CS344: Introduction to Artificial Intelligence
(associated lab: CS386)

Pushpak Bhattacharyya
CSE Dept.,
IIT Bombay

Lecture 26: # of regions; Prolog

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Number of regions founded by n hyperplanes in d-dim passing through origin is given by the following recurrence relation

$$R_{n,d} = R_{n-1,d} + R_{n-1,d-1}$$

we use generating function as an operating function

Boundary condition:

$$R_{1,d} = 2$$ 1 hyperplane in d-dim

$$R_{n,1} = 2$$ n hyperplanes in 1-dim, Reduce to n points thru origin

The generating function is

$$f(x,y) = \sum_{n=1}^{\infty} \sum_{d=1}^{\infty} R_{n,d} \cdot x^n y^d$$

From the recurrence relation we have,

$$R_{n,d} - R_{n-1,d} - R_{n-1,d-1} = 0$$

$R_{n-1,d}$ corresponds to 'shifting' n by 1 place, => multiplication by x

$R_{n-1,d-1}$ corresponds to 'shifting' n and d by 1 place => multiplication by xy

On expanding $f(x,y)$ we get

$$\begin{aligned} f(x,y) = {} & R_{1,1} \cdot xy + R_{1,2} \cdot x y^2 + R_{1,3} \cdot x y^3 + \ldots + R_{1,d} \cdot x y^d + \ldots.\infty \\ & + R_{2,1} \cdot x^2 y + R_{2,2} \cdot x^2 y^2 + R_{2,3} \cdot x^2 y^3 + \ldots + R_{2,d} \cdot x^2 y^d + \ldots.\infty \\ & \ldots.. \\ & + R_{n,1} \cdot x^n y + R_{n,2} \cdot x^n y^2 + R_{n,3} \cdot x^n y^3 + \ldots + R_{n,d} \cdot x^n y^d + \ldots.\infty \end{aligned}$$

$$f(x,y) = \sum_{n=1}^{\infty} \sum_{d=1}^{\infty} R_{n,d} \cdot x^n y^d$$

$$x \cdot f(x,y) = \sum_{n=1}^{\infty} \sum_{d=1}^{\infty} R_{n,d} \cdot x^{n+1} y^d = \sum_{n=2}^{\infty} \sum_{d=1}^{\infty} R_{n-1,d} \cdot x^n y^d$$

$$xy \cdot f(x,y) = \sum_{n=1}^{\infty} \sum_{d=1}^{\infty} R_{n,d} \cdot x^{n+1} y^{d+1} = \sum_{n=2}^{\infty} \sum_{d=2}^{\infty} R_{n-1,d-1} \cdot x^n y^d$$

$$x \cdot f(x,y) = \sum_{n=2}^{\infty} \sum_{d=2}^{\infty} R_{n-1,d} \cdot x^n y^d + \sum_{n=2}^{\infty} R_{n-1,1} \cdot x^n y$$

$$= \sum_{n=2}^{\infty} \sum_{d=2}^{\infty} R_{n-1,d} \cdot x^n y^d + 2 \cdot \sum_{n=2}^{\infty} x^n y$$

$$f(x,y) = \sum_{n=1}^{\infty}\sum_{d=1}^{\infty} R_{n,d}\cdot x^n y^d$$

$$= \sum_{n=2}^{\infty}\sum_{d=2}^{\infty} R_{n,d}\cdot x^n y^d + \sum_{d=1}^{\infty} R_{1,d}\cdot xy^d + \sum_{n=1}^{\infty} R_{n,1}\cdot x^n y - R_{1,1}\cdot xy$$

$$= \sum_{n=2}^{\infty}\sum_{d=2}^{\infty} R_{n,d}\cdot x^n y^d + 2x\cdot\sum_{d=1}^{\infty} y^d + 2y\cdot\sum_{n=1}^{\infty} x^n - 2xy$$

After all this expansion,

$$f(x,y) - x\cdot f(x,y) - xy\cdot f(x,y)$$

$$= \sum_{n=2}^{\infty}\sum_{d=2}^{\infty}(R_{n,d} - R_{n-1,d} - R_{n-1,d-1})x^n y^d$$

$$+ 2y\sum_{n=1}^{\infty} x^n - 2xy - 2y\sum_{n=2}^{\infty} x^n + 2x\sum_{d=1}^{\infty} y^d$$

$$= 2x\sum_{d=1}^{\infty} y^d$$

since other two terms become zero

This implies

$$[1 - x - xy]\, f(x, y) = 2x \sum_{d=1}^{\infty} y^d$$

$$f(x, y) = \frac{1}{[1 - x(1 - y)]} \cdot 2x \sum_{d=1}^{\infty} y^d$$

$$= 2x[y + y^2 + y^3 + \ldots + y^d + \ldots..\infty]$$

$$[1 + x(1 + y) + x^2(1 + y)^2 + \ldots + x^d(1 + y)^d + \ldots..\infty]$$

also we have,

$$f(x, y) = \sum_{n=1}^{\infty} \sum_{d=1}^{\infty} R_{n,d} \cdot x^n y^d$$

Comparing coefficients of each term in RHS we get, Comparing co-efficients we get

$$R_{n,d} = \sum_{i=0}^{d-1} C_i^{n-1}$$

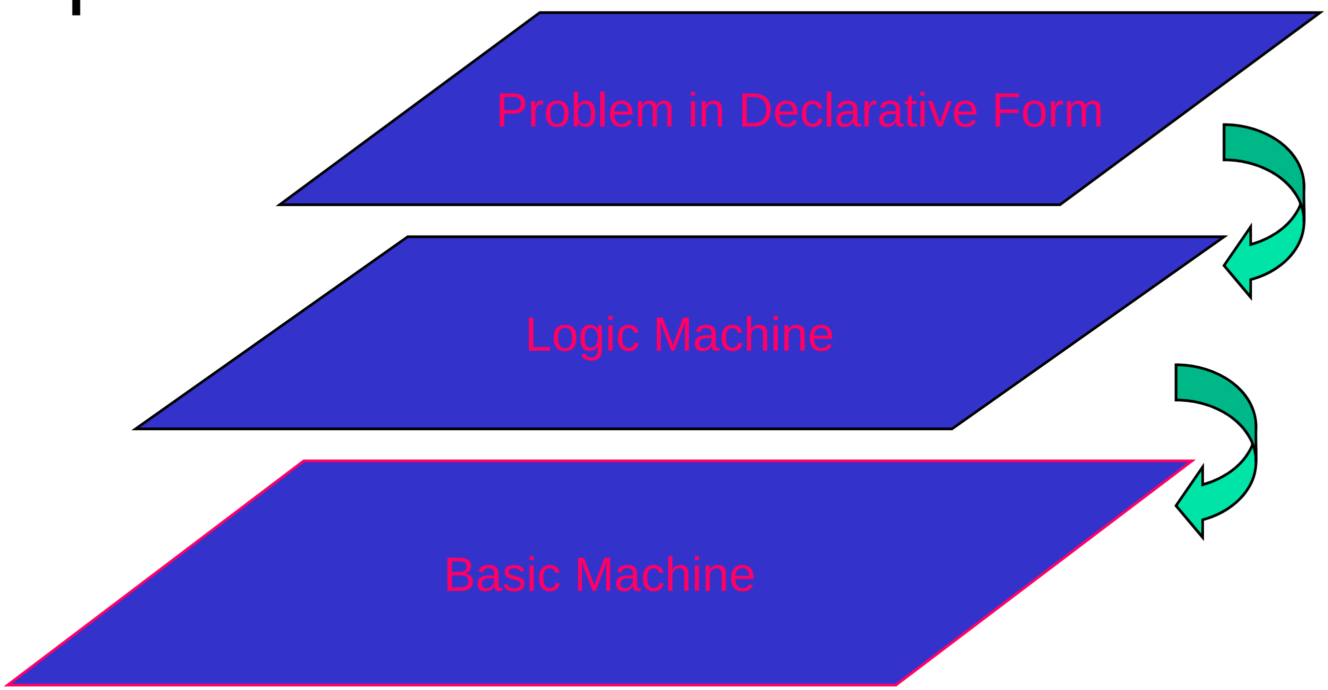
For peceptron

- n= no. of inputs
- $d= n+1$
- $R_{n,d}$ comes out to be upper bounded by $O(2^{n^2})$

Prolog

Introduction

- PROgramming in LOGic
- Emphasis on *what* rather than *how*



A Typical Prolog program

Compute_length ([],0).

Compute_length ([Head|Tail], Length):-

Compute_length (Tail,Tail_length),

Length is Tail_length+1.

<u>High level explanation:</u>

The length of a list is 1 plus the length of the tail of the list, obtained by removing the first element of the list.

This is a declarative description of the computation.

Fundamentals

(absolute basics for writing Prolog Programs)

Facts

- *John likes Mary*
  - *like(john,mary)*
- Names of relationship and objects must begin with a lower-case letter.
- Relationship is written *first* (typically the *predicate* of the sentence).
- *Objects* are written separated by commas and are enclosed by a pair of round brackets.
- The full stop character '.' must come at the end of a fact.

More facts

Predicate	**Interpretation**
valuable(gold)	Gold is valuable.
owns(john,gold)	John owns gold.
father(john,mary)	John is the father of Mary
gives (john,book,mary)	John gives the book to Mary

Questions

- *Questions* based on facts
- Answered by *matching*

Two facts *match* if their predicates are same (spelt the same way) and the arguments each are same.

- If matched, prolog answers *yes*, else *no*.
- *No* does not mean falsity.

Prolog does *theorem proving*

- When a question is asked, prolog tries to match *transitively.*
- When no match is found, answer is *no.*
- This means *not provable* from the given facts.

Variables

- Always begin with a capital letter
  - *?- likes (john,X).*
  - *?- likes (john, Something).*
- But *not*
  - *?- likes (john,something)*

Example of usage of variable

Facts:

likes(john,flowers).

likes(john,mary).

likes(paul,mary).

Question:

?- likes(john,X)

Answer:

X=flowers and wait

;

mary

;

no

Conjunctions

- Use ‘,’ and pronounce it as *and*.
- Example
  - Facts:
    - likes(mary,food).
    - likes(mary,tea).
    - likes(john,tea).
    - likes(john,mary)
- ?-
    - likes(mary,X),likes(john,X).
    - Meaning *is anything liked by Mary also liked by John?*

Backtracking *(an inherent property of prolog programming)*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. First goal succeeds. *X=food*
2. Satisfy *likes(john,food)*

Backtracking *(continued)*

Returning to a marked place and trying to resatisfy is called *Backtracking*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. Second goal fails
2. Return to marked place
   and try to resatisfy the first goal

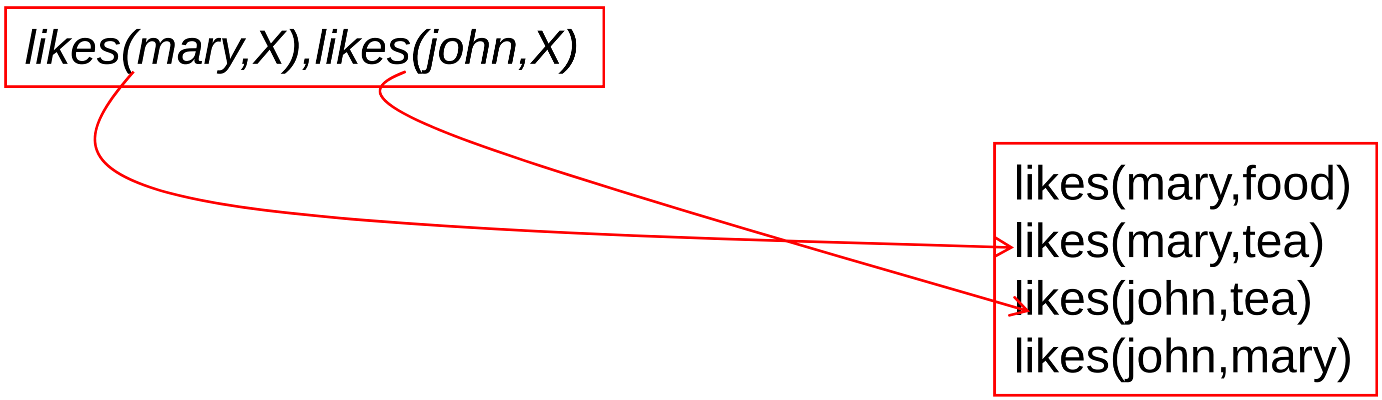
Backtracking *(continued)*

likes(mary,X),likes(john,X)

likes(mary,food)
likes(mary,tea)
likes(john,tea)
likes(john,mary)

1. First goal succeeds again, *X=tea*
2. Attempt to satisfy the *likes(john,tea)*

Backtracking *(continued)*



1. Second goal also suceeds
2. Prolog notifies success and waits for a reply

Rules

- Statements about *objects* and their *relationships*
- Expess
  - *If-then conditions*
    - *I use an umbrella if there is a rain*
    - *use(i, umbrella) :- occur(rain).*
  - *Generalizations*
    - *All men are mortal*
    - *mortal(X) :- man(X).*
  - *Definitions*
    - *An animal is a bird if it has feathers*
    - *bird(X) :- animal(X), has_feather(X).*

Syntax

- **<head> :- <body>**
- Read **‘:-’ as ‘if’.**
- E.G.
  - *likes(john,X) :- likes(X,cricket).*
  - *“John likes X if X likes cricket”.*
  - *i.e., “John likes anyone who likes cricket”.*
- Rules always end with ‘.’.

Another Example

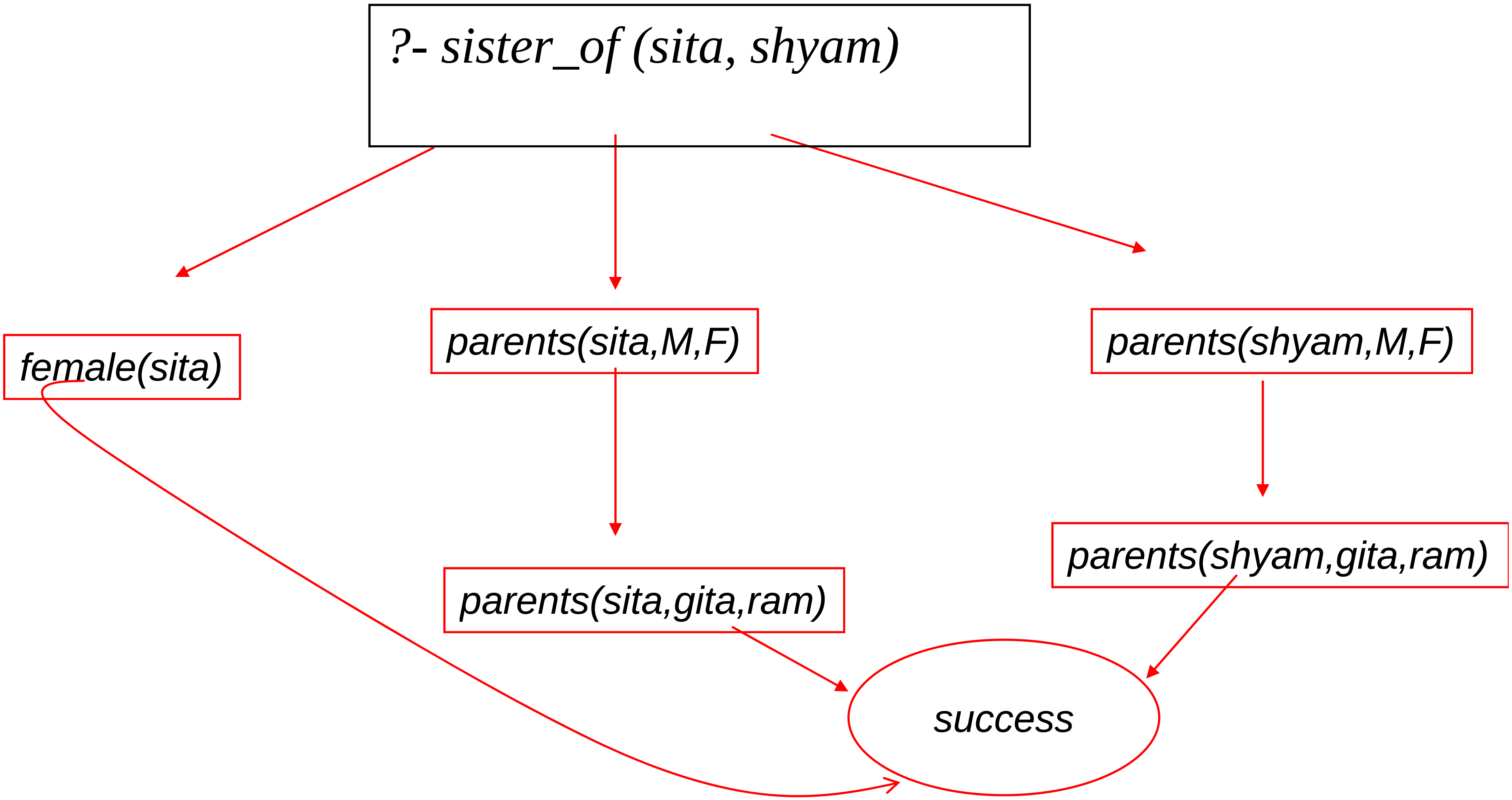
sister_of (X,Y):- female (X),
parents (X, M, F),
parents (Y, M, F).

X is a sister of Y is
X is a female and
X and Y have same parents

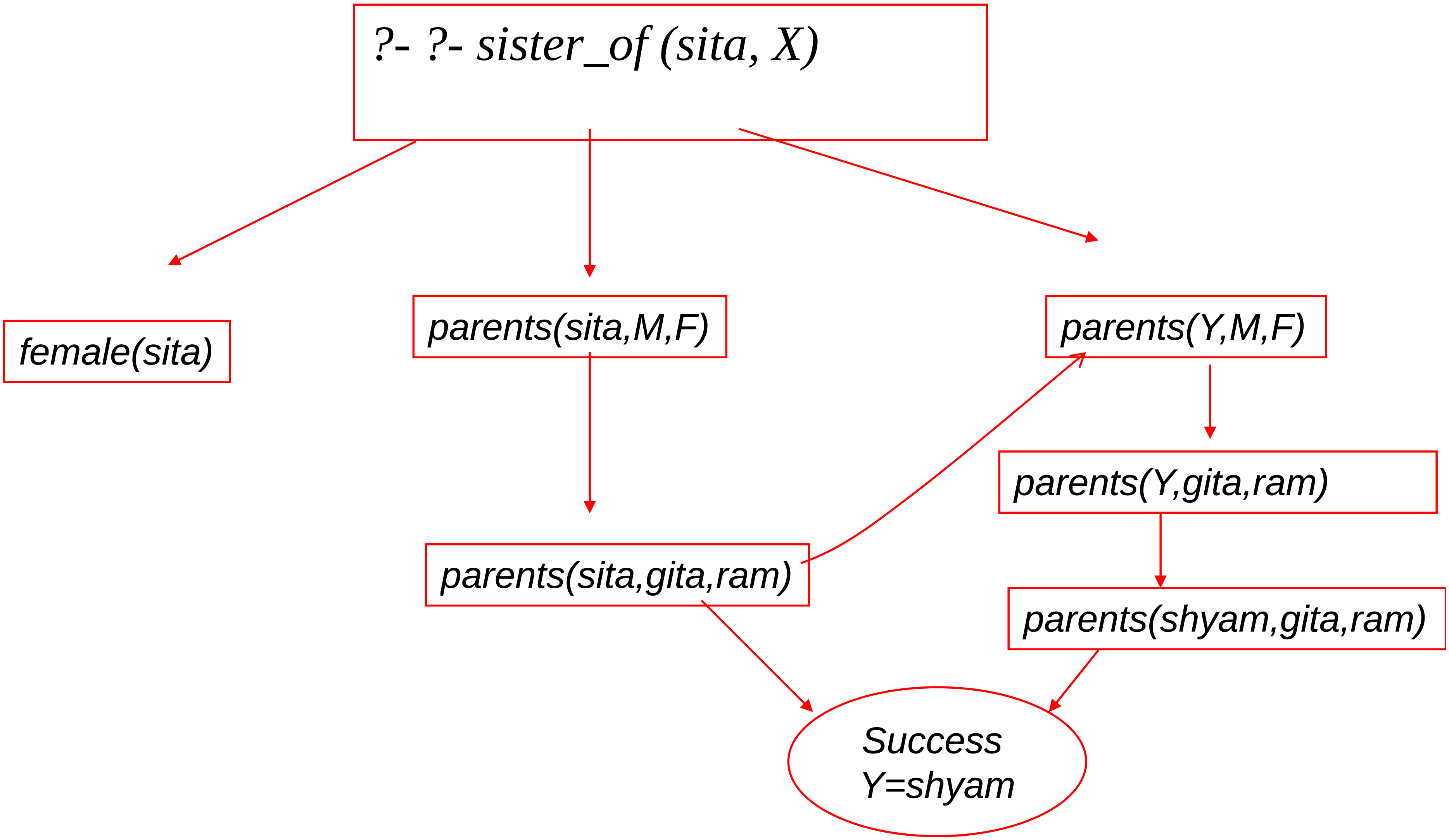
Question Answering in presence of *rules*

- Facts
  - male (ram).
  - male (shyam).
  - female (sita).
  - female (gita).
  - parents (shyam, gita, ram).
  - parents (sita, gita, ram).

Question Answering: Y/N type: *is sita the sister of shyam?*



Question Answering: wh-type: *whose sister is sita?*



Rules

- Statements about *objects* and their *relationships*
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    - *I use an umbrella if there is a rain*
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Make and Break

Fundamental to Prolog

Prolog examples using making and breaking lists

```
%incrementing the elements of a list to produce another list
incr1([],[]).
incr1([H1|T1],[H2|T2]) :- H2 is H1+1, incr1(T1,T2).

%appending two lists; (append(L1,L2,L3) is a built is
function in Prolog)
append1([],L,L).
append1([H|L1],L2,[H|L3]):- append1(L1,L2,L3).


%reverse of a list (reverse(L1,L2) is a built in function
reverse1([],[]).
reverse1([H|T],L):- reverse1(T,L1),append1(L1,[H],L).
```

Remove duplicates

Problem: to remove duplicates from a list

```
rem_dup([],[]).
rem_dup([H|T],L) :- member(H,T), !, rem_dup(T,L).
rem_dup([H|T],[H|L1]) :- rem_dup(T,L1).
```

Note: The cut ! in the second clause needed, since after succeeding at member(H,T), the 3rd clause should not be tried even if rem_dup(T,L) fails, which prolog will otherwise do.

Member (membership in a list)

```
member(X,[X|_]).
member(X,[_|L]):- member(X,L).
```

Union (lists contain unique elements)

```
union([],Z,Z).
union([X|Y],Z,W):-
  member(X,Z),!,union(Y,Z,W).
union([X|Y],Z,[X|W]):- union(Y,Z,W).
```

Intersection (lists contain unique elements)

```
intersection([],Z,[]).
intersection([X|Y],Z,[X|W]):-
    member(X,Z),!,intersection(Y,Z,W).
intersection([X|Y],Z,W):-
    intersection(Y,Z,W).
```

Prolog Programs are close to Natural Language

Important Prolog Predicate:

member(e, L) / true if e is an element of list L*

member(e,[e|L1). / e is member of any list which it starts*

*member(e,[_|L1]):- member(e,L1) /*otherwise e is member of a list if the tail of the list contains e*

Contrast this with:

P.T.O.

Prolog Programs are close to Natural Language, C programs are not

```
For (i=0;i<length(L);i++){
    if (e==a[i])
        break(); /*e found in a[]
}
If (i<length(L){
    success(e,a); /*print location where e appears in
        a[]/*
else
    failure();
}
```

What is *i* doing here? Is it natural to our thinking?

Machine should ascend to the level of man

- A prolog program is an example of reduced man-machine gap, unlike a C program
- That said, a very large number of programs far outnumbering prolog programs gets written in C
- The demand of practicality many times incompatible with the elegance of ideality
- But the ideal should nevertheless be striven for

Prolog Program Flow, BackTracking and Cut

Controlling the program flow

Prolog’s computation

- **Depth First Search**
  - **Pursues a goal till the end**
- **Conditional AND; *falsity* of any goal prevents satisfaction of further clauses.**
- **Conditional OR; *satisfaction* of any goal prevents further clauses being evaluated.**

Control flow (top level)

Given

g:- a, b, c. (1)

g:- d, e, f; g. (2)

If prolog cannot satisfy (1), control will automatically fall through to (2).

Control Flow within a rule

Taking (1),

g:- a, b, c.

If *a* succeeds, prolog will try to satisfy *b,* succeding which *c* will be tried.

For ANDed clauses, control flows forward till the '.', iff the current clause is *true.*

For ORed clauses, control flows forward till the '.', iff the current clause evaluates to *false.*

What happens on failure

- **REDO the immediately preceding goal.**

Fundamental Principle of prolog programming

- **Always place the more general rule AFTER a specific rule.**

CUT

- **Cut tells the system that**

  IF YOU HAVE COME THIS FAR

  DO NOT BACKTRACK

  EVEN IF YOU FAIL SUBSEQUENTLY.

  ‘CUT’ WRITTEN AS ‘!’ ALWAYS SUCCEEDS.

Fail

- This predicate always fails.
- *Cut* and *Fail* combination is used to produce negation.
- Since the LHS of the neck cannot contain any operator, *A* → *~B* is implemented as

  B :- A, !, Fail.