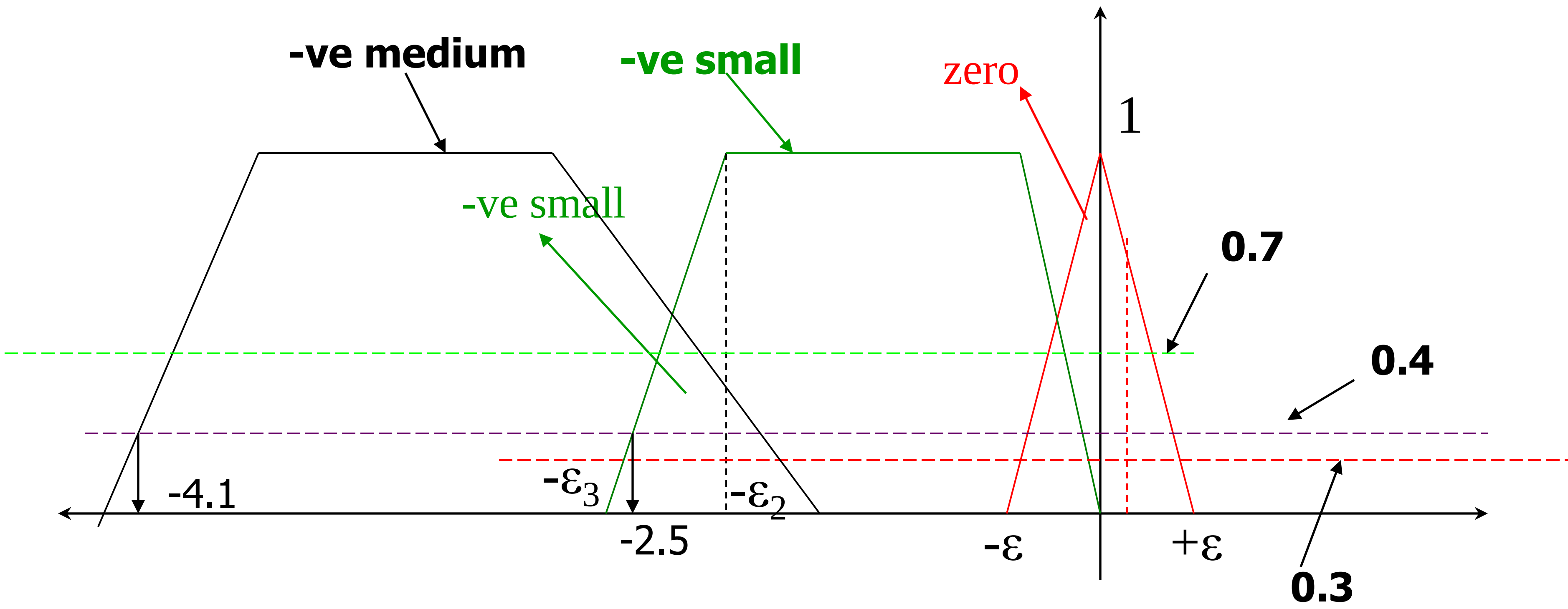


Fuzzification

Suppose θ is 1 radian and dθ/dt is 1 rad/sec
$\mu_{zero}(\theta = 1) = 0.8$ (say)
$\mu_{+ve\text{-}small}(\theta = 1) = 0.4$ (say)
$\mu_{zero}(d\theta/dt = 1) = 0.3$ (say)
$\mu_{+ve\text{-}small}(d\theta/dt = 1) = 0.7$ (say)

if θ is Zero and dθ/dt is Zero then i is Zero
min(0.8, 0.3)=0.3
hence $\mu_{zero}(i) = 0.3$
if θ is Zero and dθ/dt is +ve small then i is –ve small
min(0.8, 0.7)=0.7
hence $\mu_{-ve\text{-}small}(i) = 0.7$
if θ is +ve small and dθ/dt is Zero then i is –ve small
min(0.4, 0.3)=0.3
hence *μ-ve-small(i)=0.3*
if θ +ve small and dθ/dt is +ve small then i is -ve medium
min(0.4, 0.7)=0.4
hence $\mu_{-ve\text{-}medium}(i) = 0.4$

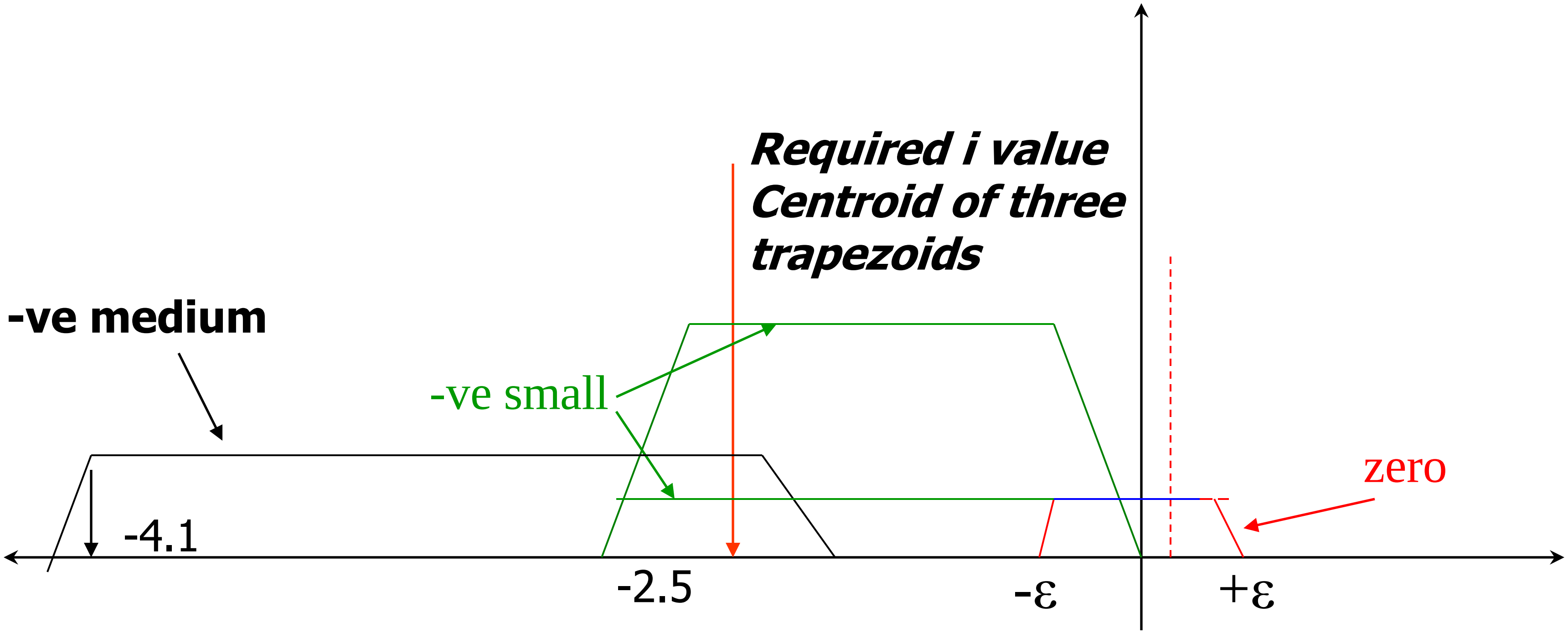
Finding *i*



Possible candidates:

i=0.5 and -0.5 from the "zero" profile and μ=0.3
i=-0.1 and -2.5 from the "-ve-small" profile and μ=0.3
i=-1.7 and -4.1 from the "-ve-small" profile and μ=0.3

Defuzzification: Finding *i* by the *centroid* method



Possible candidates:

i is the x-coord of the centroid of the areas given by the blue trapezium, the green trapeziums and the black trapezium

Propositional Calculus and Puzzles

Propositions

- Stand for facts/assertions
- Declarative statements
  - As opposed to interrogative statements (questions) or imperative statements (request, order)

Operators

$$AND(\wedge), OR(\vee), NOT(\neg), IMPLICATION(\Rightarrow)$$

=> and ¬ form a minimal set (can express other operations)

- Prove it.

Tautologies are formulae whose truth value is always T, whatever the assignment is

Model

In propositional calculus any formula with n propositions has 2^n models (assignments)
- Tautologies evaluate to T in all models.

Examples:

1) $$P \vee \neg P$$

2) $$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$$

-e Morgan with AND

Semantic Tree/Tableau method of proving tautology

Start with the negation of the formula

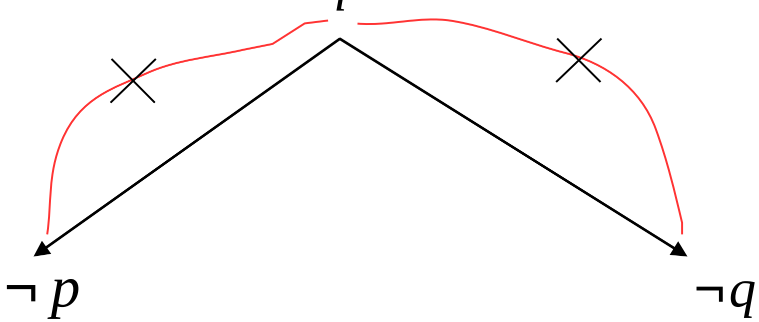
$$\neg[\neg(P \wedge Q) \Rightarrow (\neg P \vee \neg Q)]$$ - α - formula

$$\neg(P \wedge Q)$$ - β - formula

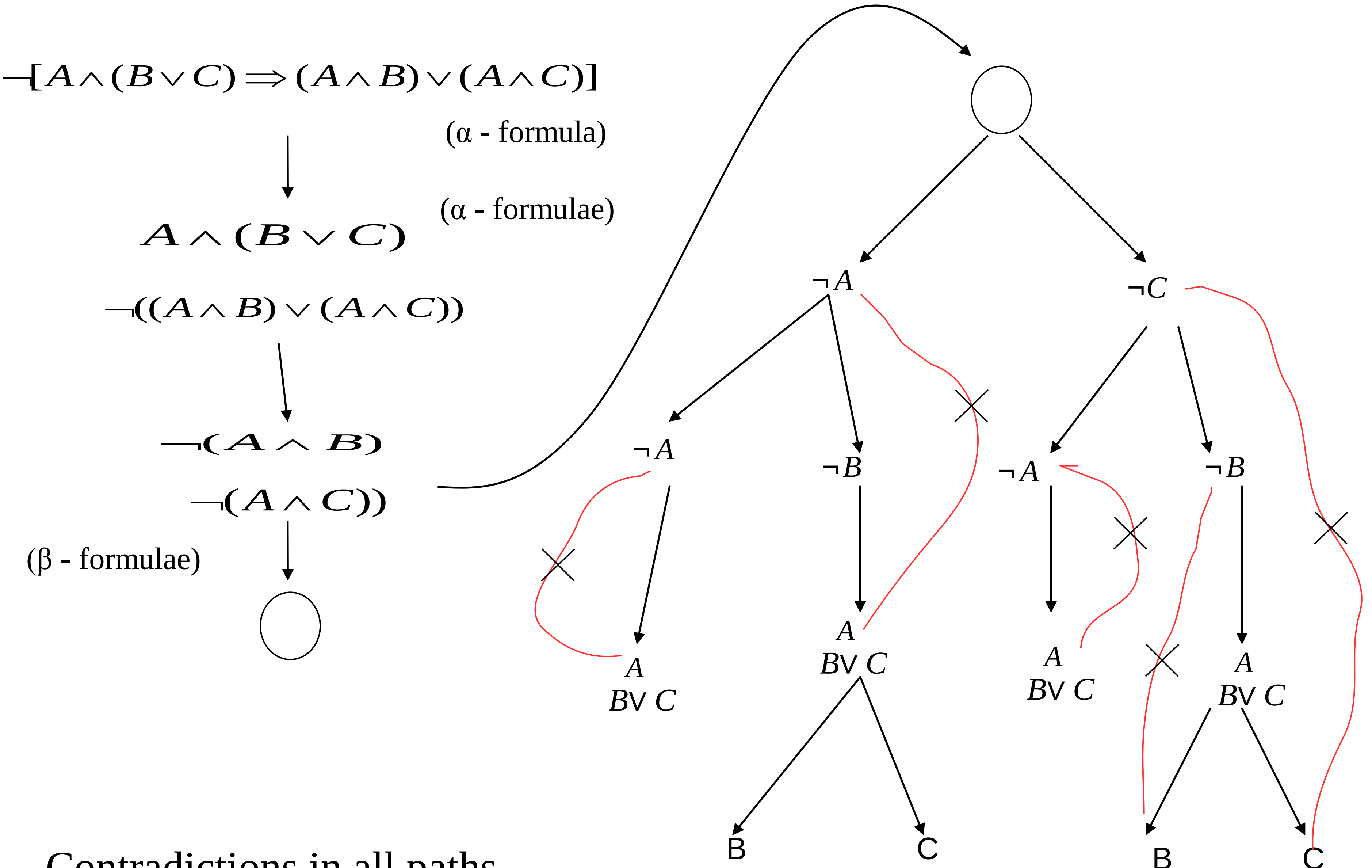
$$\neg(\neg P \vee \neg Q)$$ - α - formula

p

q



Example 2:



Contradictions in all paths

A puzzle

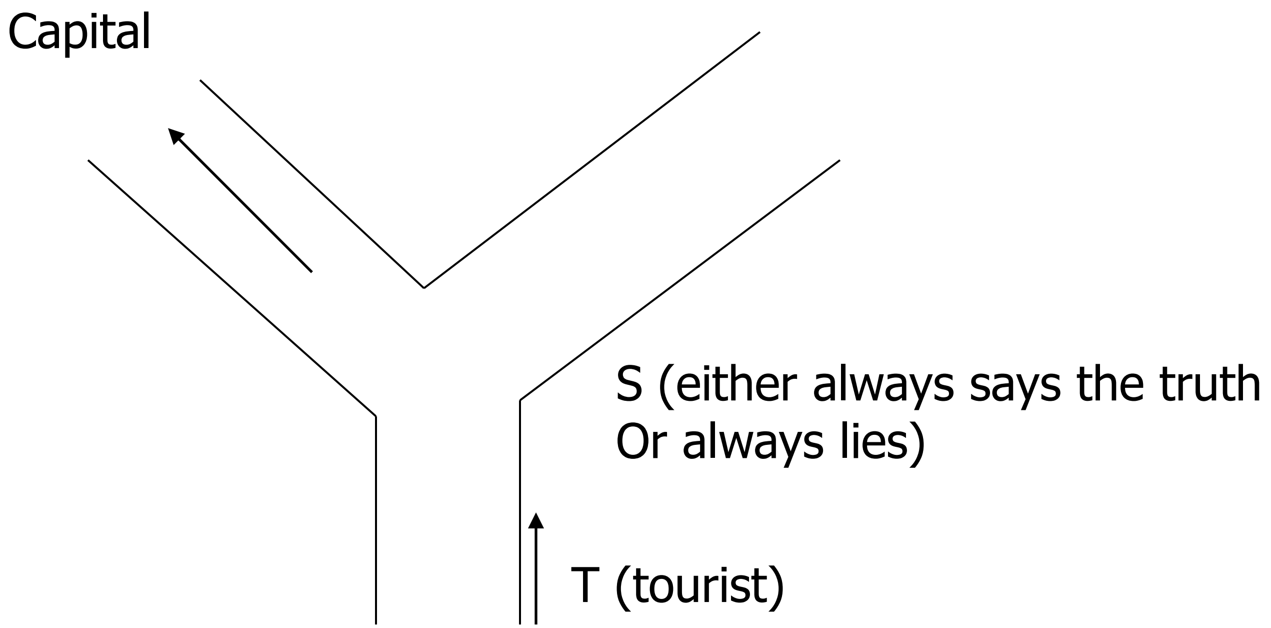
(Zohar Manna, Mathematical Theory of Computation, 1974)

From Propositional Calculus

Tourist in a country of truth-sayers and liers

- Facts and Rules: In a certain country, people **either always** speak the truth **or always** lie. A tourist T comes to a junction in the country and finds an inhabitant S of the country standing there. One of the roads at the junction leads to the capital of the country and the other does not. S can be asked only **yes/no** questions.
- Question: What **single** yes/no question can T ask of S, so that the direction of the capital is revealed?

Diagrammatic representation



Deciding the Propositions: a very difficult step- needs human intelligence

- P: Left road leads to capital
- Q: S always speaks the truth

Meta Question: What question should the tourist ask

- The **form** of the question
- Very difficult: needs human intelligence
- The tourist should ask
  - ***Is R true?***
  - ***The answer is "yes" if and only if the left road leads to the capital***
  - ***The structure of R to be found as a function of P and Q***

A more mechanical part: use of truth table

P	**Q**	**S's Answer**	**R**
T	T	Yes	T
T	F	Yes	F
F	T	No	F
F	F	No	T

Get form of R: quite mechanical

- From the truth table
  - ***R is of the form (P x-nor Q) or (P ≡ Q)***

Get *R* in English/Hindi/Hebrew…

- Natural Language Generation: non-trivial
- The question the tourist will ask is
  - ***Is it true that the left road leads to the capital if and only if you speak the truth?***
- Exercise: A more well known form of this question asked by the tourist uses the X-OR operator instead of the X-Nor. What changes do you have to incorporate to the solution, to get that answer?