CS344: Introduction to Artificial Intelligence (associated lab: CS386)

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Lecture 15, 16: Predicate Calculus

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Predicate Calculus: well known examples

- Man is mortal : rule

  $\forall x[man(x) \rightarrow mortal(x)]$

- shakespeare is a man
  man(shakespeare)

- To infer shakespeare is mortal
  mortal(shakespeare)

Predicate Calculus: origin

- Predicate calculus originated in language

Sentence

Subject Predicate

Grass is green

Subject Predicate

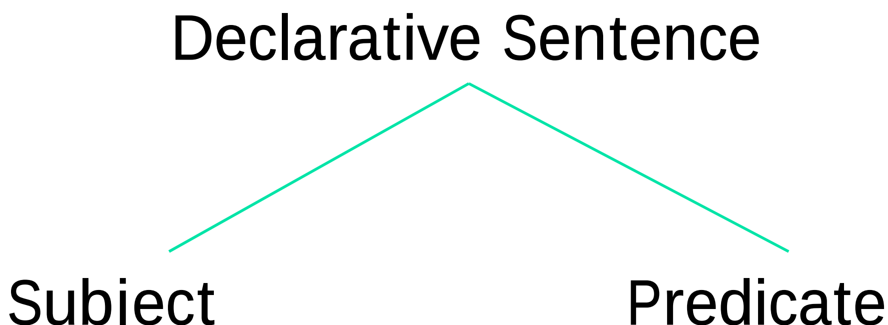
green(grass)

$$P : D \rightarrow \{T, F\}$$

D : Domain

Predicate Calculus: only for declarative sentences

- Is grass green ?
- Oh, grass is green!



- Grass which is supple is green

$$\textit{supple(grass)} \longrightarrow \textit{green(grass)}$$

$$\forall x(\mathrm{grass}(x)) \wedge \mathrm{supple}(x) \rightarrow \mathrm{green}(x))$$

PC primitive: N-ary Predicate

$$P(a_1, \ldots a_n)$$

$$P : D^n \rightarrow \{T, F\}$$

- Arguments of predicates can be variables and constants
- Ground instances : Predicate all whose arguments are constants

N-ary Functions

$$f : D^n \rightarrow D$$

president(India) : Prathiba Patel

- Constants & Variables : Zero-order objects
- Predicates & Functions : First-order objects

President of India is a lady

lady(president(India))

Prime minister of India is older than the president of India

older(prime_minister(India), president(India))

Operators

$$\wedge \ \vee \ \sim \ \oplus \forall \ \rightarrow \ \exists$$

- Universal Quantifier $\exists$
- Existential Quantifier

All men are mortal

$$\forall x[man(x) \rightarrow mortal(x)]$$

Some men are rich

$$\exists x[man(x) \wedge rich(x)]$$

Tautologies

$$\sim \forall x(p(x)) \rightarrow \exists x(\sim p(x))$$

$$\sim \exists x(p(x)) \rightarrow \forall x(\sim p(x))$$

- 2^{nd} tautology in English:
  - *Not a single man in this village is educated implies all men in this village are uneducated*
- Tautologies are important instruments of logic, but uninteresting statements!

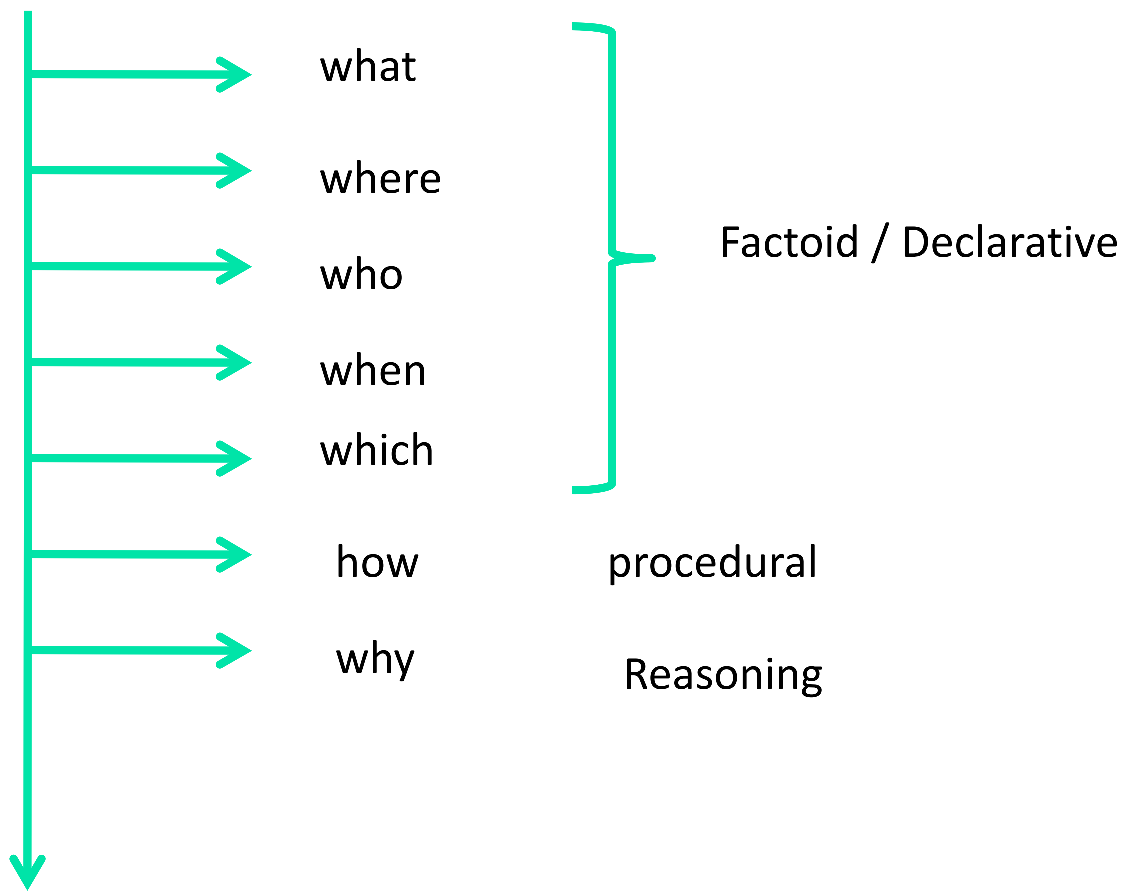
Inferencing: Forward Chaining

- *man(x)* $\rightarrow$ *mortal(x)*
  - *Dropping the quantifier, implicitly Universal quantification assumed*
  - *man(shakespeare)*
- Goal mortal(shakespeare)
  - Found in one step
  - x = shakespeare, unification

Backward Chaining

- $man(x) \rightarrow mortal(x)$
- Goal mortal(shakespeare)
  - x = shakespeare
  - Travel back over and hit the fact asserted
  - man(shakespeare)

Wh-Questions and Knowledge



Fixing Predicates

- Natural Sentences

<Subject> <verb> <object>

↓

Verb(subject,object)

↓

predicate(subject)

Examples

- Ram is a boy
  - Boy(Ram)?
  - Is_a(Ram,boy)?

- Ram Playes Football
  - Plays(Ram,football)?
  - Plays_football(Ram)?

Knowledge Representation of Complex Sentence

- *"In every city there is a thief who is beaten by every policeman in the city"*

$$\forall x[\text{city}(x) \rightarrow \{\exists y((\text{thief}(y) \wedge \text{lives\_in}(y,x)) \wedge \forall z(\text{poleceman}(z,x) \rightarrow \text{beaten\_by}(z,y)))\}]$$

Himalayan Club example

- Introduction through an example *(Zohar Manna, 1974):*
  - Problem: A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - Facts
  - Rules

Example contd.

- Let *mc* denote mountain climber and *sk* denotes skier. Knowledge representation in the given problem is as follows:
    1. *member(A)*
    2. *member(B)*
    3. *member(C)*
    4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
    5. $\forall x[mc(x) \rightarrow \sim like(x,rain)]$
    6. $\forall x[sk(x) \rightarrow like(x, snow)]$
    7. $\forall x[like(B, x) \rightarrow \sim like(A, x)]$
    8. $\forall x[\sim like(B, x) \rightarrow like(A, x)]$
    9. *like(A, rain)*
    10. *like(A, snow)*
    11. *Question:* $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$
- We have to infer the 11^{th} expression from the given 10.
- Done through Resolution Refutation.

Club example: Inferencing

1. *member(A)*
2. *member(B)*
3. *member(C)*
4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - Can be written as
     $[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - $\sim member(x) \vee mc(x) \vee sk(x)$
5. $\forall x[sk(x) \rightarrow lk(x, snow)]$
   - $\sim sk(x) \vee lk(x, snow)$
6. $\forall x[mc(x) \rightarrow \sim lk(x, rain)]$
   - $\sim mc(x) \vee \sim lk(x, rain)$
7. $\forall x[like(A, x) \rightarrow \sim lk(B, x)]$
   - $\sim like(A, x) \vee \sim lk(B, x)$

8. $\forall x[\sim lk(A,x) \rightarrow lk(B,x)]$

– $lk(A,x) \vee lk(B,x)$

9. $lk(A,rain)$

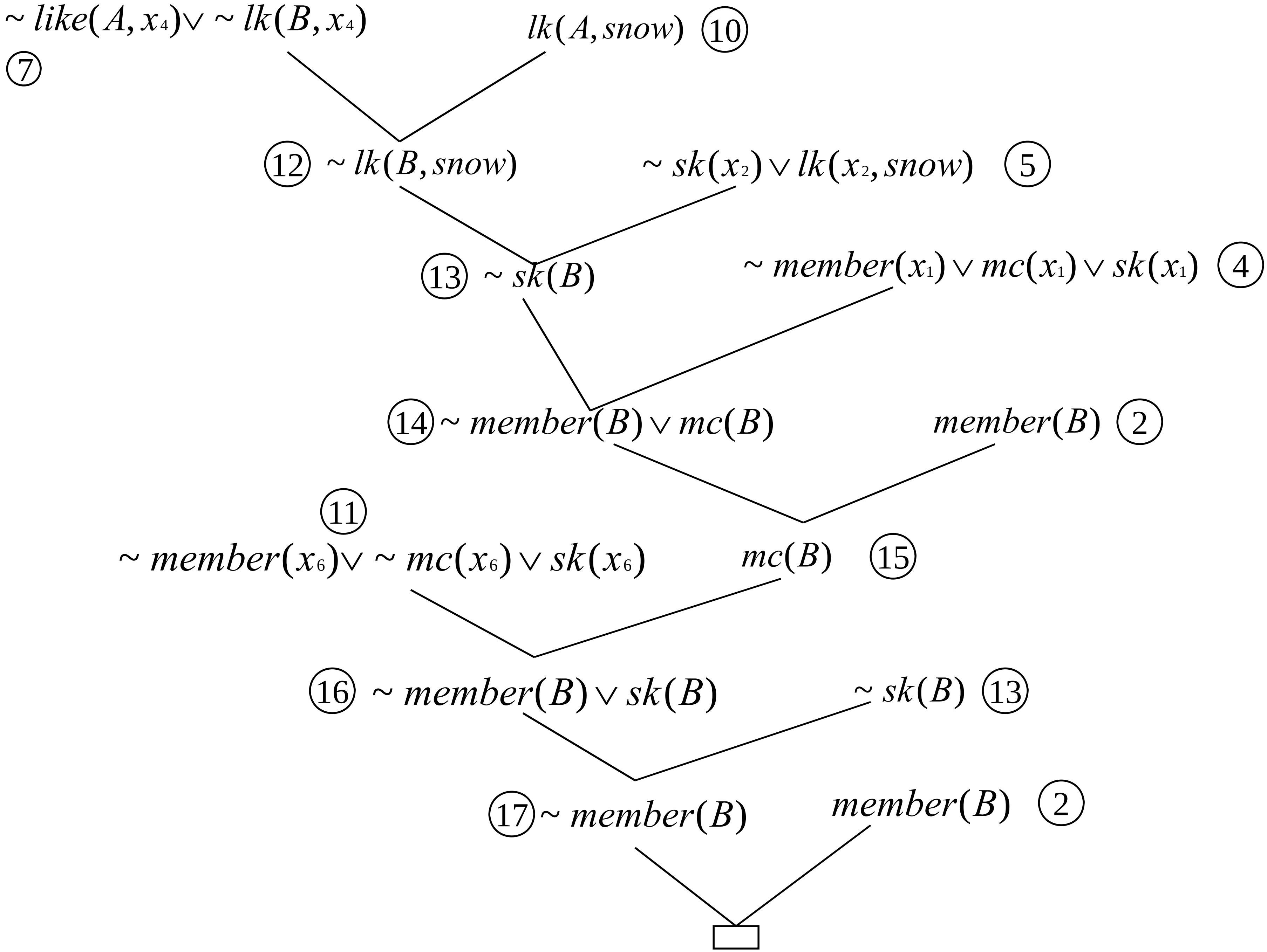
10. $lk(A,snow)$

11. $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$

– Negate– $\forall x[\sim member(x) \vee \sim mc(x) \vee sk(x)]$

- Now standardize the variables apart which results in the following

1. $member(A)$
2. $member(B)$
3. $member(C)$
4. $\sim member(x_1) \vee mc(x_1) \vee sk(x_1)$
5. $\sim sk(x_2) \vee lk(x_2, snow)$
6. $\sim mc(x_3) \vee \sim lk(x_3, rain)$
7. $\sim like(A, x_4) \vee \sim lk(B, x_4)$
8. $lk(A, x_5) \vee lk(B, x_5)$
9. $lk(A, rain)$
10. $lk(A, snow)$
11. $\sim member(x_6) \vee \sim mc(x_6) \vee sk(x_6)$


~ like(A, x4) ∨ ~ lk(B, x4) ⑦
lk(A, snow) ⑩
⑫ ~ lk(B, snow)
~ sk(x2) ∨ lk(x2, snow) ⑤
⑬ ~ sk(B)
~ member(x1) ∨ mc(x1) ∨ sk(x1) ④
⑭ ~ member(B) ∨ mc(B)
member(B) ②
⑪
~ member(x6) ∨ ~ mc(x6) ∨ sk(x6)
mc(B) ⑮
⑯ ~ member(B) ∨ sk(B)
~ sk(B) ⑬
⑰ ~ member(B)
member(B) ②