CS344: Introduction to Artificial Intelligence (associated lab: CS386)

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Lecture 14: AI, Logic, and Puzzle Solving

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A puzzle

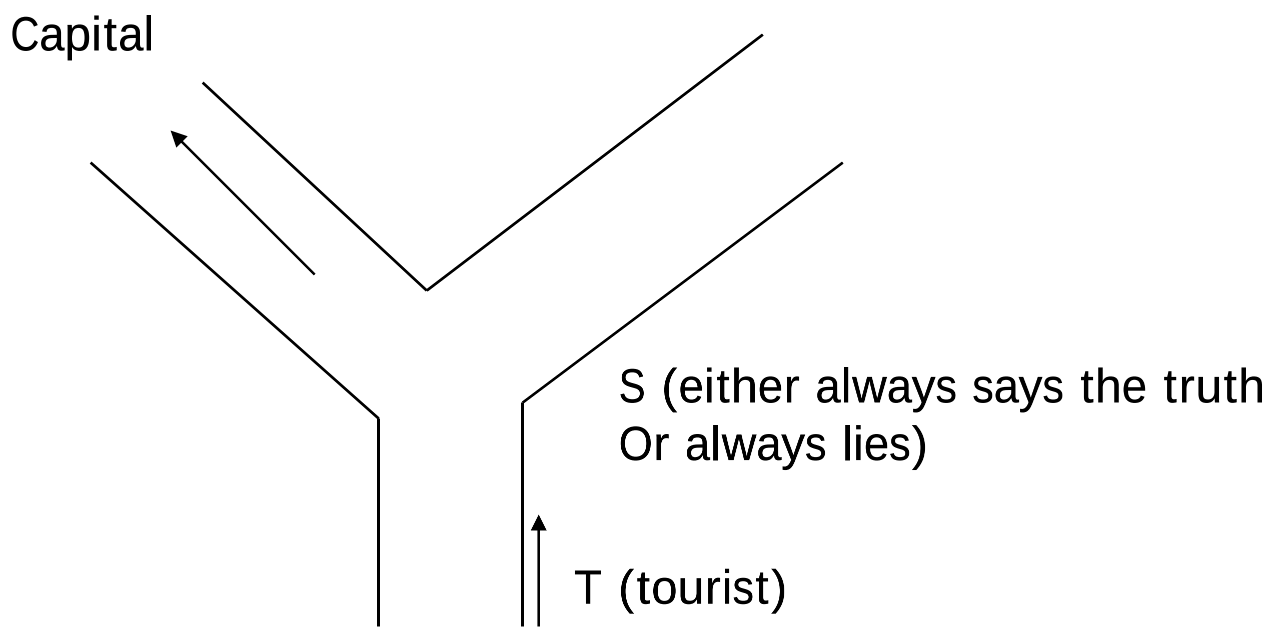
(Zohar Manna, Mathematical Theory of Computation, 1974)

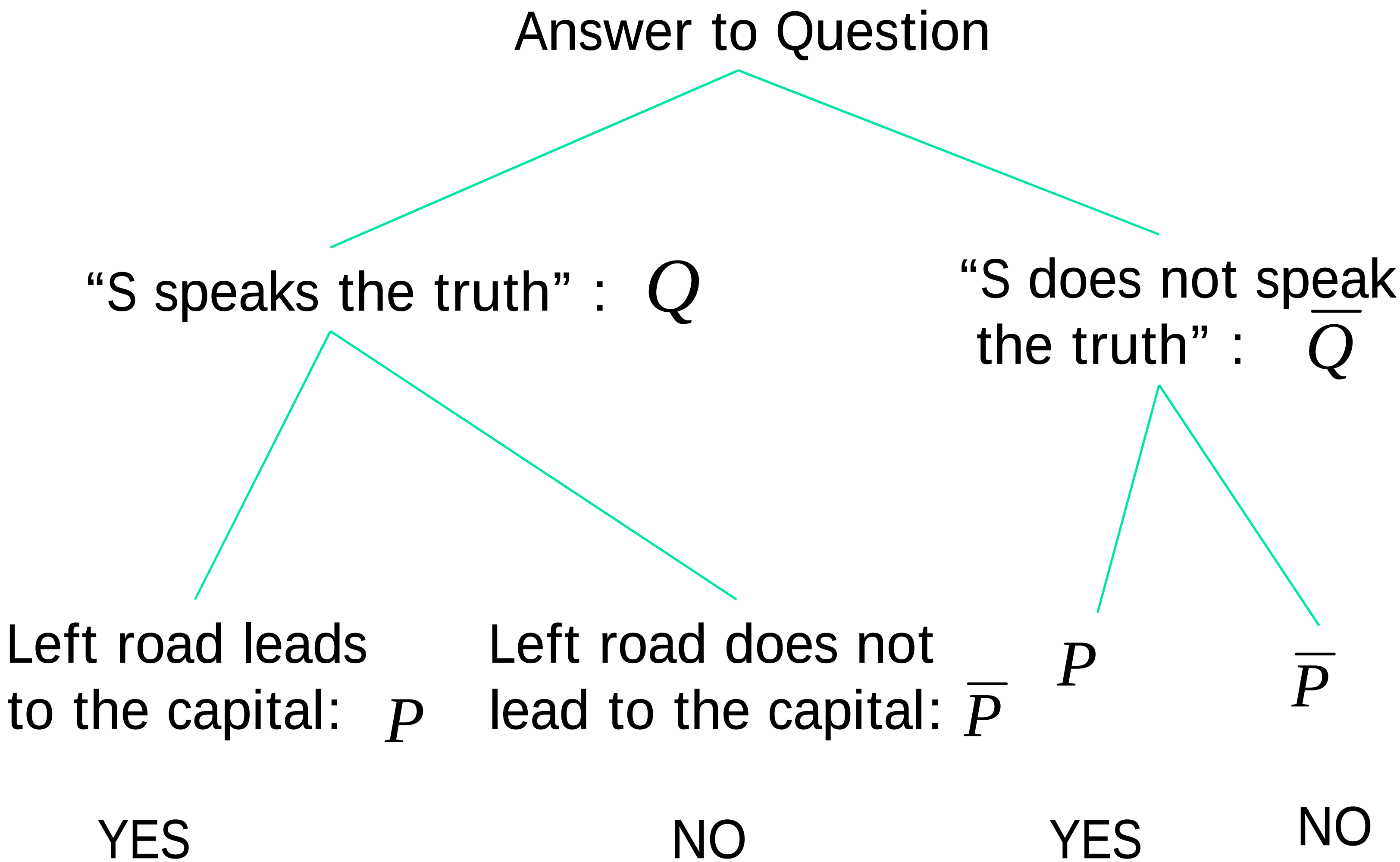
From Propositional Calculus

Tourist in a country of truth-sayers and liers

- Facts and Rules: In a certain country, people **either always** speak the truth **or always** lie. A tourist T comes to a junction in the country and finds an inhabitant S of the country standing there. One of the roads at the junction leads to the capital of the country and the other does not. S can be asked only **yes/no** questions.
- Question: What **single** yes/no question can T ask of S, so that the direction of the capital is revealed?

Diagrammatic representation




Answer to Question
"S speaks the truth" : Q
"S does not speak the truth" : $\overline{Q}$
Left road leads to the capital: P
Left road does not lead to the capital: $\overline{P}$
P
$\overline{P}$
YES
NO
YES
NO

Deciding the Propositions: a very difficult step- needs human intelligence

- P: Left road leads to capital
- Q: S always speaks the truth

Meta Question: What question should the tourist ask

- The **form** of the question
- Very difficult: needs human intelligence
- The tourist should ask
  - ***Is R true?***
  - ***The answer is "yes" if and only if the left road leads to the capital***
  - ***The structure of R to be found as a function of P and Q***

A more mechanical part: use of truth table

P	**Q**	**S's Answer**	**R**
T	T	Yes	T
T	F	Yes	F
F	T	No	F
F	F	No	T

Get form of R: quite mechanical

- From the truth table
  - ***R is of the form (P x-nor Q) or (P ≡ Q)***

Get *R* in English/Hindi/Hebrew...

- Natural Language Generation: non-trivial
- The question the tourist will ask is
  - ***Is it true that the left road leads to the capital if and only if you speak the truth?***
- Exercise: A more well known form of this question asked by the tourist uses the X-OR operator instead of the X-Nor. What changes do you have to incorporate to the solution, to get that answer?

Himalayan Club example

- Introduction through an example *(Zohar Manna, 1974):*
  - Problem: A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - Facts
  - Rules

Example contd.

- Let *mc* denote mountain climber and *sk* denotes skier. Knowledge representation in the given problem is as follows:
  1. *member(A)*
  2. *member(B)*
  3. *member(C)*
  4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
  5. $\forall x[mc(x) \rightarrow \sim like(x,rain)]$
  6. $\forall x[sk(x) \rightarrow like(x, snow)]$
  7. $\forall x[like(B, x) \rightarrow \sim like(A, x)]$
  8. $\forall x[\sim like(B, x) \rightarrow like(A, x)]$
  9. *like(A, rain)*
  10. *like(A, snow)*
  11. *Question:* $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$
- We have to infer the 11th expression from the given 10.
- Done through Resolution Refutation.

Club example: Inferencing

1. *member(A)*
2. *member(B)*
3. *member(C)*
4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - Can be written as $[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - $\sim member(x) \vee mc(x) \vee sk(x)$
5. $\forall x[sk(x) \rightarrow lk(x, snow)]$
   - $\sim sk(x) \vee lk(x, snow)$
6. $\forall x[mc(x) \rightarrow \sim lk(x, rain)]$
   - $\sim mc(x) \vee \sim lk(x, rain)$
7. $\forall x[like(A, x) \rightarrow \sim lk(B, x)]$
   - $\sim like(A, x) \vee \sim lk(B, x)$

8. $\forall x[\sim lk(A,x) \rightarrow lk(B,x)]$

– $lk(A,x) \vee lk(B,x)$

9. $lk(A, rain)$

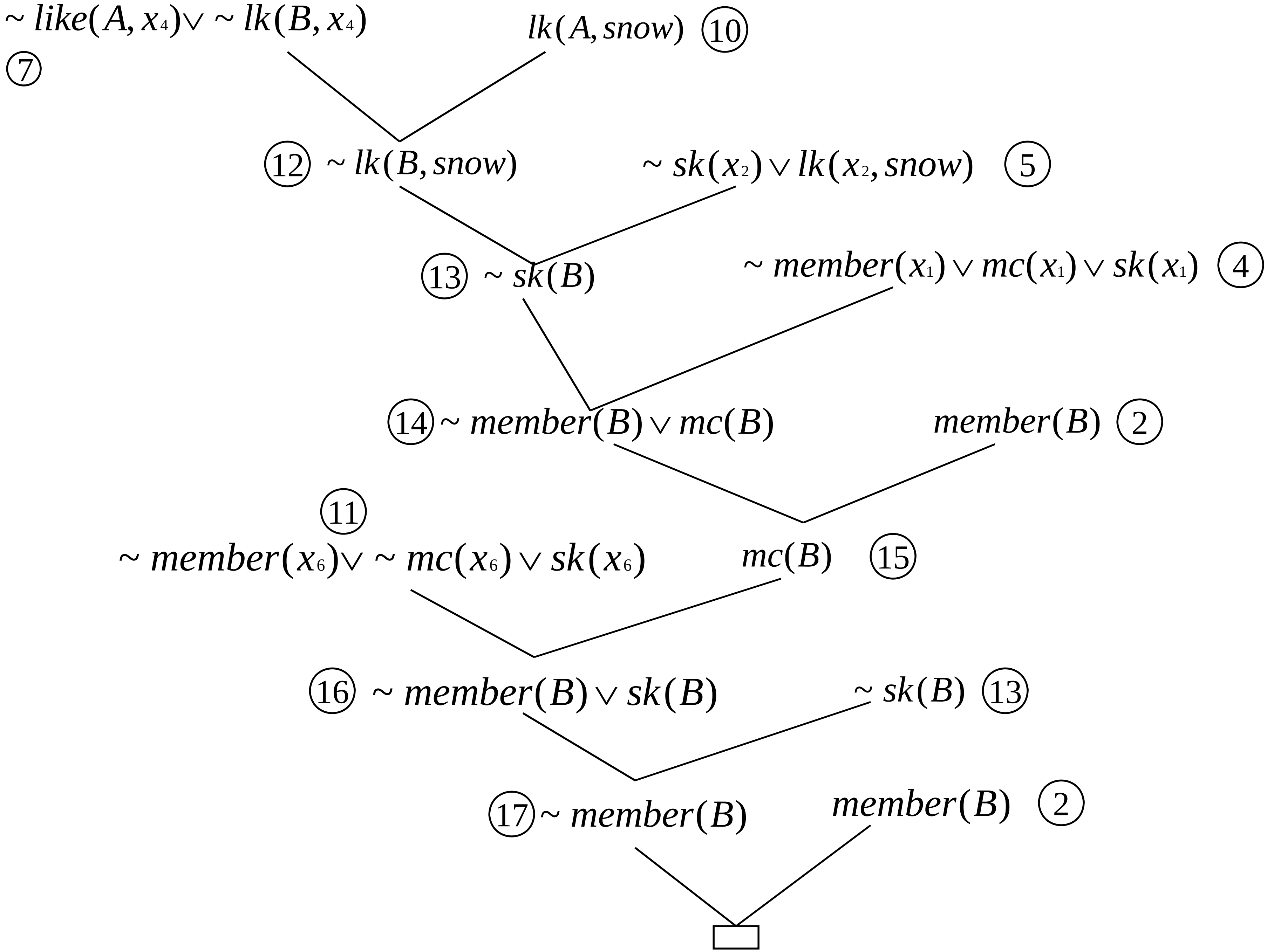
10. $lk(A, snow)$

11. $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$

– Negate– $\forall x[\sim member(x) \vee \sim mc(x) \vee sk(x)]$

- Now standardize the variables apart which results in the following

1. $member(A)$
2. $member(B)$
3. $member(C)$
4. $\sim member(x_1) \vee mc(x_1) \vee sk(x_1)$
5. $\sim sk(x_2) \vee lk(x_2, snow)$
6. $\sim mc(x_3) \vee \sim lk(x_3, rain)$
7. $\sim like(A, x_4) \vee \sim lk(B, x_4)$
8. $lk(A, x_5) \vee lk(B, x_5)$
9. $lk(A, rain)$
10. $lk(A, snow)$
11. $\sim member(x_6) \vee \sim mc(x_6) \vee sk(x_6)$


~ like(A, x4) ∨ ~ lk(B, x4)
7
lk(A, snow)
10
12
~ lk(B, snow)
~ sk(x2) ∨ lk(x2, snow)
5
13
~ sk(B)
~ member(x1) ∨ mc(x1) ∨ sk(x1)
4
14
~ member(B) ∨ mc(B)
member(B)
2
11
~ member(x6) ∨ ~ mc(x6) ∨ sk(x6)
mc(B)
15
16
~ member(B) ∨ sk(B)
~ sk(B)
13
17
~ member(B)
member(B)
2