Dijkstra's Self stabilization

CS 451 Lecture 5
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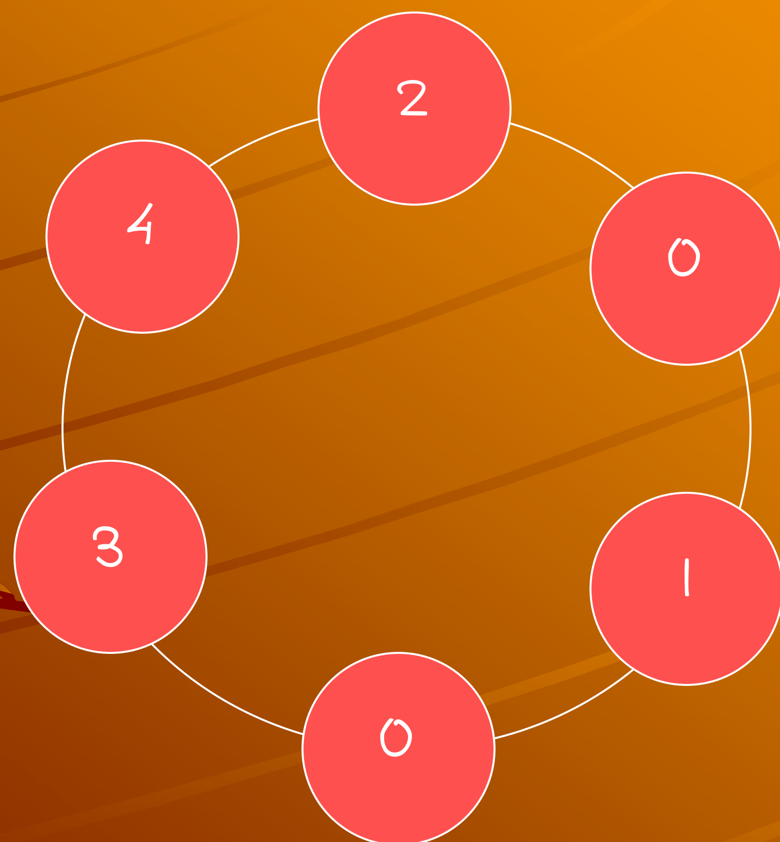
Self Stabilization

- Self correcting systems
- Local nodes may corrupt their local states
- System heals itself/stabilizes after a bounded number of iterations/steps

Connected network of processors, and their states



Privileges

- Boolean functions of its own state and of its neighbours' states
- When such a boolean function is true, we say that a privilege is 'present'
- Example

  If (S = L) S = S+1

Selecting Privileges for execution

- From among many priviledges which may be present in the entire system, only one is selected for execution
- i.e. imagin a central scheduler that picks up any one privilege from among those present (nondeterministically)

- After the execution of selected privilege, the system again continues the same way

Legitimate State

- Is the system as a whole in 'legitimate state'?
- Requirements:

  In each legitimate state, one or more privileges are present

  In each legitimate state, each possible move brings the system back to a legitimate state

  Each privilege must be present at least in one legitimate state

  For any pair of legitimate states, there exists a sequence of moves transferring the system from one into another

When is a system self stabilizing?

- If and only if
  Regardless of the privilege selected each time for the next move, at least one privilege is always present

  And

  The system is a guaranteed to enter legitimate state in a finite number of moves

Do non-trivial Self stabilizing systems exist?

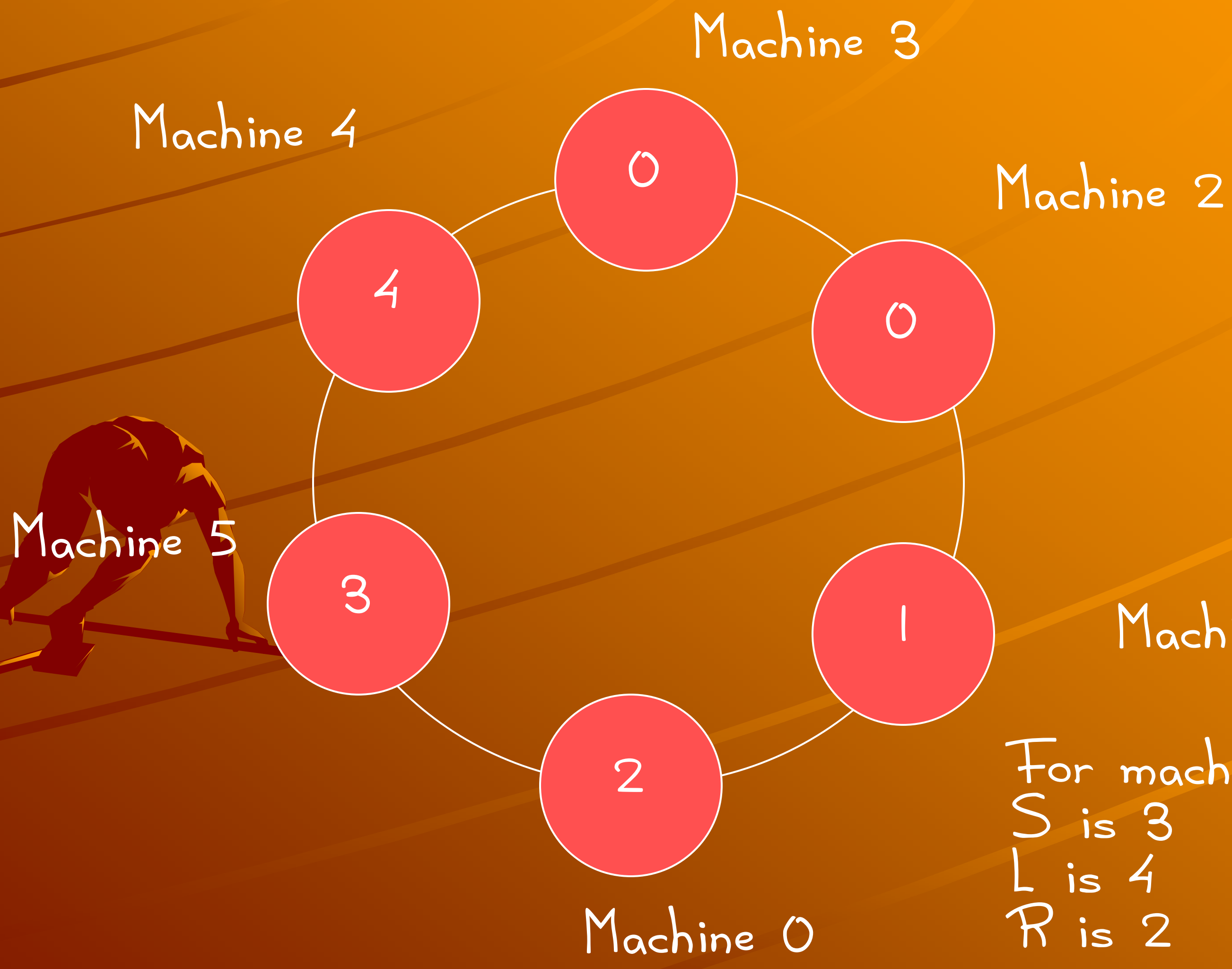
- How about all states as legitimate states?
- Not obvious whether local moves lead to can assure convergence to global ligitimacy
- Edsger Dijkstra presented 3 machines in his paper (CACM November 1974)

Machine description

- N+1 machines (0..N)
- L: State of left neighbour
  (i-i) mod (N+1)

- R: State of right neighbour
  (i+1) mod (N+1)
- S: State of itself

Connected network of processors, and their states



For machine 5,
S is 3
L is 4
R is 2

Privilege description

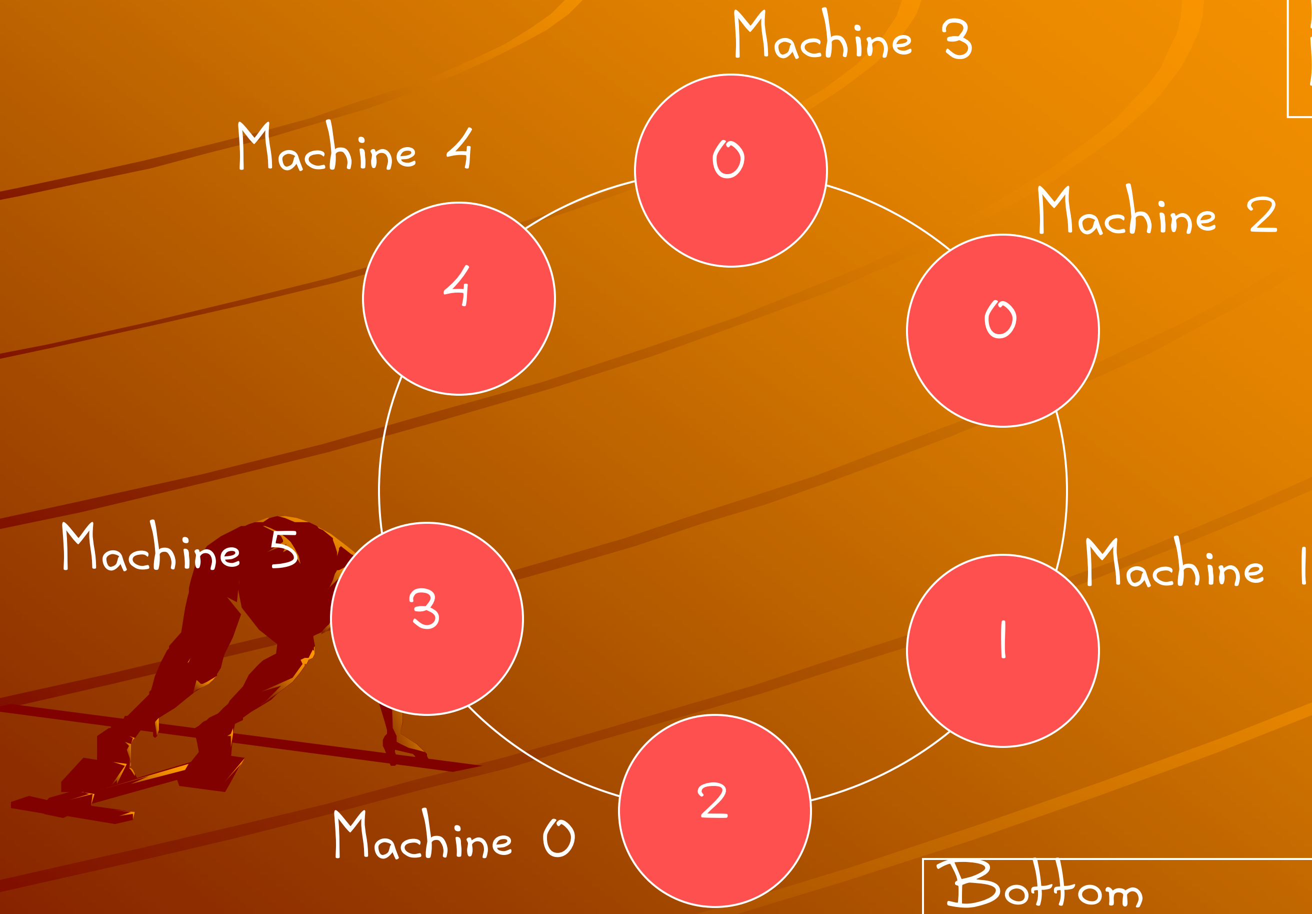
If _privilege_ then _corresponding move_

K State machine with exactly one privilege per machine

- Bottom (machine 0)
  If L=S then S=s+1 mod K

- Others
  If L != S then S=L

K State machine

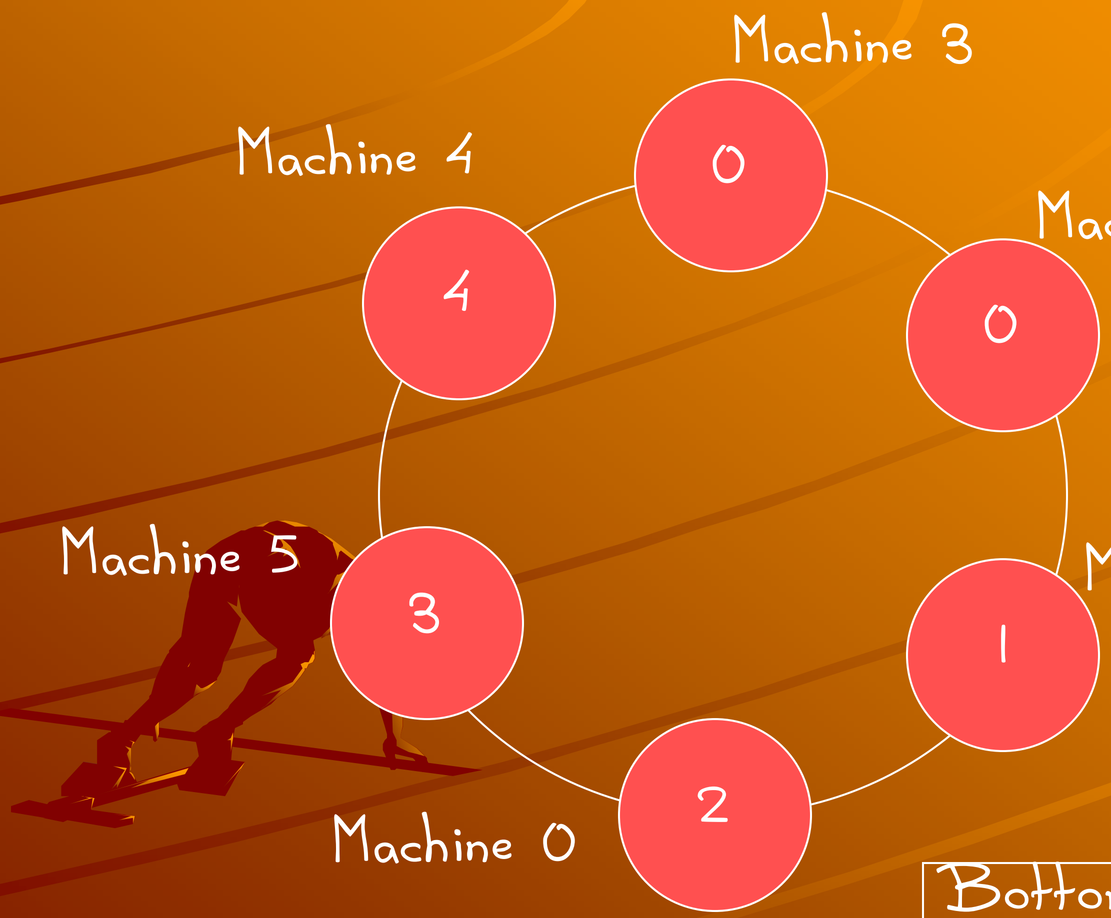


N=5
K=5

Bottom
if (S=L) S=L+1 mod K fi
Others
if (S!=L) S=L fi

Who has privileges?

N=4
K=5



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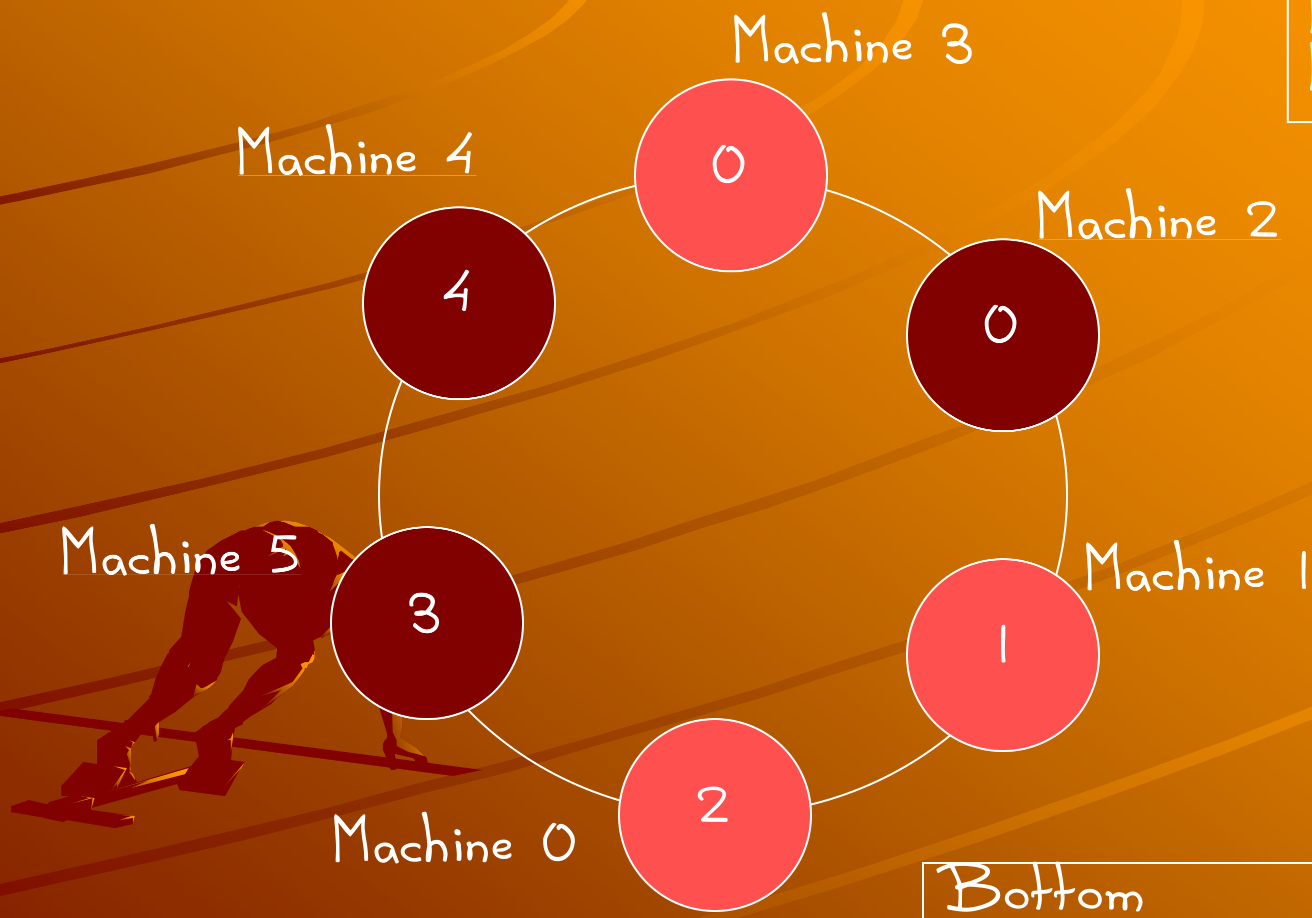
Who has privileges?



N=4
K=5

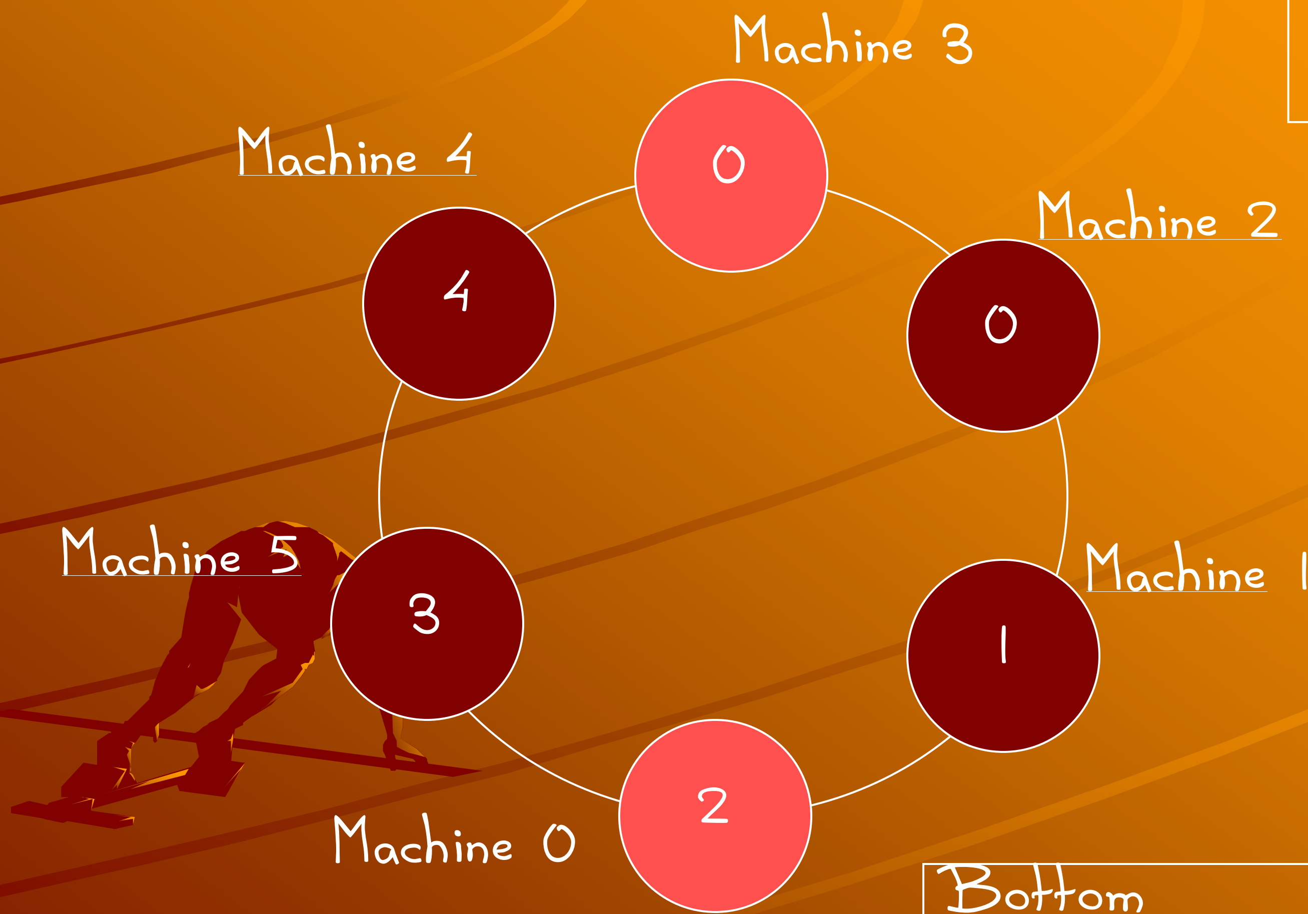
Is this correct?

Bottom
if (S=L) S=L+1 mod K fi

Others
if (S!=L) S=L fi

Who has privileges?

N=4
K=5

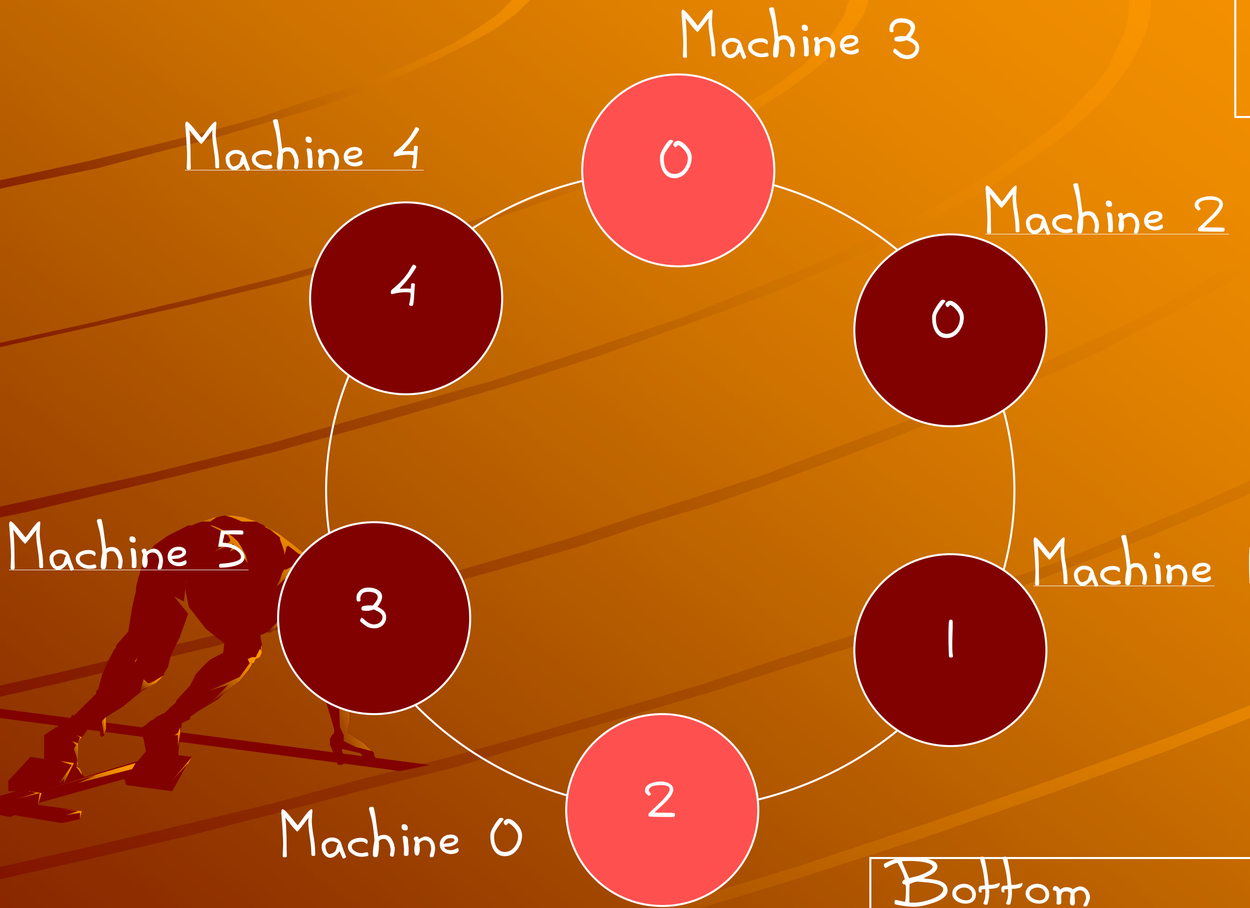


Bottom
    if (S=L) S=L+1 mod K fi
Others
    if (S!=L) S=L fi

Let's select one of the privileges nondeterministically through a central daemon

Selecting machine 2's move..

N=4
K=5



Bottom
if (S=L) S=L+1 mod K fi
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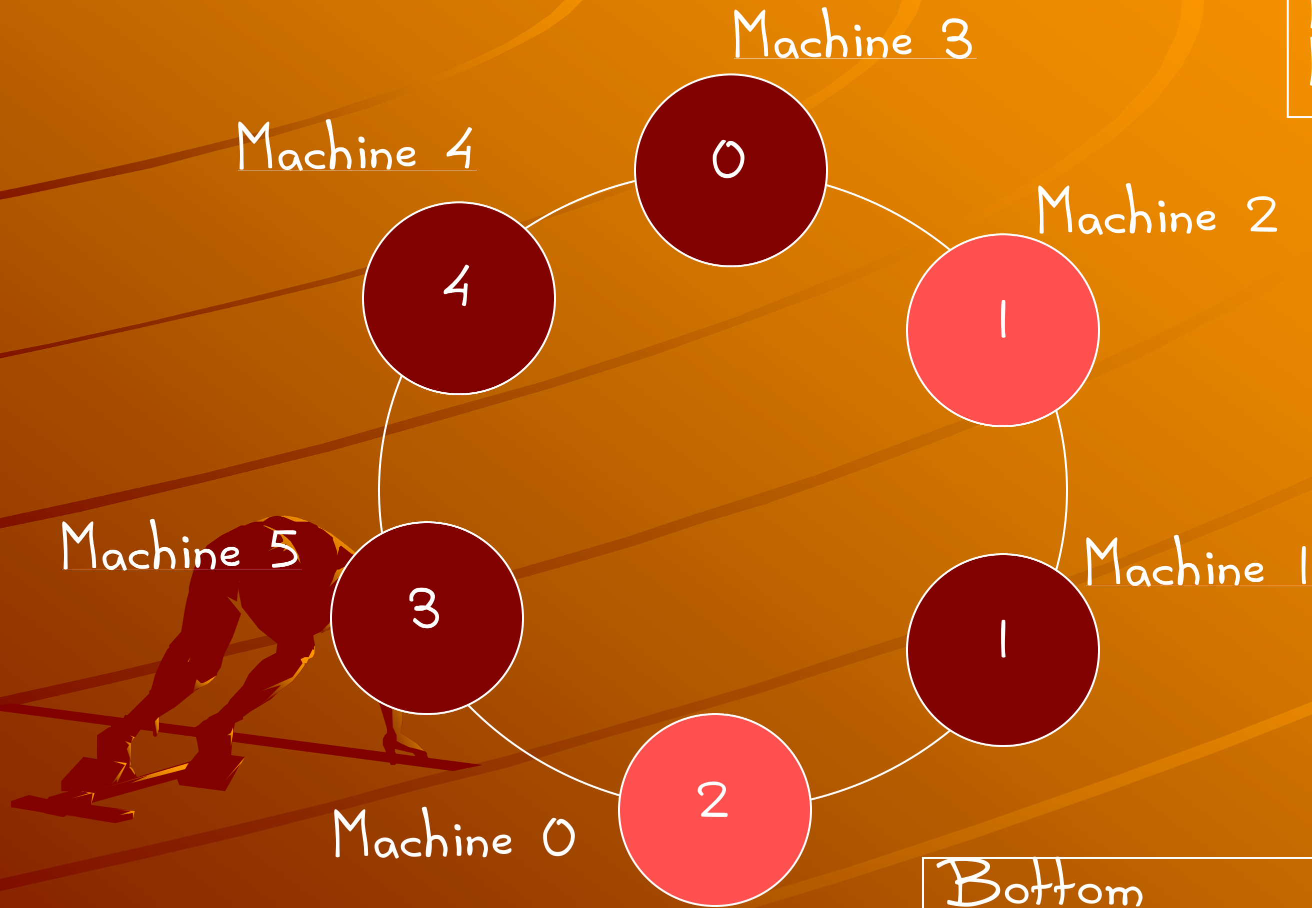
After machine 2's move

N=4
K=5



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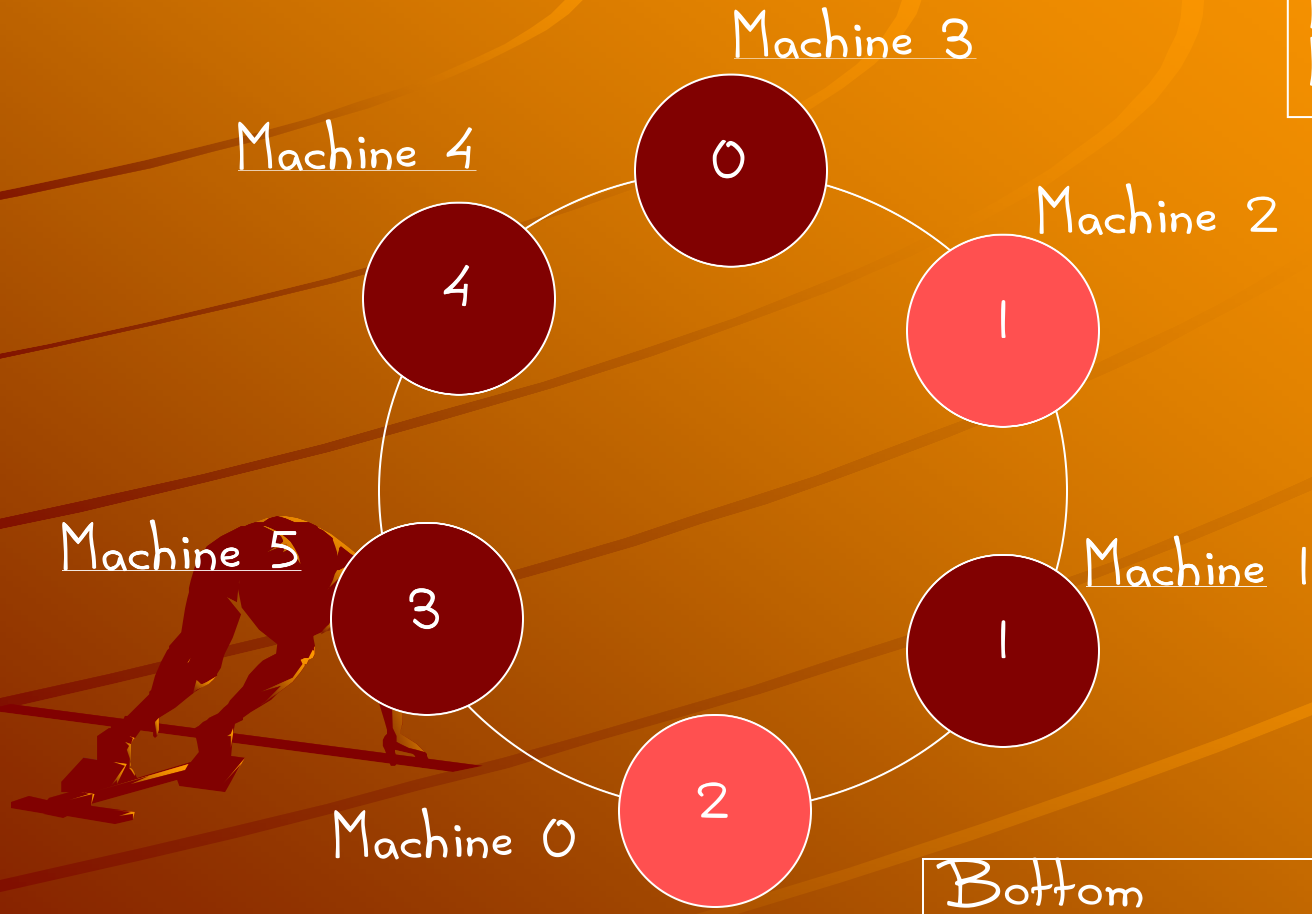
Selecting machine 5's move..

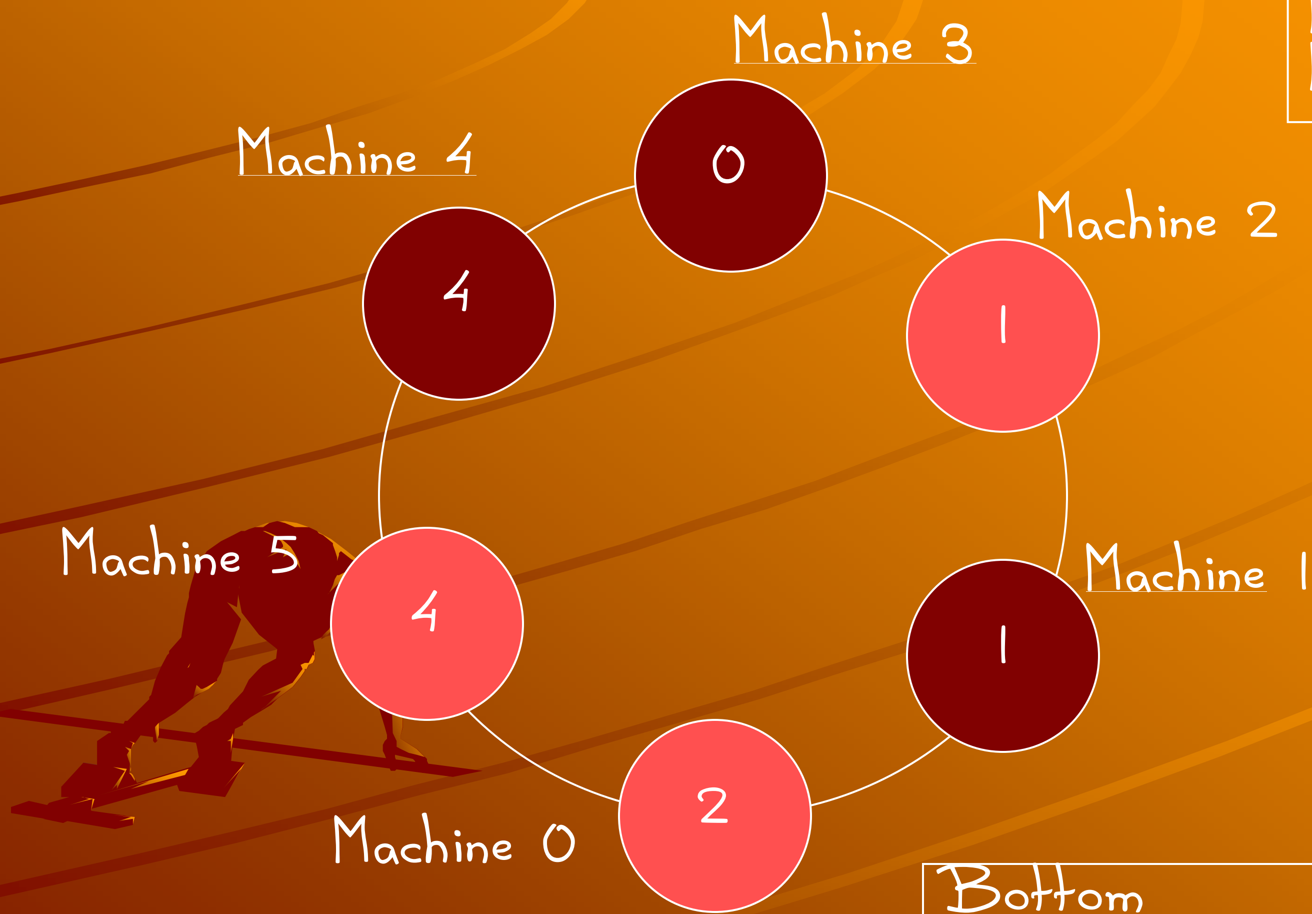


N=4
K=5

Bottom
if (S=L) S=L+1 mod K fi
Others
if (S!=L) S=L fi

After machine 5's move..

N=4
K=5



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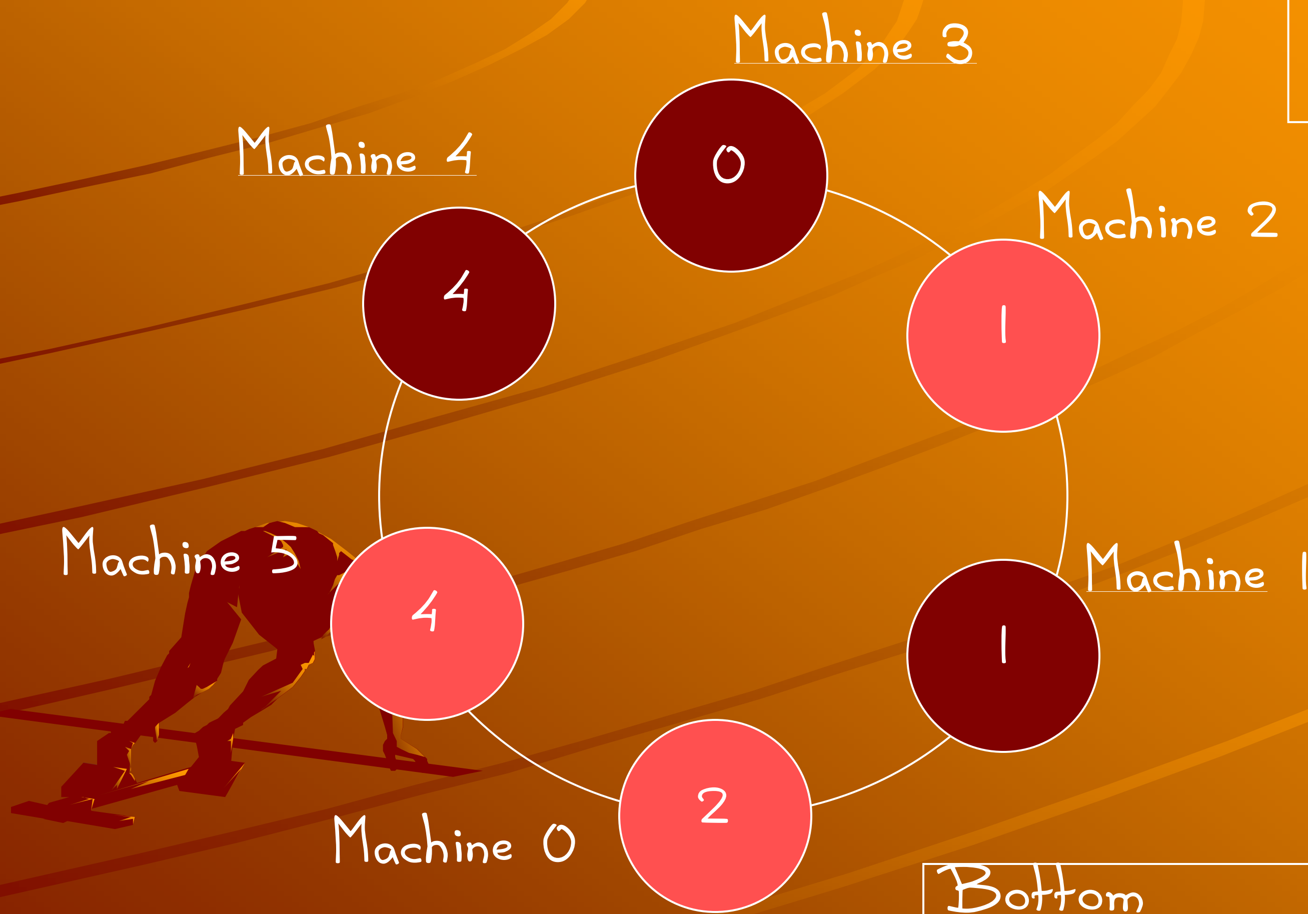
Selecting machine 1's move..



N=4
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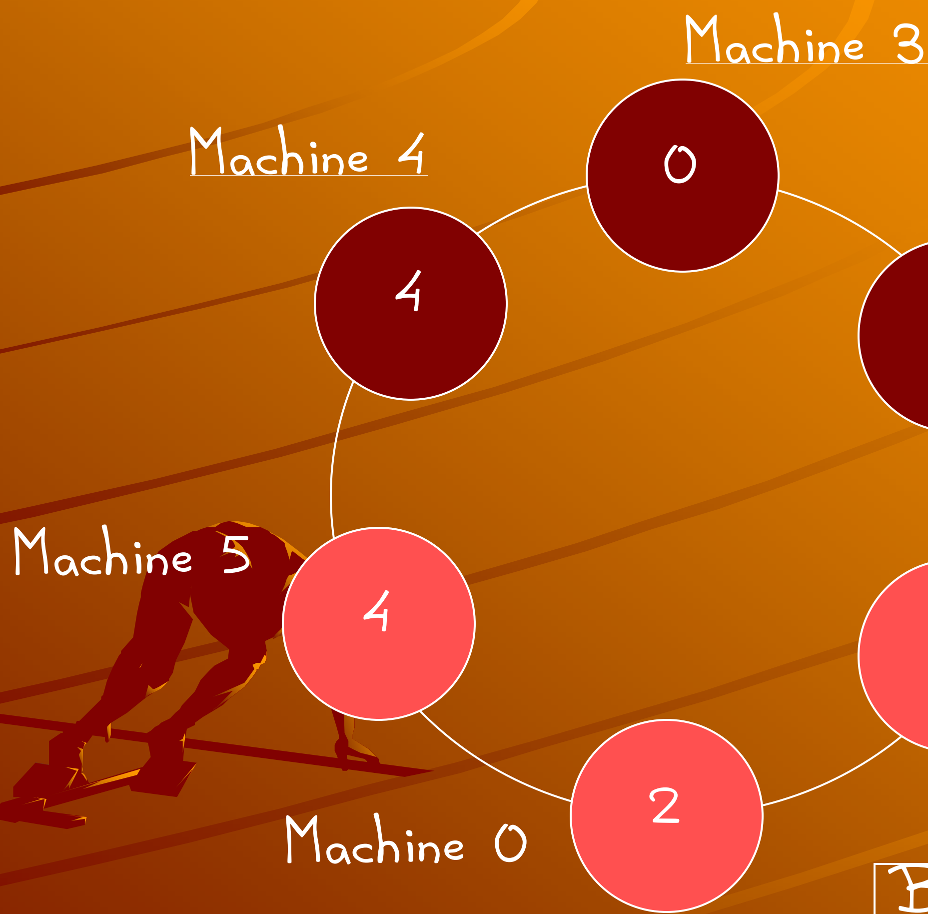
After machine 1's move..

N=4
K=5



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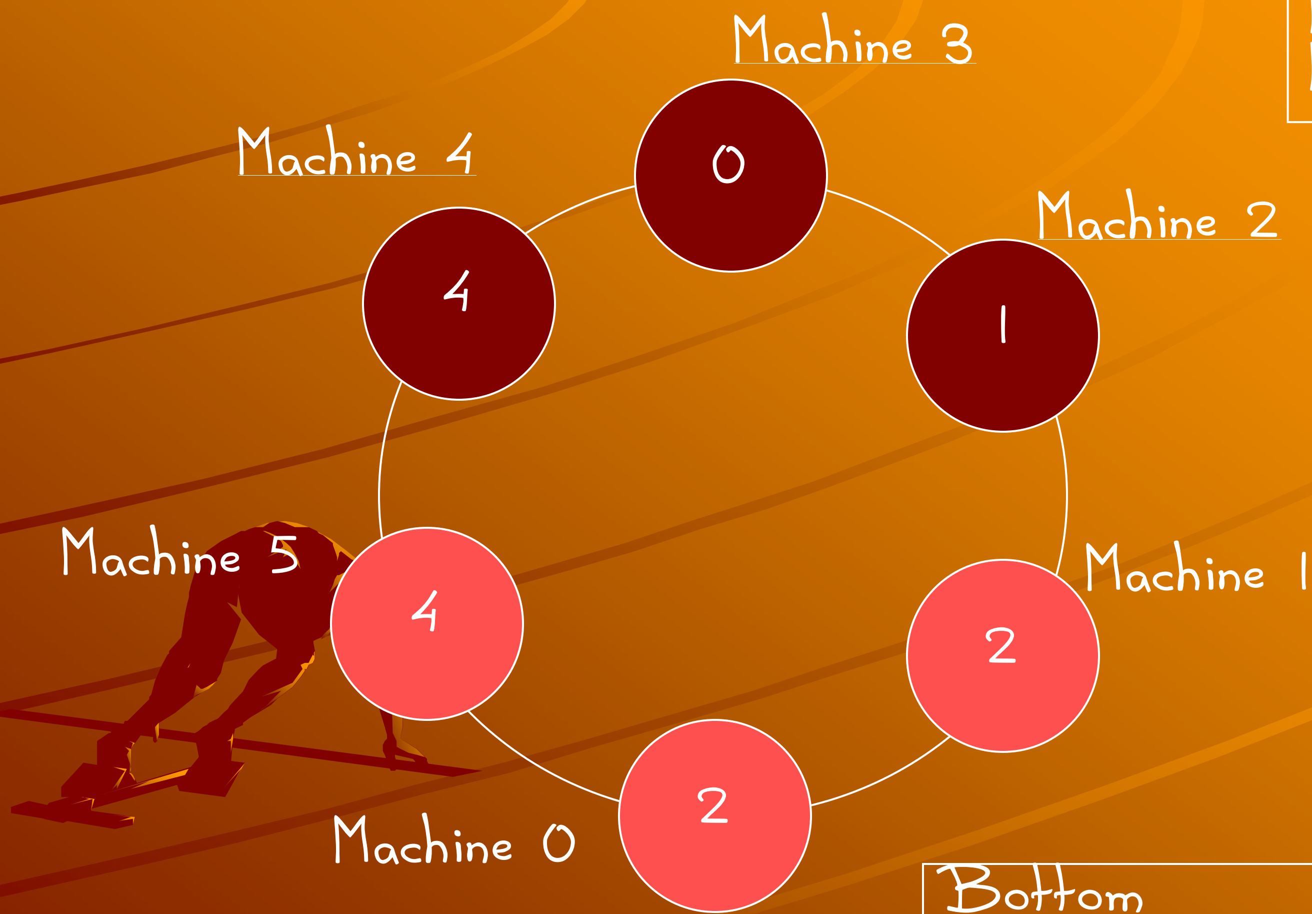
if (S=L) S=L+1 mod K fi

Others
if (S!=L) S=L fi

Selecting machine 4's move..

N=4
K=5

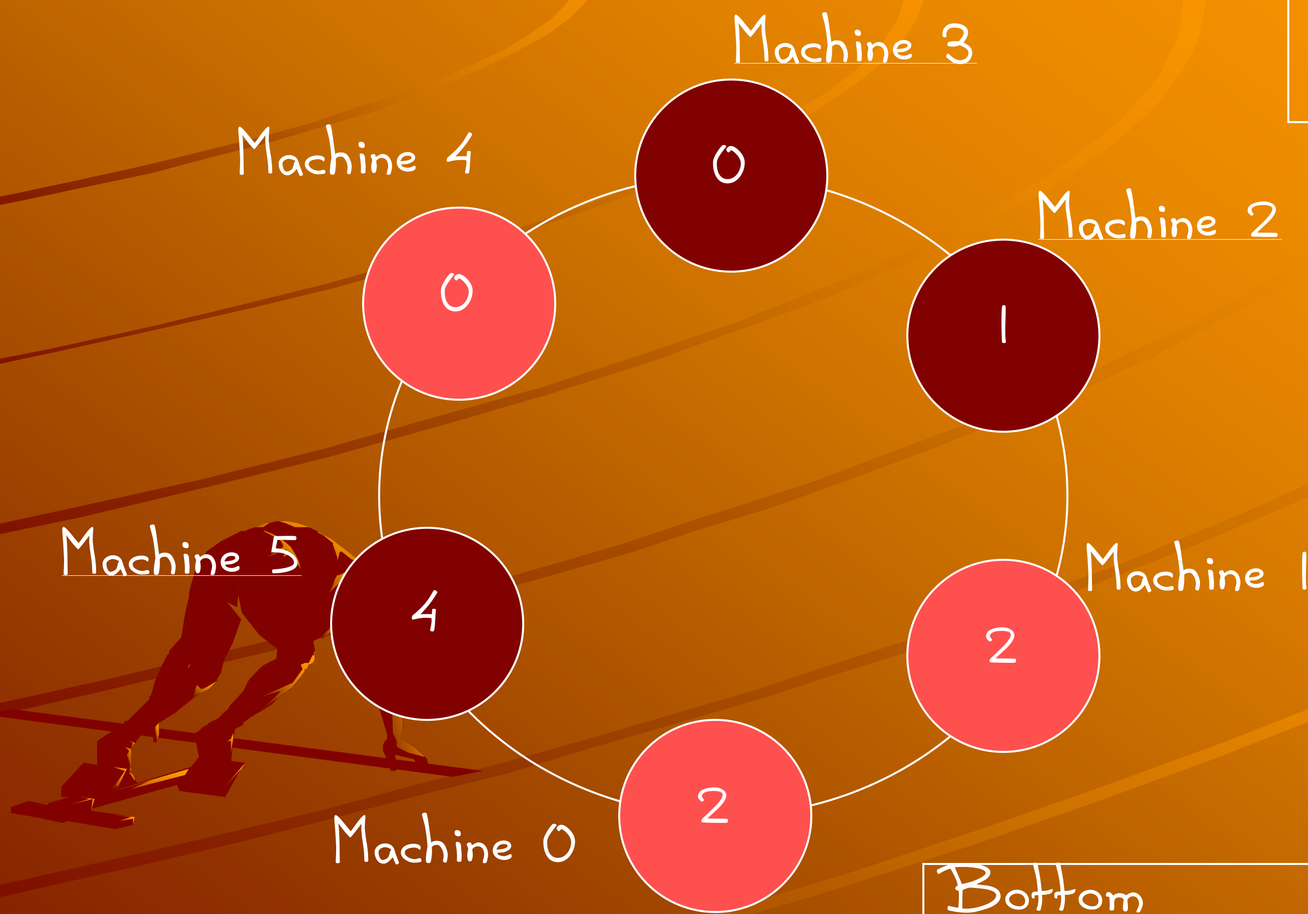


Bottom
if (S=L) S=L+1 mod K fi

Others
if (S!=L) S=L fi

After machine 4's move..

N=4
K=5



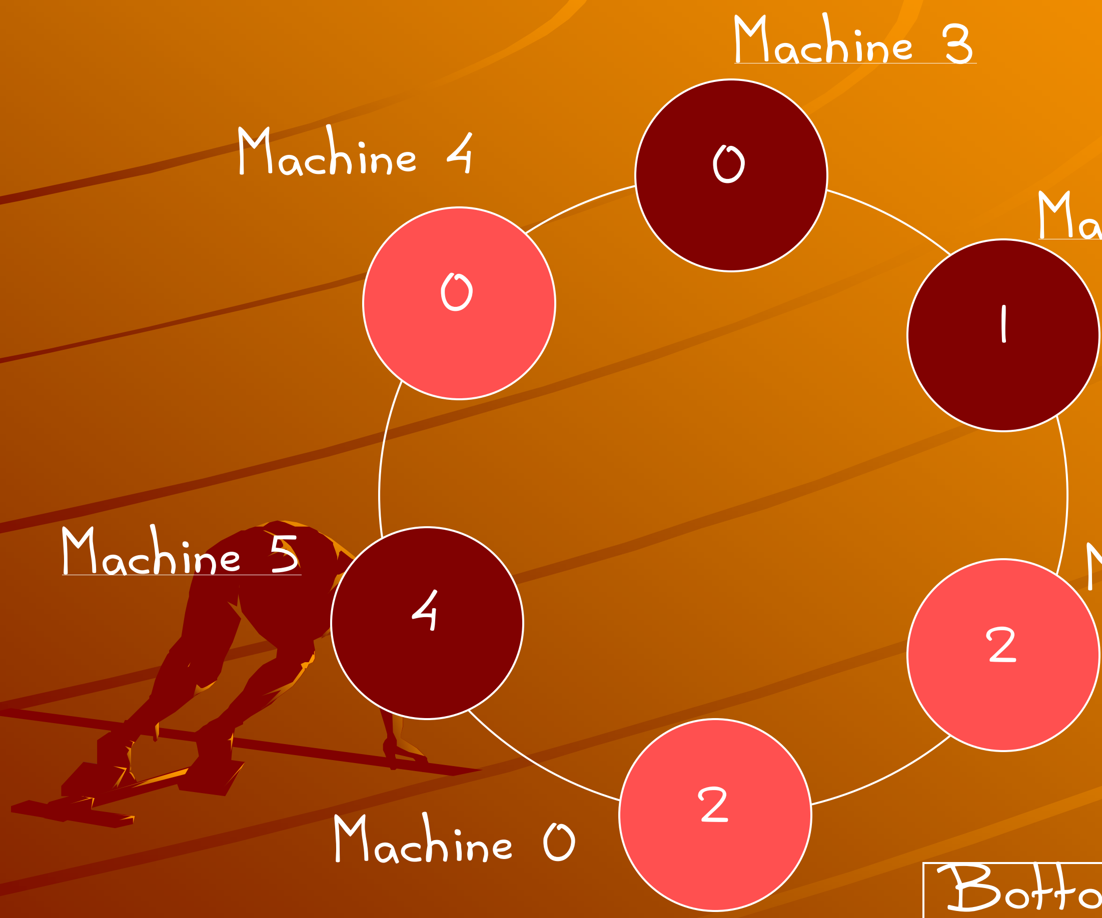
Bottom

if (S=L) S=L+1 mod K fi

Others

if (S!=L) S=L fi

Selecting machine 2's move..



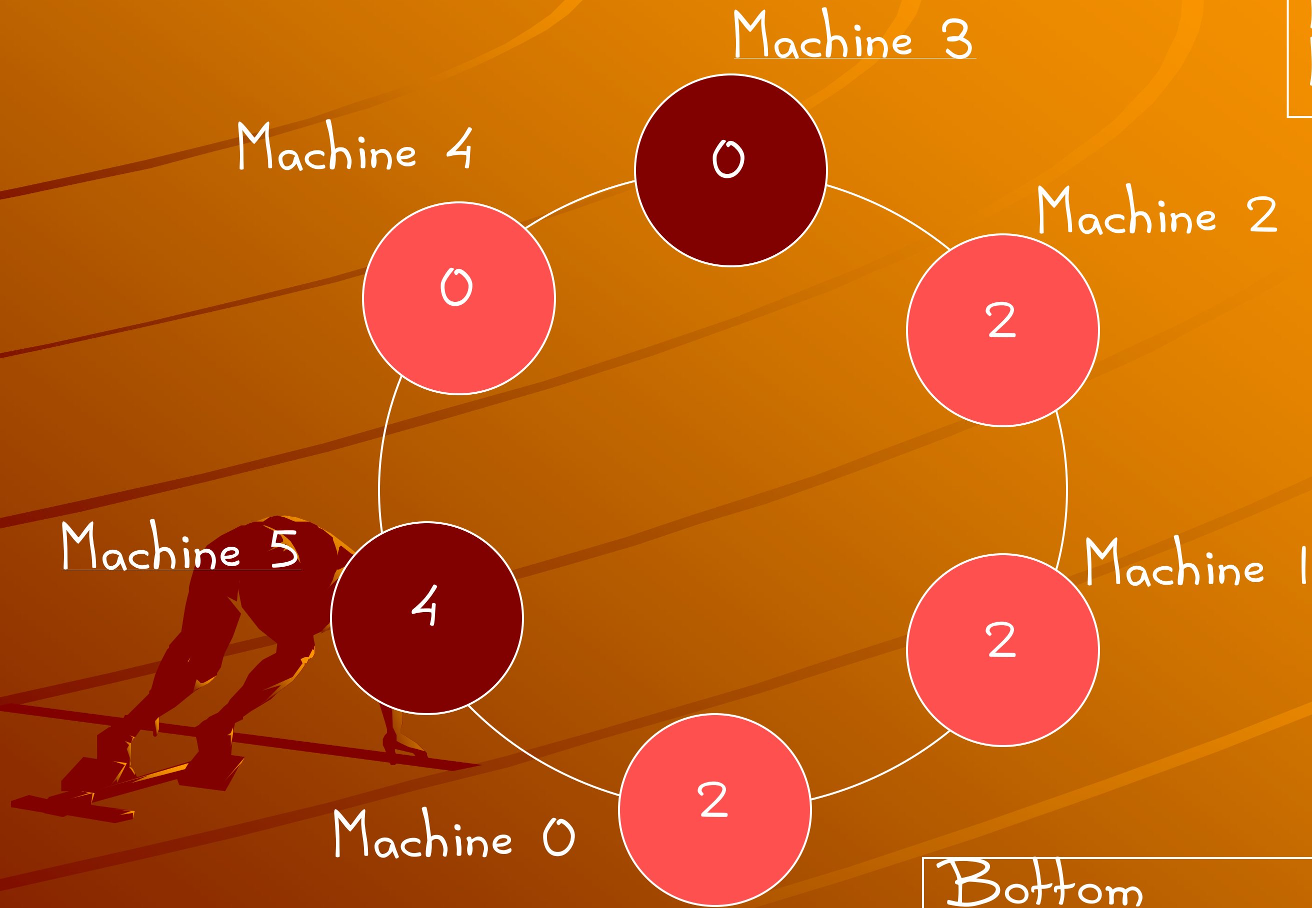
N=4
K=5

Bottom
if (S=L) S=L+1 mod K fi

Others
if (S!=L) S=L fi

After machine 2's move..

N=4
K=5



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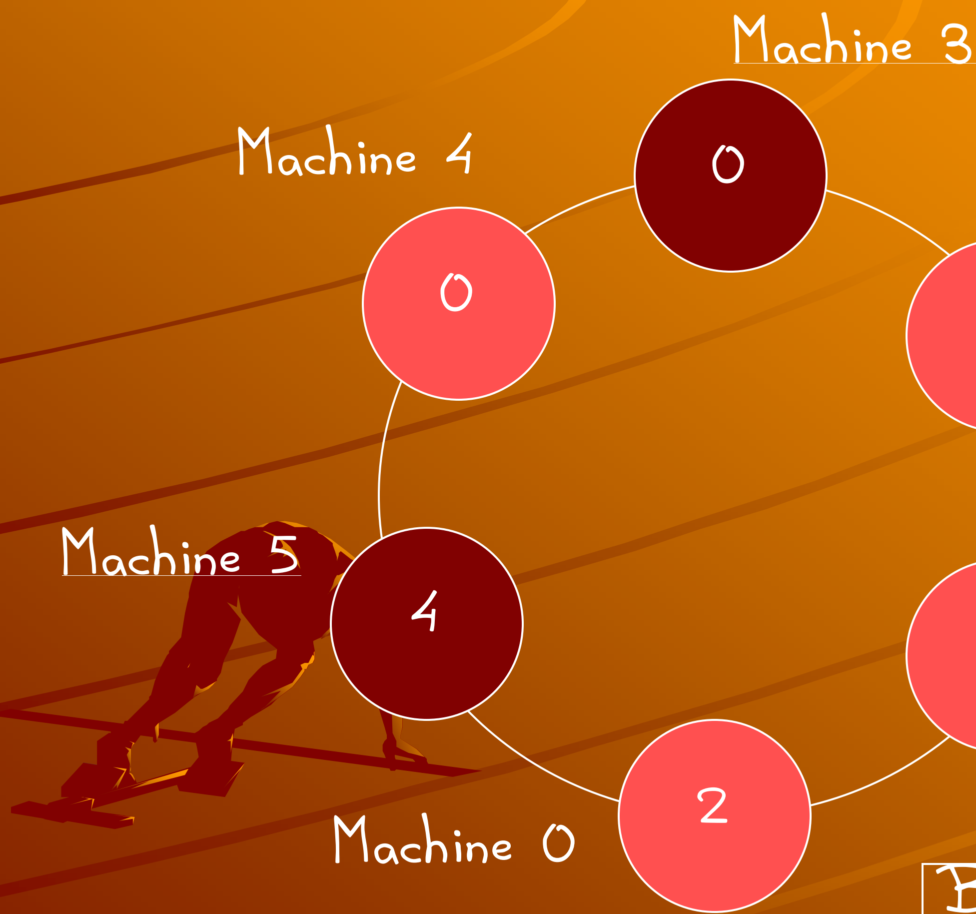
if (S!=L) S=L fi

Selecting machine 3's move..

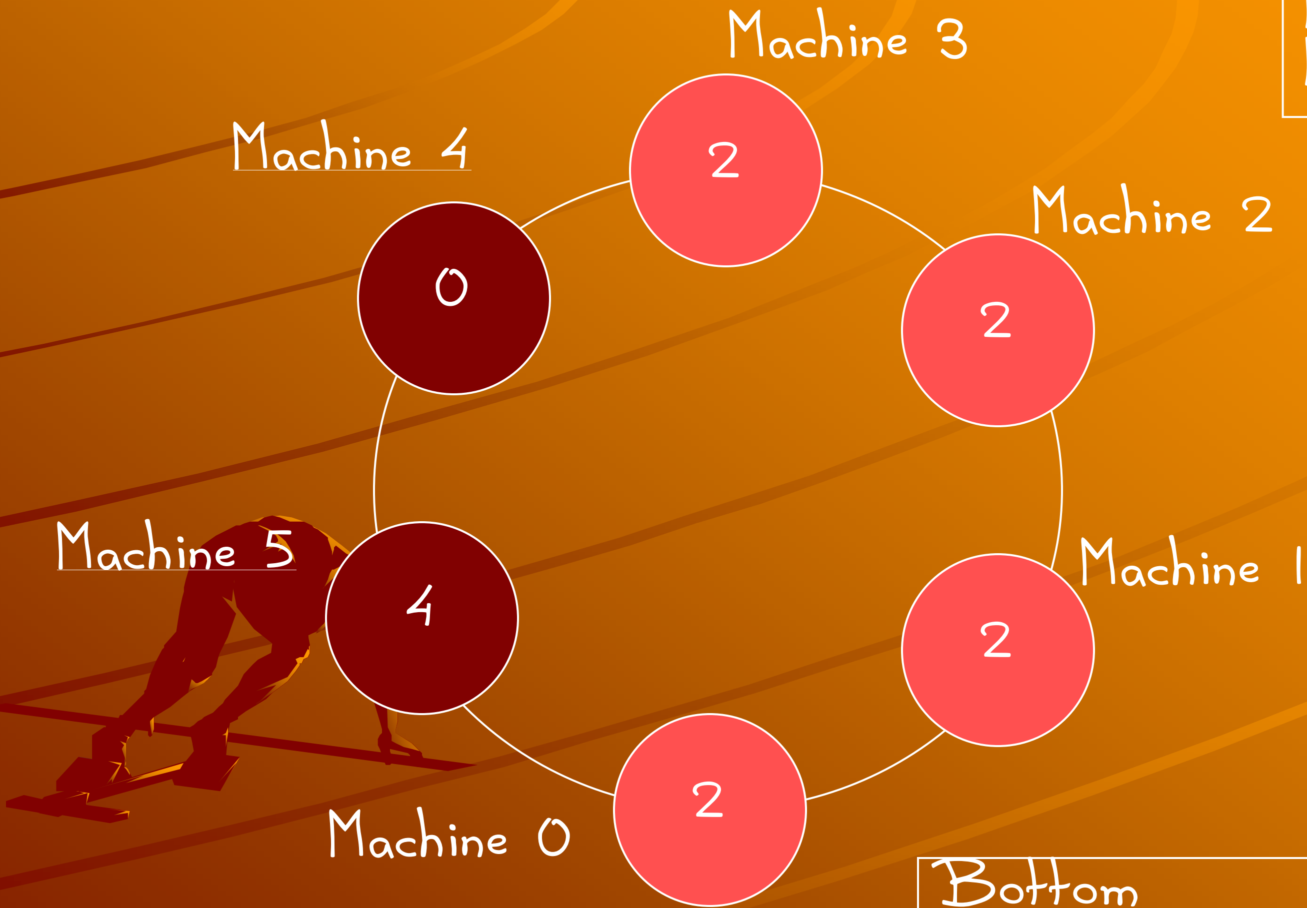


N=4
K=5

Bottom
if (S=L) S=L+1 mod K fi
Others
if (S!=L) S=L fi

After machine 3's move..

N=4
K=5



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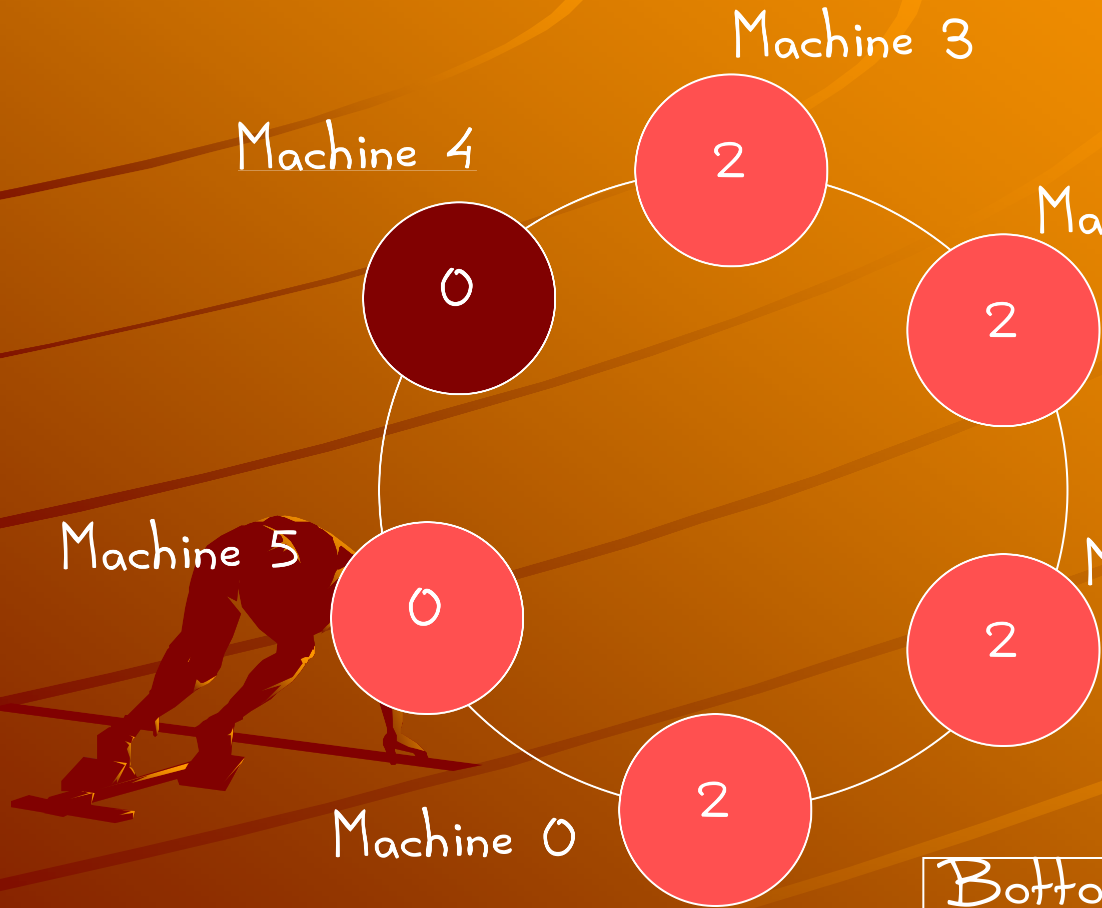
Others
if (S!=L) S=L fi

After machine 5's move..

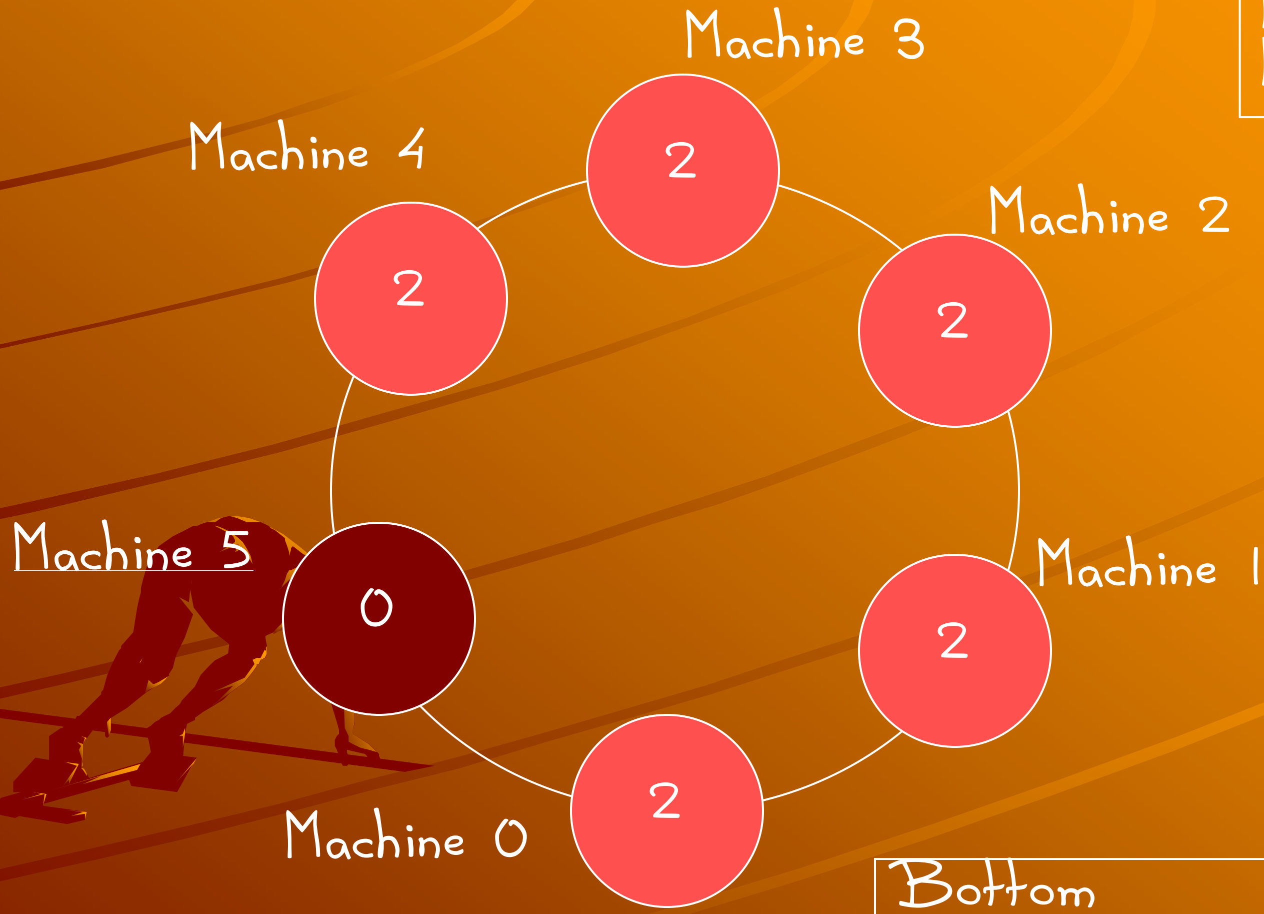


N=4
K=5

Bottom
if (S=L) S=L+1 mod K fi
Others
if (S!=L) S=L fi

After machine 4's move..

N=4
K=5



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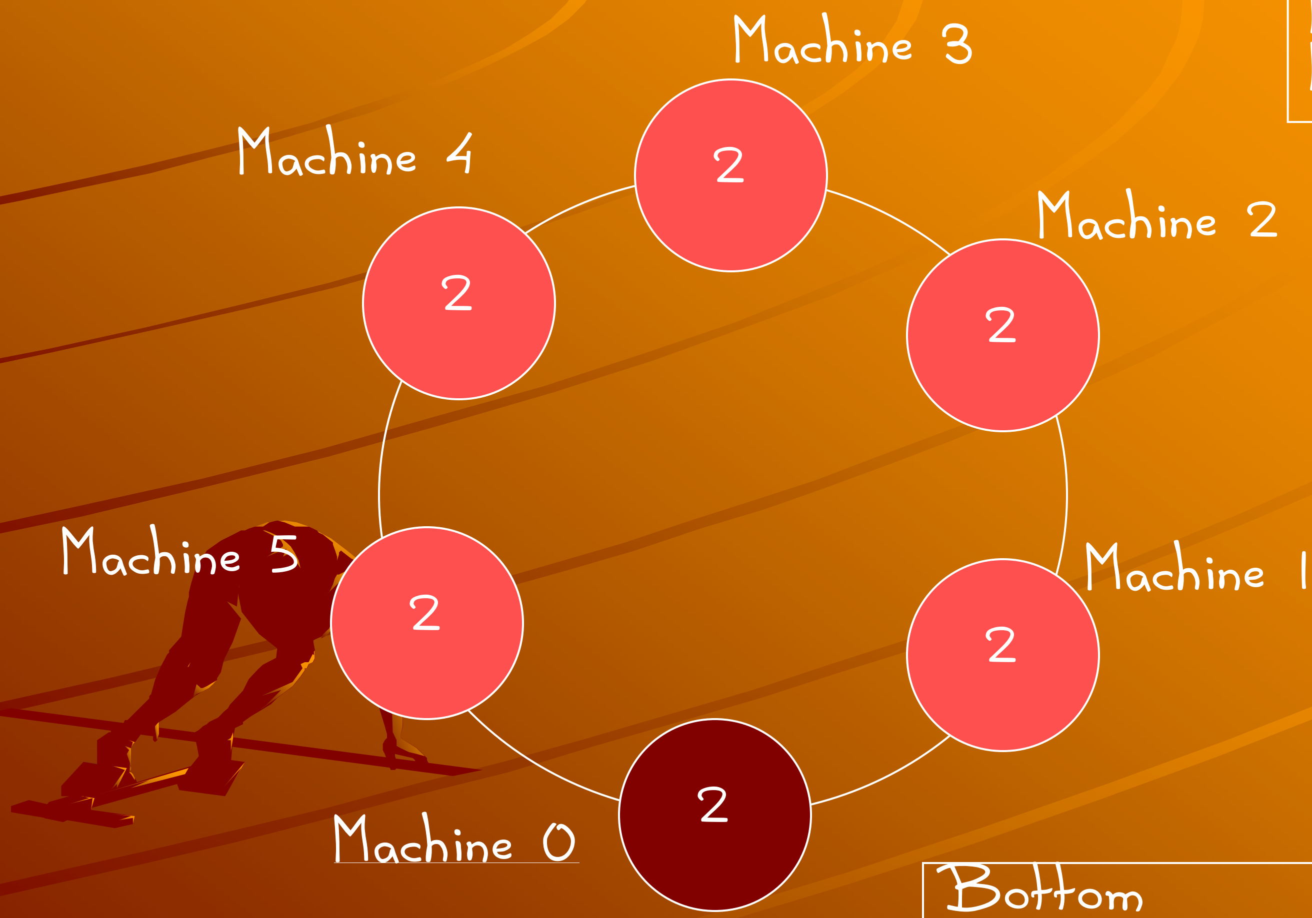
After machine 5's move..

N=4
K=5



Bottom

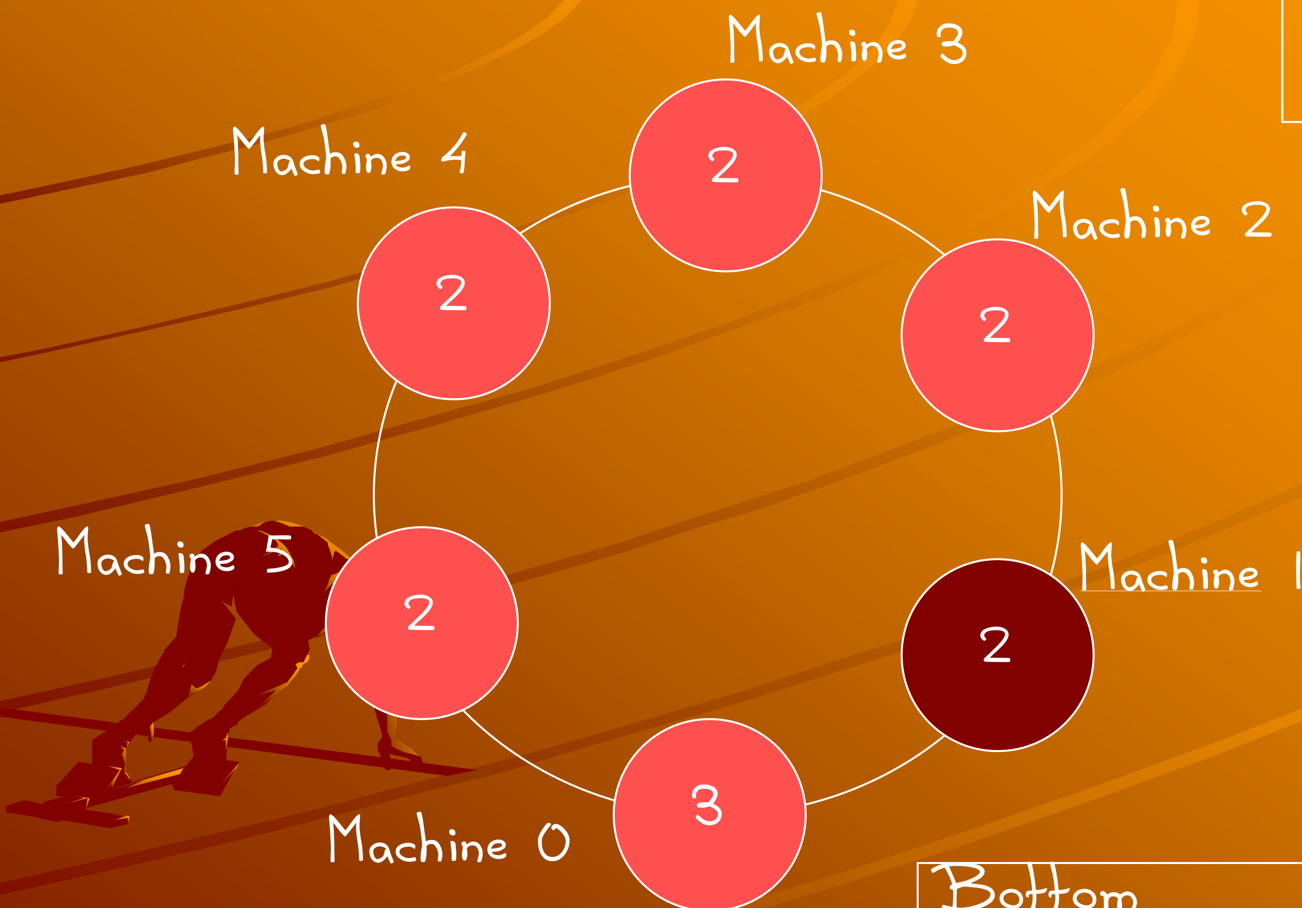
if (S=L) S=L+1 mod K fi

Others

if (S!=L) S=L fi

After machine 0's move..

N=4
K=5



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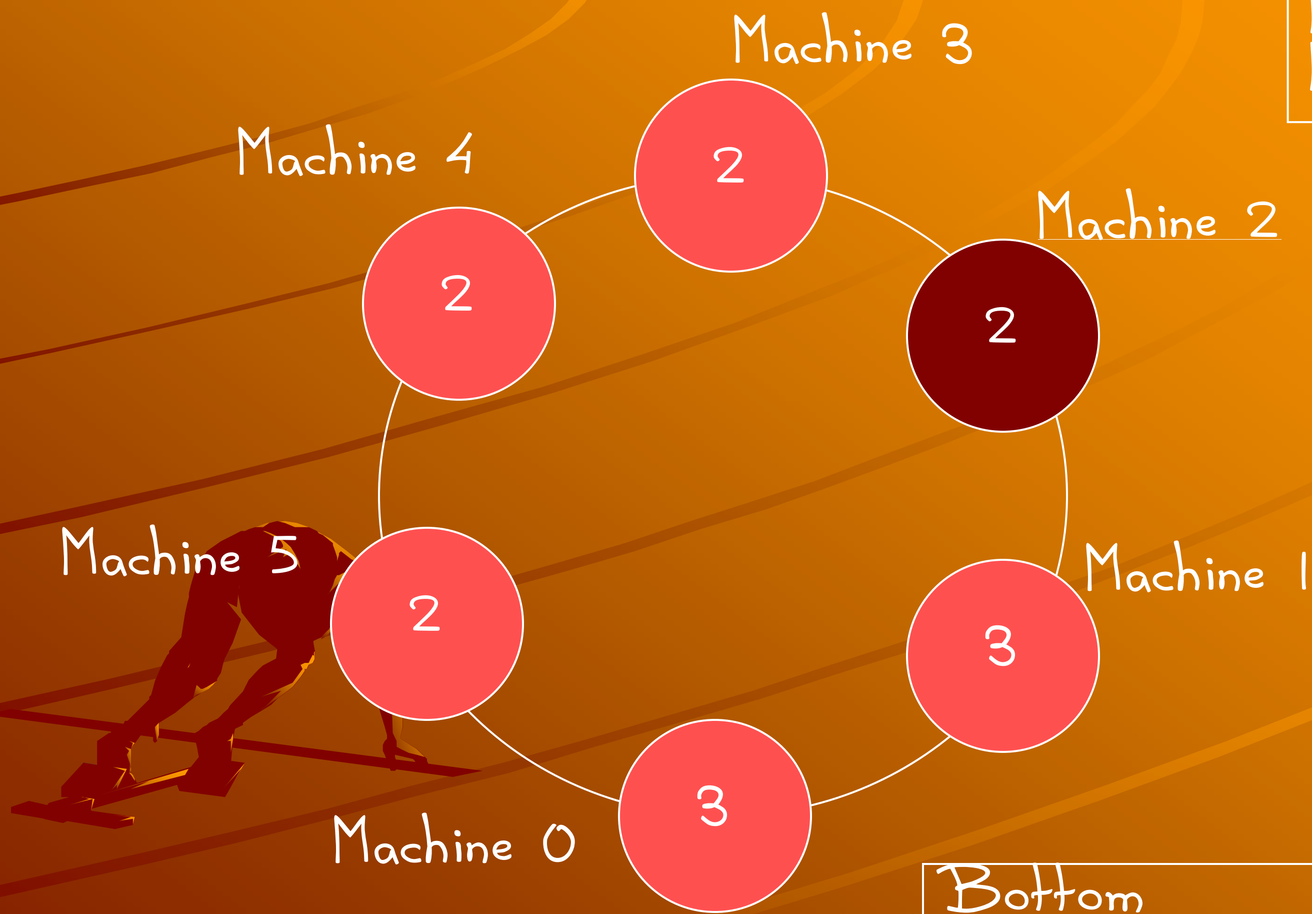
After machine 1's move..

N=4
K=5



Bottom
if (S=L) S=L+1 mod K fi
Others
if (S!=L) S=L fi

The system is trapped in legitimate state

- In this machine the legitimate state was defined as exactly one privilege in the whole system
- You can observe that this has been reached
- The privilege will now make rounds in the ring
- Application- The legitimacy criterion is that of mutual exclusion (only 1 privilege at a time). The algorithm is token ring.
- But notice that the system is self stabilizing i.e. it recovers from any error state (non legitimate state) on its own in finite number of steps

Why does the idea work?

i.e. what's the intution behind this machine?

- discussed in the class

Exercise problem: Lift. Is it possible?

Reference Reading for other 2 machines

- E W Dijkstra, Self stabilizing systems in spite of distributed control: CACM, Nov 1974, pp. 643-644