CS344: Introduction to Artificial Intelligence
(associated lab: CS386)

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Lecture 19: Interpretation in Predicate Calculus

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Interpretation in Logic

- Logical expressions or formulae are “FORMS” (placeholders) for whom contents are created through interpretation.
- Example:

$$\exists F\left[\left\{F(a)=b\right\}\wedge\forall x\left\{P(x)\rightarrow\left(F(x)=g\left(x,F\left(h(x)\right)\right)\right)\right\}\right]$$

- This is a Second Order Predicate Calculus formula.
- Quantification on ‘F’ which is a function.

Examples

- Interpretation:1

  $D=N$ (natural numbers)

  $a = 0$ and $b = 1$

  $x \in N$

  $P(x)$ stands for $x > 0$

  $g(m,n)$ stands for $(m \text{ x } n)$

  $h(x)$ stands for $(x - 1)$

- Above interpretation defines **Factorial**

Examples (contd.)

- Interpretation:2

  D={strings}

  $a = b = \lambda$

  $P(x)$ stands for "x is a non empty string"

  $g(m, n)$ stands for "append head of m to n"

  $h(x)$ stands for $tail(x)$
- Above interpretation defines "reversing a string"

More Examples

- $\forall x \, [\, P(x) \rightarrow Q(x)]$
- Following interpretations conform to above expression:

  $man(x) \rightarrow mortal(x)$

  $dog(x) \rightarrow mammal(x)$

  $prime(x) \rightarrow 2\_or\_odd(x)$

  $CS(x) \rightarrow bad\_hand\_writing(x)$

Structure of Interpretation

- All interpretations begin with a domain 'D', constants (0-order functions) and functions pick values from there.
- With respect to -

  $\forall x\, [\, P(x) \rightarrow Q(x)]$

  $D = \{living\ beings\}$

  $P: D \rightarrow \{T, F\}$

  $Q: D \rightarrow \{T, F\}$
- *P* can be looked upon as a table shown here:

Elements of *D* i.e. *x*	***P(x)***
Ram	T
Pushpak	T
$Virus_{1201}$	F

Factorial interpretation of the structure

- $D = \{0, 1, 2, ..., \infty\}$
- $a = 0, b = 1$
- $g(m, n) = m \times n$ and $g: D \times D \rightarrow D$
- $h(x) = (x - 1)$ and $h(0) = 0, h: D \rightarrow D$
- $P(x)$ *is* $x > 0$ and $P: D \rightarrow \{T, F\}$

Steps in Interpretation

1. Fix Domain *D*
2. Assign values to constants
3. Define functions
4. Define predicates

- An expression which is true for <u>all</u> interpretations is called <u>valid</u> or tautology.
- Interpretations and their validity in “Herbrand’s Universe” is sufficient for proving validity in Predicate Calculus.

- Note:- Possible seminar topic – “Herbrand’s Interpretation and Validity”

Some examples…

- Valid for any domain

$$\forall x \exists y \left[P(x) \rightarrow P(y) \right]$$

- Valid for any domain with less than 3 elements but invalid for any domain with more than 2 elements

$$\forall x_1 x_2 x_3 \begin{bmatrix} \{P(x_1, x_1) \wedge P(x_2, x_2) \wedge P(x_3, x_3)\} \\ \rightarrow \{P(x_1, x_2) \vee P(x_2, x_3) \vee P(x_3, x_1)\} \end{bmatrix}$$

Definitions

1. An **interpretation** is also called **model**.
2. A formula is called a **tautology** or is said to be **valid** if it is true in **all** models.
3. A formula is called **satisfiable** if there exists **at least one** model where it is **true**.

Explanation

- The example

$$\forall x_1 x_2 x_3 \begin{bmatrix} \{P(x_1,x_1) \wedge P(x_2,x_2) \wedge P(x_3,x_3)\} \\ \rightarrow \{P(x_1,x_2) \vee P(x_2,x_3) \vee P(x_3,x_1)\} \end{bmatrix}$$

can be written as

$$\begin{bmatrix} \{\neg P(x_1,x_1) \vee \neg P(x_2,x_2) \vee \neg P(x_3,x_3)\} \\ \vee \{\neg P(x_1,x_2) \vee P(x_2,x_3) \vee P(x_3,x_1)\} \end{bmatrix}$$

- Domains

$$D1 = \{a\} (Valid)$$

$$D2 = \{a,b\} (Valid)$$

$$D3 = \{a,b,c\} (Invalid)$$