A Computational Framework for the Boundary Representation of Solid Sweeps

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Solid Sweep

Given a solid M in brep format and a one parameter family of rigid motions h, compute the volume $\mathcal{V}$ swept by M as a brep.

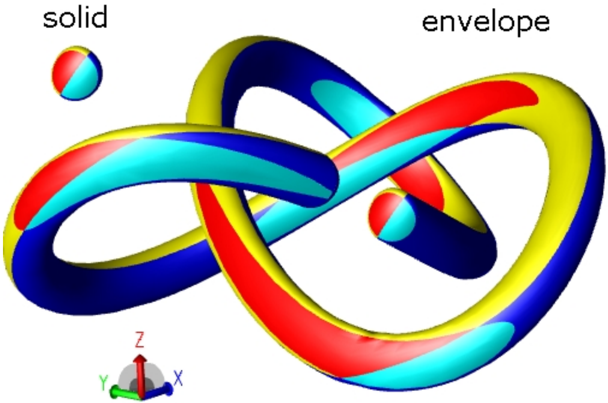


Figure: A solid swept along a trefoil knot.

Application in product handling

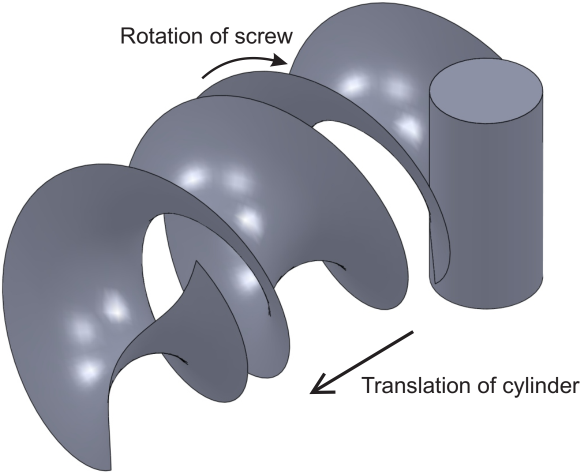
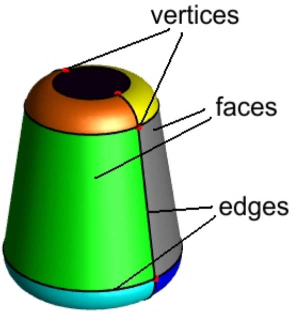


Figure: Conveyor screw.

The boundary representation



- **Geometric data:** Parametric definitions of faces, edges and vertices.
- **Topological data:** Orientation of faces and edges. Ajdacency relations amongst geometric entities.

Outline of the talk

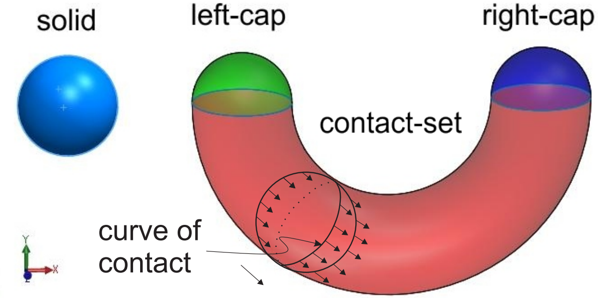
- When introducing a new surface type in a CAD kernel
  - Parametrization: Local aspects
  - Topology: Global aspects
  - Self-intersection: Global aspects
- Parametrization: **Funnel**
- Self-intersection: **Trim curves**.
- Topology: **Local homeomorphism** between solid and envelope.
- We focus on parametrization and topology in this talk.

The envelope condition

- **Trajectory**
  $h : I \to (SO(3), \mathbb{R}^3)$, $h(t) = (A(t), b(t))$.
- **Trajectory of a point** x under h
  $\gamma_x : I \to \mathbb{R}^3$, $\gamma_x(t) = A(t) \cdot x + b(t)$.
- Define $g : \partial M \times I \to \mathbb{R}$ as $g(x, t) = \langle A(t) \cdot N(x), \gamma_x'(t) \rangle$.
- **Curve of contact** at t
  $C(t) = \{\gamma_x(t) \in \partial M(t) | g(x, t) = 0\}$.
- For $I = [t_0, t_1]$, the necessary condition for $\gamma_x(t)$ to belong to envelope $\mathcal{E}$:
  - If $t = t_0$ then $g(x, t) \leq 0$: **Left-cap**
  - If $t = t_1$ then $g(x, t) \geq 0$: **Right-cap**
  - If $t \in (t_0, t_1)$ then $g(x, t) = 0$: **Contact set**

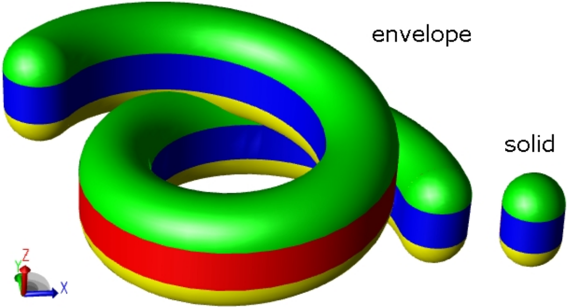
The envelope condition

A point $\gamma_x(t)$ belongs to the contact-set only if the velocity $\gamma'_x(t)$ is tangent to ∂M at $\gamma_x(t)$.



Simple sweeps

In general, the contact set needs to be trimmed to obtain the envelope.



Assume sweep (M, h) to be *simple*, i.e., no trimming of contact set required to obtain $\mathcal{E}$.

Algorithm 1 Solid sweep

for all faces F in ∂M **do**
    for all co-edges e in ∂F **do**
        for all vertices z in ∂e **do**
            Compute vertices C^z generated by z
        end for
        Compute co-edges C^e generated by e
        Orient co-edges C^e
    end for
    Compute $C^F(t_0)$ and $C^F(t_1)$
    Compute loops bounding faces C^F generated by F
    Compute faces C^F generated by F
    Orient faces C^F
end for
for all F_i, F_j adjacent in ∂M **do**
    Compute adjacencies between faces in C^{F_i} and C^{F_j}
end for

Parametrization

Parametrization of envelope

- Parametric surface $S : \mathbb{R}^2 \to \mathbb{R}^3$, $S(D) = F \subseteq \partial M$.
- Define $f : D \times I \to \mathbb{R}$ as $f(u, v, t) = g(S(u, v), t)$
- **Funnel**: $\mathcal{F}^F = \{(u, v, t) \in D \times I | f(u, v, t) = 0\}$.
- **Parametrization map**: $\sigma^F : \mathcal{F}^F \to C^F$,
  $\sigma(u, v, t) = A(t) \cdot S(u, v) + b(t)$.

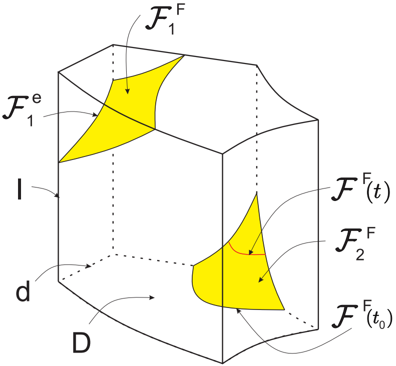


Figure: In this example, the funnel has two components, shaded in yellow.

The natural correspondence between $\mathcal{E}$ and ∂M

Correspondence $\pi : \mathcal{E} \to \partial M$, $\pi(y) = x$ such that $\gamma_x(t) = y$ for some $t \in I$.

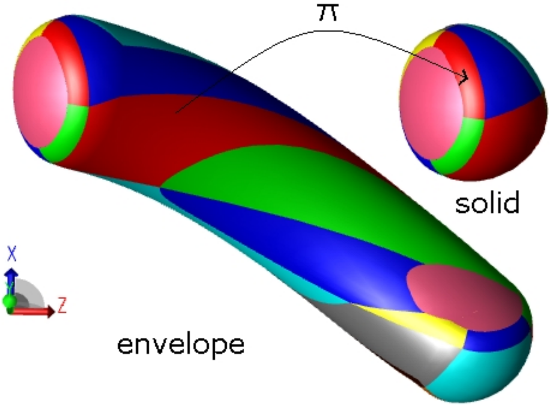


Figure: The points y and $\pi(y)$ are shown in same color.

Adjacency relations

Issues related to brep: Adjacency relations

A face F of ∂M may give rise to multiple faces on $\mathcal{E}$.

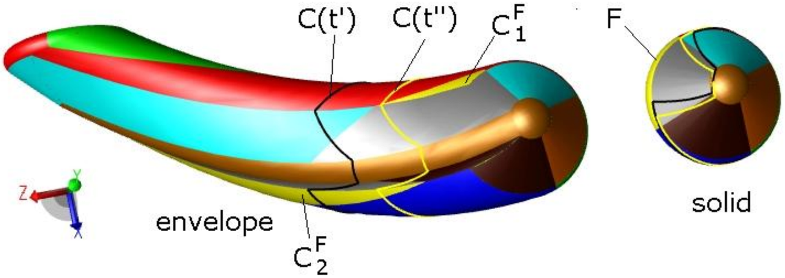
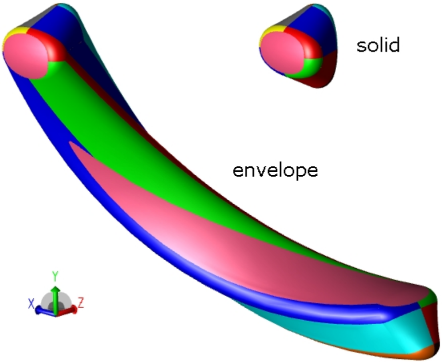


Figure: The face $F \subset \partial M$ generates two faces, viz., C_1^F and C_2^F on envelope. Curves of contact at two time instants are shown imprinted on $\mathcal{E}$ and ∂M.

Issues relate to brep: Adjacency relations

Theorem: The map $\pi : \mathcal{E} \to \partial M$ is a local homeomorphism almost everywhere on $\mathcal{E}$. If C_i^F and $C_j^{F'}$ are adjacent in $\mathcal{E}$, then F and F' are adjacent in ∂M. If a co-edge C_i^e bounds a face C_j^F in C then the co-edge e bounds the face F in ∂M.



While the global brep structures of $\mathcal{E}$ and ∂M are different, locally they are similar.

Computing loops bounding faces C^F

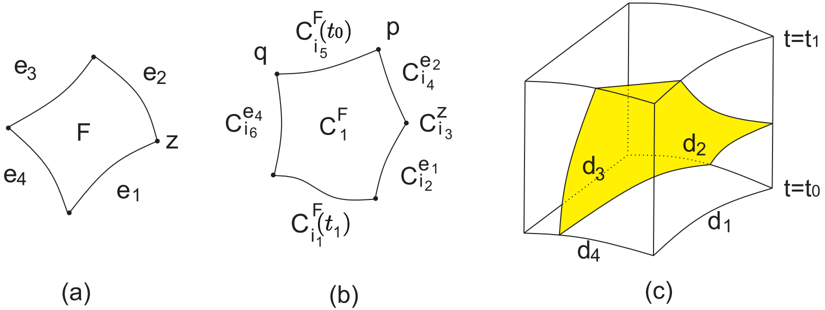


Figure: (a) A face F of ∂M bound by four co-edges. (b) A corresponding face C^F_1. (c) Prism with domains d_i for co-edges e_i.

Orientation

Issues related to brep: Orientation

The map $\pi : \mathcal{E} \to \partial M$ is orientation preserving if $-f_t > 0$ and reversing if $-f_t < 0$.

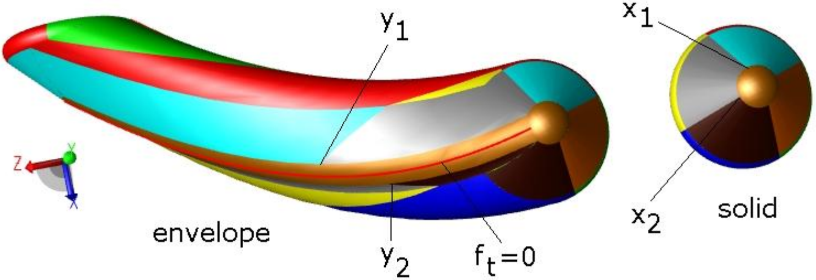


Figure: Here $\pi(y_i) = x_i$. The map π is orientation preserving at y_2 and reversing at y_1. The curve $f_t = 0$ is shown in red.

Orienting co-edges

For a co-edge e bounding a face F of ∂M, let $y \in C_i^e \subset C^F$ and $\pi^F(y) = x \in e$. Let $p \in \mathcal{F}^e \subset \mathcal{F}^F$ such that $\sigma^F(p) = y$ and $\bar{z}$ be the orientation of e. If $-f_t^F(p) > 0$ then $J_{\pi^F}^{-1} \cdot \bar{z}$ is the orientation of C_i^e and if $-f_t^F(p) < 0$ then $-J_{\pi^F}^{-1} \cdot \bar{z}$ is the orientation of C_i^e.

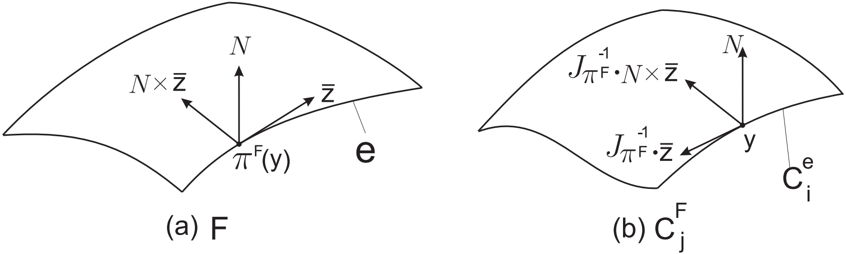
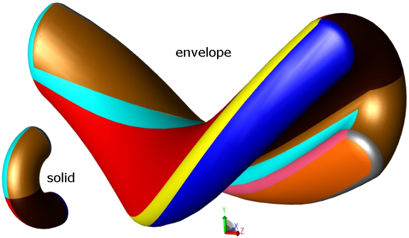
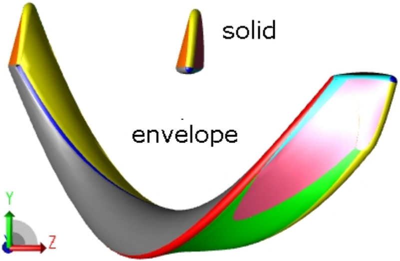
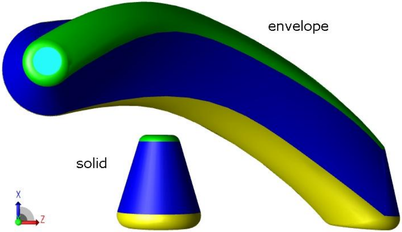


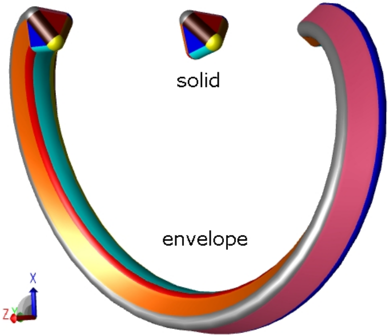
Figure: In this examples, $-f_t^F$ is negative at the point y.

Examples


envelope
solid


solid
envelope
Y
Z


envelope
solid


solid
envelope
X
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Trimming in non-simple sweeps

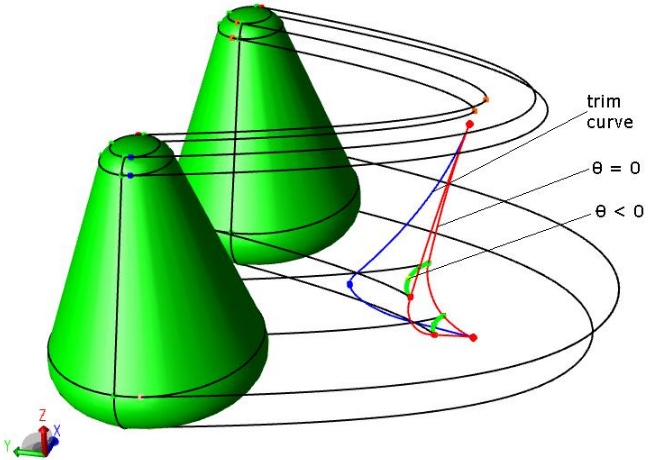

Figure: A cone being swept along a parabola. The trim curve, shown in blue, meets the zero locus of an invariant function θ, shown in red.

Sweeps with sharp features

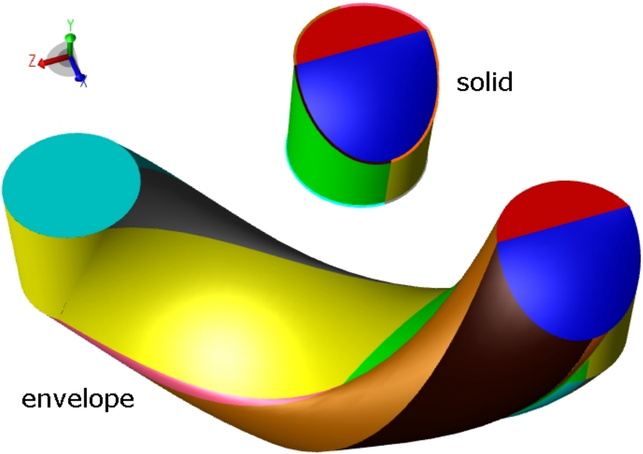

Figure: A sharp edge will generate a set of faces and a sharp vertex will generate a set of edges on the envelope.

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Thank You