Competitive Exams and the Indian Society

A game theoretic and economic analysis

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The Outline

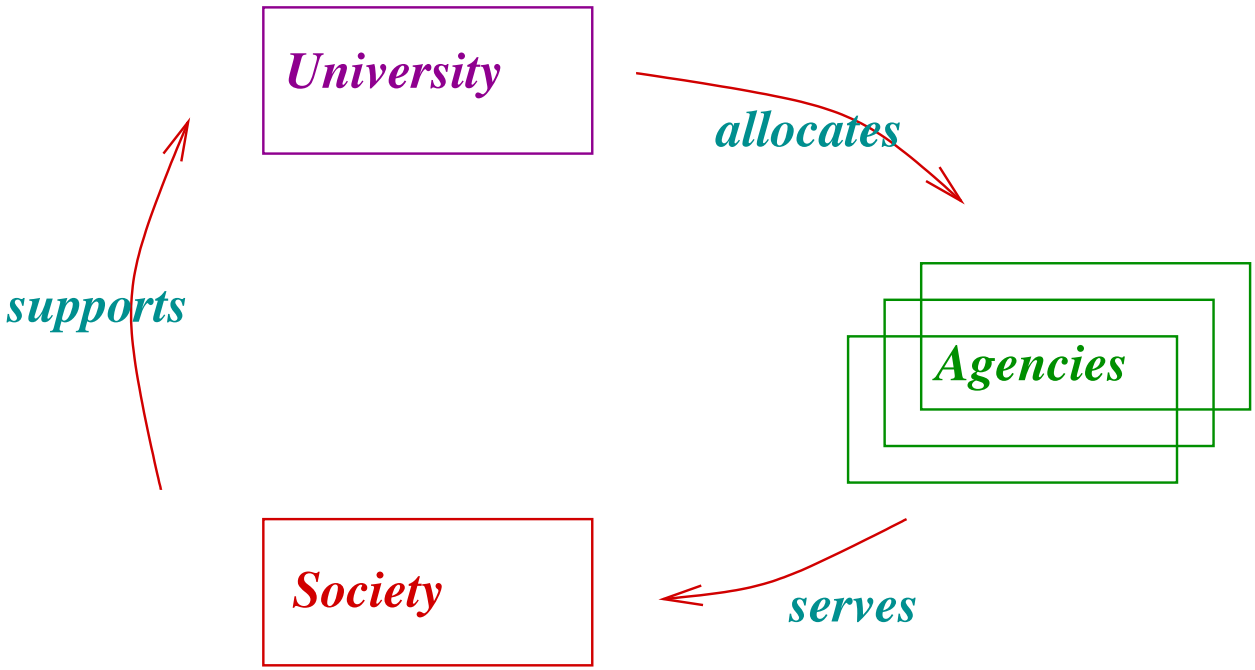
Competitive Exams

- The University, the Elite University and the CE.
- Some data and questions.
- The Game -Theoretic Formulations.
- Strategies and what do they tell us?
- What are broader implications.

Science, Society and Excessive Selectivity

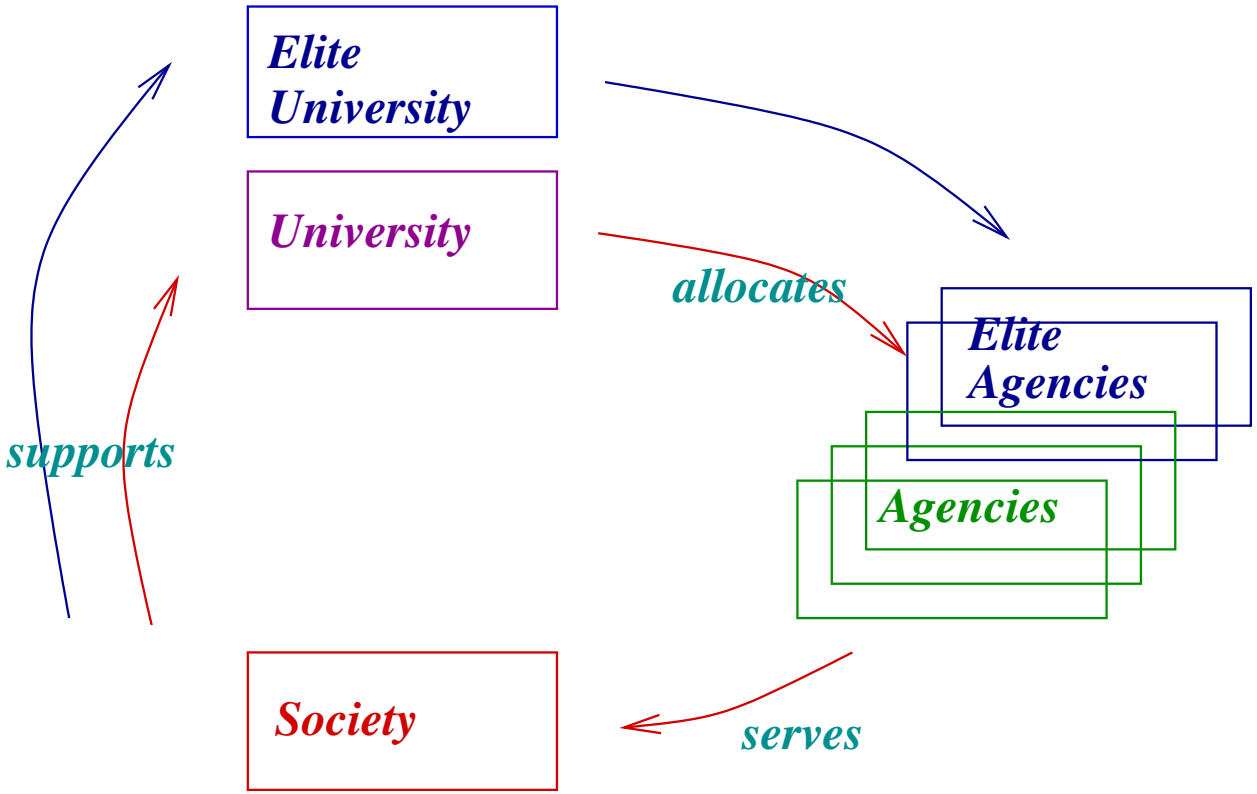
- The role of science in the progress of societies.
- The Elite Belief: Our way of doing science and engineering.
- Consequences.
- Where do we go from here.

The University Loop



- The University: perhaps the greatest human invention.
- Repository of *knowledge and practice*. Preparing pre-adults for various roles. New jobs, new technologies.
- Agencies: Civil society, Companies, State.
- In return: the society gives salaries, gives mandate and prestige.

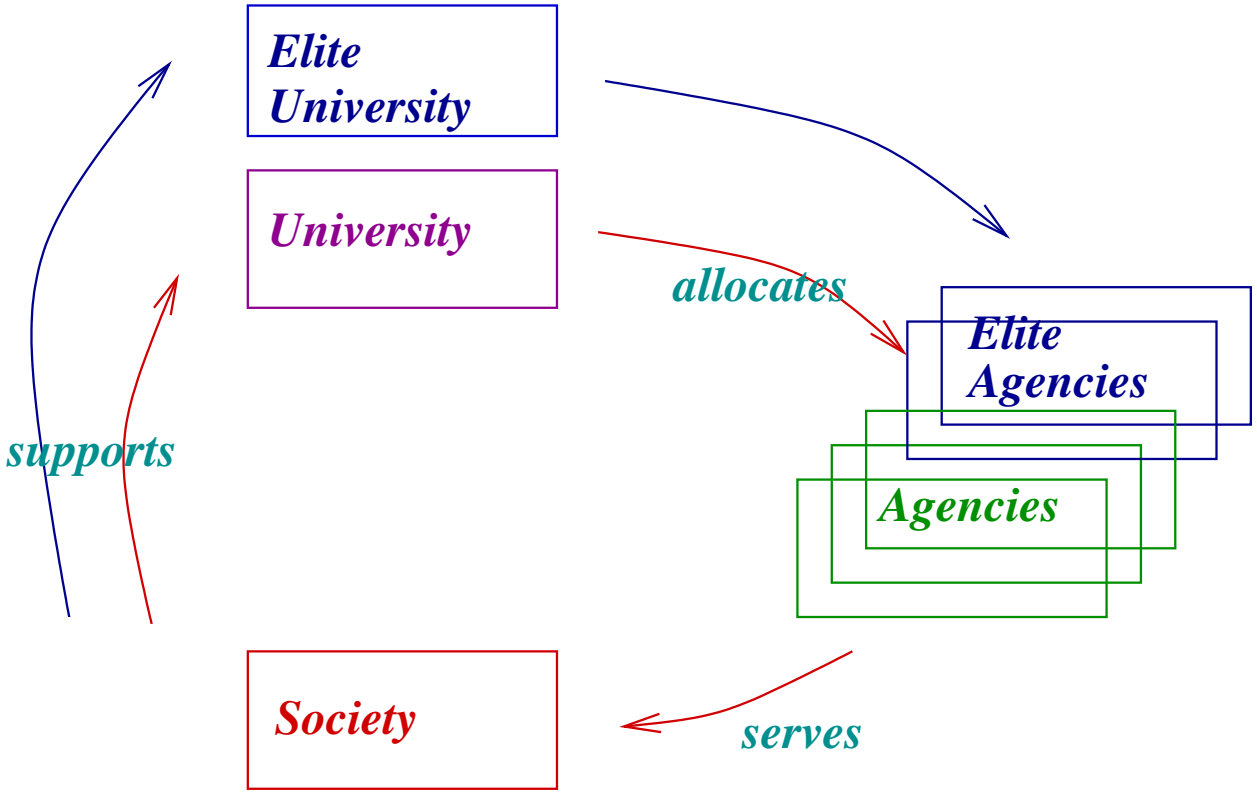
The University Loop



The Elite University

- Thought leadership, the arts, long-term challenges, *destiny*.
- Symbolic of what a society values!

The Elite University in India



- The Elite Belief. : Gifted people with more power will move the nation.
- Special status, super resources. Special positions of power.
- *Right to conduct exams in the name of higher ideals*.

The Indian Elite University

- Long history-right after independence. Periods of scarcity. The emerging world of science.
- The IITs, IISc, ISIs, IIMs, TIFR, JNU, Delhi School of Economics
  - the new IISERs, new IITs
- Key areas: Science, Technology, Engineering, Economics, Mathematics, Architecture, Medicine, Bureaucracy

Typical features:

- Centrally funded, autonomous
- Research orientation, international faculty
- focus on excellence and global standing
- transparent and highly selective admissions

Money, more than money- intellectual space.

The Input side

At the UG level:

- Two layer process-JEE and advanced JEE - Multiple Choice Exams.
- In 2013, 12 lakh students sat for JEE of which 1.5 lakh were allowed to write advanced JEE.
- JEE admits to NITs (roughly 7000-10000 seats)
- Advanced JEE admits into IIT (another 7000 seats)
- odds of roughly 1 in 200. Selectivity varies dramatically with discipline.

At the PG level:

- Disciplinary GATE exams. Separate admissions.
- Roughly 10 lakh sat for about 5000 seats. Selectivity roughly more constant.

The Output at the IITs

Engineering Placements 2013 (IIT Bombay)

Sector	Engg.	Finance	Consulting	IT
Super-GG	25 (27.7)	10 (35.0)	8 (49.6)	41 (52.1)
GG	116 (7.9)	82 (11.7)	110 (9.6)	102 (10.0)
IG	52 (6.5)	19 (7.2)	11 (5.8)	28 (7.2)
GI	24 (9.3)	10 (14.2)	10 (5.2)	5 (9.3)
II	64 (6.5)	13 (9.5)	8 (5.8)	22 (7.9)

Table: Numbers by sector and profile and average annual salary in Rs. lakhs

So, why are our graduates not doing engineering?

Boys and Girls

JEE 2012			
	Registered	Qualified	%-age
Boys	337916	21226	6.28
Girls	168568	2886	1.71
%-age of Girls	33.2	11.9	-

JEE Advanced 2013			
	Appeared	Qualified	%-age
Boys	103660	18468	17.8
Girls	23089	2366	10.2
%-age of Girls	18.2	11.4	-

Table 7. Rural–urban statistics for JEE

	JEE 2011		JEE 2012		JEE (Advanced) 2014	
Cohort	Registered (%)	Qualified (%)	Registered (%)	Qualified (%)	Registered (%)	Qualified (%)
Village	18	10	19	11	13	10
Town	29	25	30	26	19	14
City	53	65	51	63	68	76

Are we doing this right?

Are testing for aptitude for science? Or speed and alacrity? Or the smartest coaching class? Or some other proxy?

How do students prepare for competitive exams?

- Are the more suitable more likley to be selected?
- Do motives such as placements play a role in deciding who has applied?

What is the impact on society?

Games

- Agents $\{a_1, \ldots, a_n\}$ and a strategy set S available to all agents.
- A play is $\bar{x} = (x_1, \ldots, x_n)$ with each $x_i \in S$.
- Outcomes or pay-offs $f_i(\bar{x}) \to \mathbb{R}$.

Key questions:

- How do agents choose strategies?
- Are there steady states - Nash Equilibria

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Example

- 2 agents. Paper-Stone-Scissor $S = \{\square, \circ, \times\}$.
- pay-off: $\square \to \circ \to \times \to \square$. An example play: $(\times, \circ)$.
- $f_1(\times, \circ) = -1$ and $f_2(\times, \circ) = +1$.
- Nash Equilibrium: Each agent plays one of the three *equi-probably*. Expected Pay-off=0.

Solving Games

- Agents $\{a_1, \ldots, a_n\}$ and a strategy set S available to all agents.
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How to solve:

- Best response: Two states: $\bar{x} = (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n)$ $\bar{x}' = (x_1, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_n)$. If $f_i(\bar{x}) < f_i(\bar{x}')$. Then a_i will shift!
- Stability of $\bar{x}$ gives us n equations.
- Nash Equilibria may not exist, may not be desirable, may not be equitable.

The Competitive Exam Game (CEG)

The Simplest Game

- n: number of students, k: number who pass.
- *Pass fraction* $c = k/n$, selectivity $= 1 - c$.
- There are n contestants $A = \{a_1, \ldots, a_n\}$ and $S = [0, 1]$.
- There is a common prize of value 1. Each agent bids $x_i \in [0, 1]$.
- The highest k bids win, others lose.

$$u_i(\bar{x}) = \begin{cases} 1 - x_i & \text{if agent wins} \\ -x_i & \text{if agent loses} \end{cases}$$

- Symmetric Mixed Strategy: Bid close to 1 with probability c and close to 0 with probability $1 - c$.
- Conservative: i.e. winnings equal losses.

Does the exam generate value?: Important attribute to understand.

Personal Valuations

- Contestant a_i has a valuation β_i, where these β_i are independent and come from a distribution F.
- This models the fact that different agents may have different motivations in attempting the exam and may value winning it differently.
- Each contestant must make a bid x_i. Of these n bids, the top k are declared as winners and obtain a pay-off of $\beta_i - x_i$. The losers lose their bids, i.e., have a pay-off of $-x_i$.

Thus, the pay-off function $u_i(\beta_i, \bar{x})$ is given by:

$$u_i(\beta_i, \bar{x}) = \begin{cases} \beta_i - x_i & \text{if agent wins} \\ -x_i & \text{if agent loses} \end{cases}$$

Nash Equilibrium

What should an agent with valuation β play?

- Assume the symmetric mixed strategy is given by a monotonic increasing function $s_1 : \mathbb{R} \to \mathbb{R}$, i.e., agent with β plays $s_1(\beta)$.
- Assuming that all agents follow s, the probability of an agent with valuation β winning in the competitition is exactly the probability that there are at least $n - k$ distinct agents a_j with $\beta_j < \beta_i$.
- This number is

$$P(n, k, \beta) = \sum_{r=n-k}^{n-1} \binom{n-1}{r} F(\beta)^r (1 - F(\beta))^{n-1-r}$$

- The expected earnings by this agent, assuming that all agents follow s correctly are:

$$\begin{aligned} E(\beta) &= P(n, k, \beta) \cdot (\beta - s_1(\beta)) + (1 - P(k, n, \beta)) \cdot -s_1(\beta 0) \\ &= \beta P(n, k, \beta) - s_1(\beta) \end{aligned}$$

Whence...

On the other hand, if the agent had reported her valuation to be ζ instead of β, her earnings would have been:

$$\begin{aligned} E(\beta,\zeta) &= P(n,k,\zeta)\cdot(\beta - s_1(\zeta)) + (1 - P(n,k,\zeta))\cdot(-s_1(\zeta)) \\ &= \beta P(n,k,\zeta) - s_1(\zeta) \end{aligned}$$

The condition that s is a symmetric NE gives us that the expression $E(\beta,\zeta)$ is maximum at $\zeta = \beta$. This gives us:

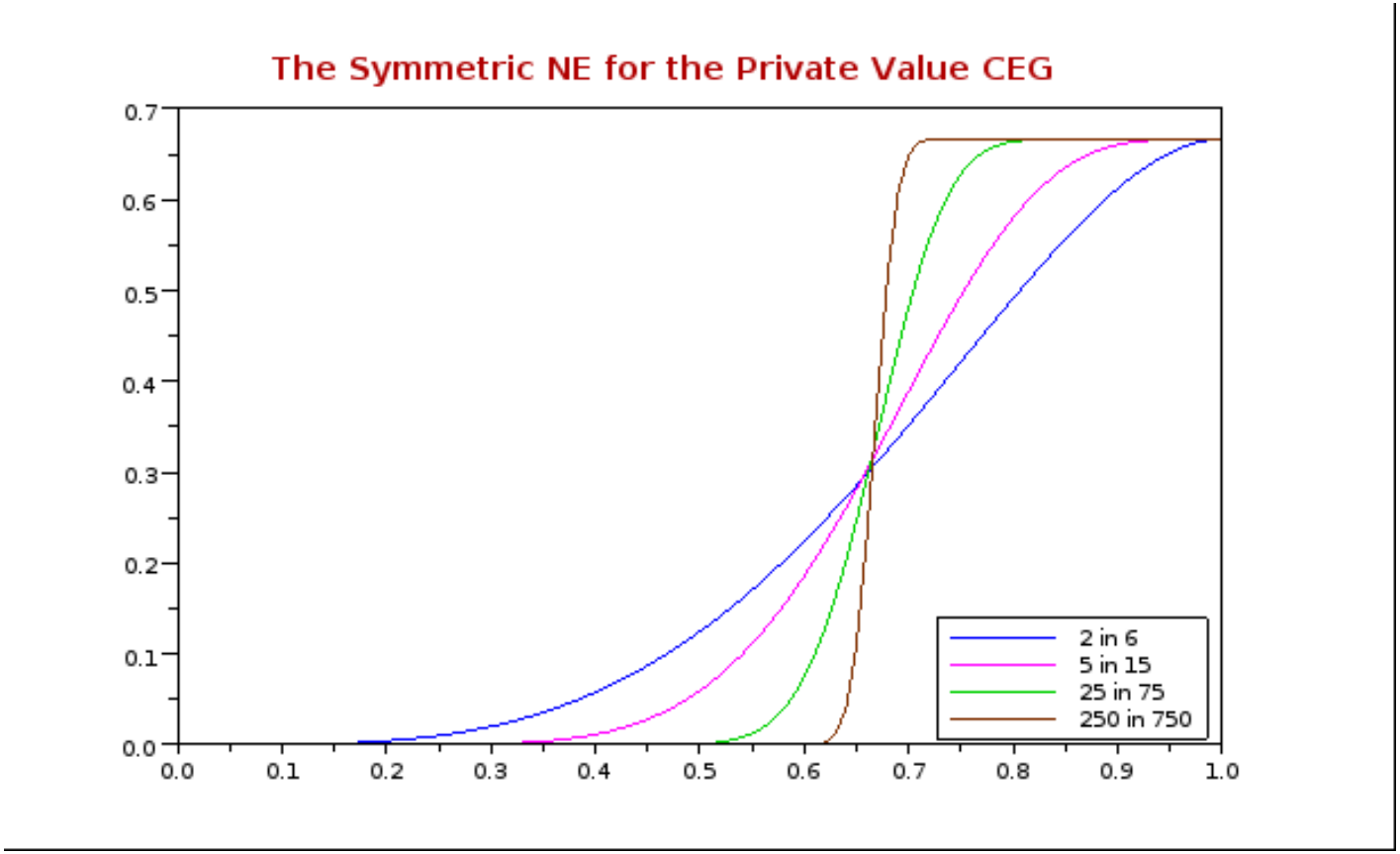
$$\beta P'(k,n,\zeta) - s'(\zeta)|_{\zeta=\beta} = 0$$

This simplifies to the equation $\beta P'(n,k,\beta) = s_1'(\beta)$. Using the condition that $s_1(0) = 0$, we have:

$$s_1(\beta) = \int_0^\beta xP'(n,k,x)dx = \beta P(n,k,\beta) - \int_0^\beta P(n,k,x)dx$$

For a simple case

- Assume that β is uniformly drawn from the interval $[0, 1]$ and thus $F(\beta) = \beta$ for $\beta \in [0, 1]$.
- This gives us $s_1(\beta) = \frac{n-k}{n} \sum_{j=n-k+1}^{n} B(n, j, \beta)$



We conclude...

Let us assume that $k = cn$, where c is the *pass fraction*.

- For large n, we have:

$$s_1(\beta) = \begin{cases} 0 & \text{if } \beta < 1 - c \\ 1 - c & \text{if } \beta > 1 - c \end{cases}$$

- The net output/input ratio is $\frac{2-c}{2-2c}$ which is greater than 1. For $c = 0.3$ it is 1.2 but for $c = 0.01$, it is 1.005.
- Thus, in theory, for large c, the exam creates social value.
- What students do after they graduate will increasingly decide who works hard and gets in. *What they do once they get in!*
- Changing ethos of institutions.

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- Thus, in theory, for large c, the exam creates social value.
- What students do after they graduate will increasingly decide who works hard and gets in. *What they do once they get in!*
- Changing ethos of institutions. Good old days...☹

Raising tuition fees?

The new pay-off function $u_i(\beta_i, \bar{x})$ is given by:

$$u_i(\beta_i, \bar{x}) = \begin{cases} \beta_i - \beta_0 - x_i & \text{if agent wins} \\ -x_i & \text{if agent loses} \end{cases}$$

- Applicants with $\beta < \beta_0$ will not participate.
- x's will reduce by β_0, i.e., coaching class fees will reduce.

Key Conclusion

Must think through placements if we want the right students!

- Selected companies? Fixed service? Strengthened collaborations? Fellowship programs?

Initial endowment

Suppose that contestants start at different levels of initial endowments.
We keep the winning pay-off to be constant at 1. The basic game is as follows.

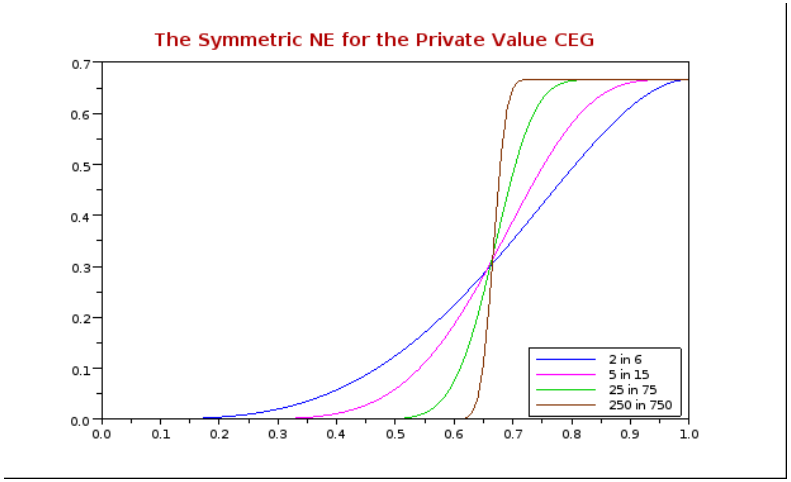
- Each contestant has a private type α_i, which is drawn independently from a common distribution F.
- Based on this α_i, the contestants puts in an additional effort $x_i = e(\alpha_i)$.
- The winning is decided on the rankings of the n numbers $\bar{\alpha} + \bar{x} = (\alpha_i + x_i)_i$ with the highest k as being declared the winners.

$$u_i(\bar{x}) = \begin{cases} 1 - x_i & \text{if agent wins} \\ -x_i & \text{if agent loses} \end{cases}$$

A more complicated calculation

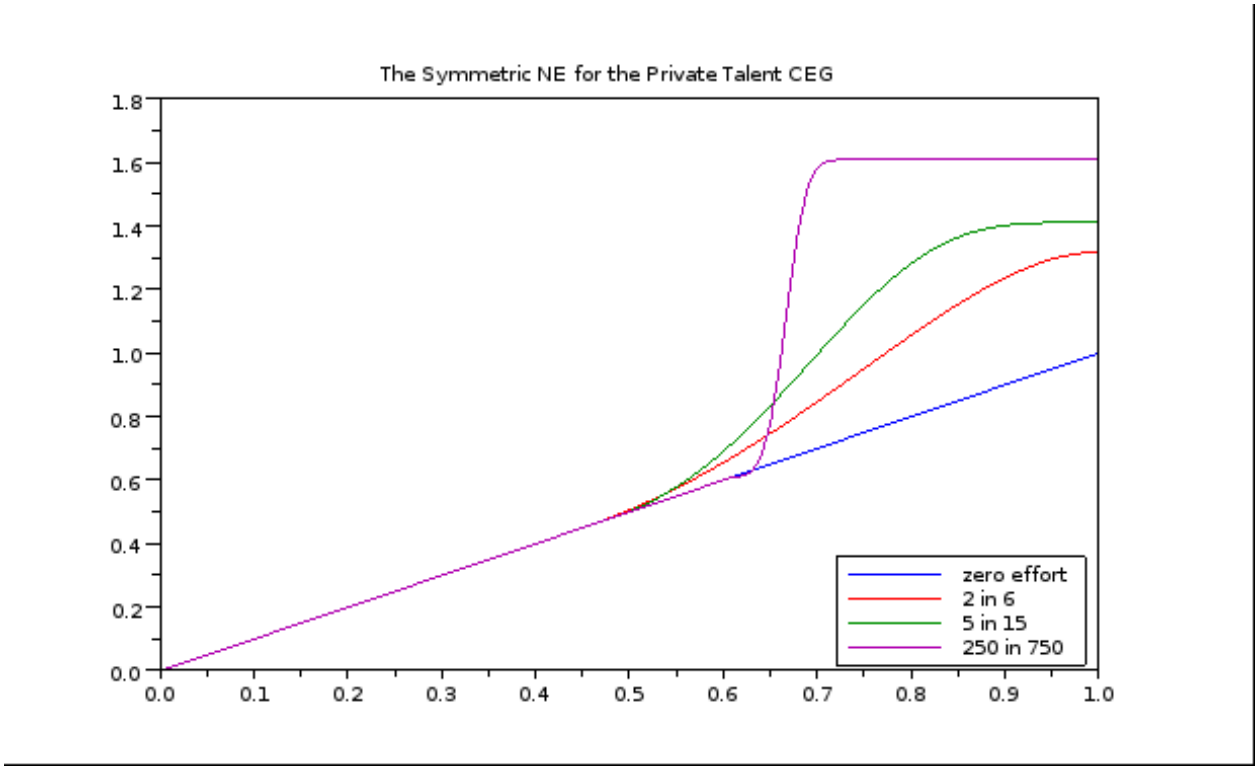
- Let us compute a symmetric NE $e(\alpha)$ for the above situation where $e(\alpha) \geq 0$ for all α.
- Let $s_3(\alpha) = \alpha + e(\alpha)$, where $e(\alpha)$ is the additional effort that is recommended. Our assumption implies that $s_3(\alpha) \geq \alpha$ for all α.
- We claim that $s_3(\alpha)$ is given by the following definition.

- Recall that $P(n, k, \alpha)$ is the probability that the value α is in the top k values for n samples drawn from F.
- For F uniform between $[0, 1]$, $P(n, k, \beta)$ is the sum of the last k Bernstein polynomials.



Finally

$$s_3(\alpha) = \begin{cases} \alpha & \text{if } \alpha \leq \alpha_0 \\ aP(n,k,\alpha) + b & \text{if } \alpha \geq \alpha_0 \end{cases}$$



Clever people may reduce their x.

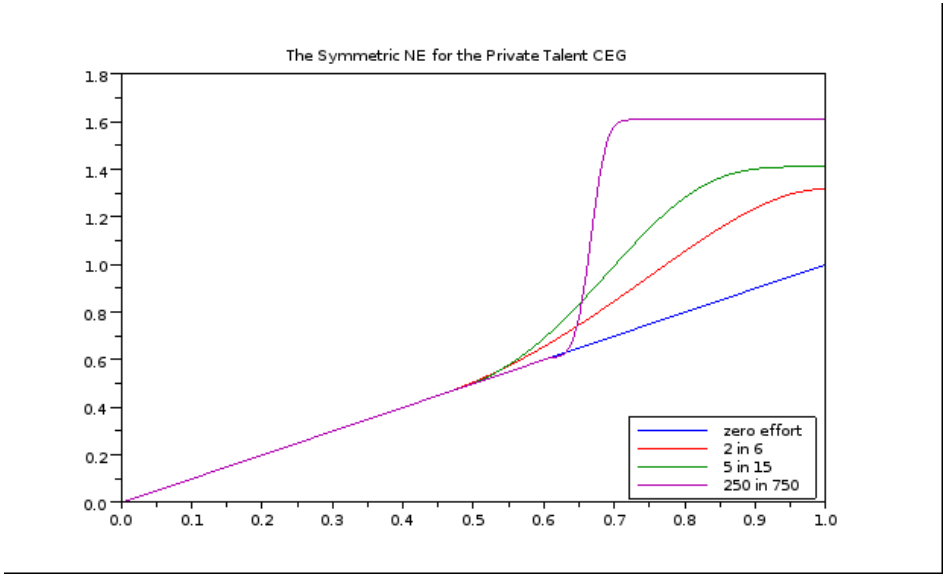
Whence...

- Competitive exams are efficient, provided all α and β are known to all. In this case, it is a small fraction who face uncertainty.
  - The information assumption is rarely true. Also better information does benefit agents.
  - Coaching Class: May provide high quality information on α. False negatives $\Rightarrow$ False positives.
- If substantially more are working (than who pass) then this points to information and belief inefficiencies.
- For large c, there is sufficient surplus and errors of information may be smaller.

Stylized conclusions

In the given conditions:

- If you want to do engineering for Indian people, you are better off not investing in the JEE.
- If you are from a small school in rural area, go to a town or city, before your start preparing.
- If you are poor, you better be brilliant and it should be a tough JEE Advanced that year.



Some Data

Boys and Girls Again

JEE Advanced 2013			
	Appeared	Qualified	%-age
Boys	103660	18468	17.8
Girls	23089	2366	10.2
%-age of Girls	18.2	11.4	-

JEE 2012			
	Registered	Qualified	%-age
Boys	337916	21226	6.28
Girls	168568	2886	1.71
%-age of Girls	33.2	11.9	-

Boys and Girls

CBSE 2013					
	Appeared	90% marks	%-age	95% marks	%-age
Boys	512210	22596	4.4	2855	0.56
Girls	376410	22053	5.8	3237	0.86
%-age of G.	42.4	49.3	-	53.1	-

- Clearly, girls should know they are smart, so α is well-understood.
- What are their βs, i.e., their aspirations?
- What is their ability to play the required x?

Other attributes

JEE 2011	JEE 2012
Total Appeared=468280	506484
Total Qualified=13196	24112
$c = 2.8\%$	$c = 4.7\%$

	2011		2012	
Cohort	Reg.	Qual.	Reg.	Qual.
Village	0.19	0.10	0.19	0.11
Town	0.29	0.25	0.29	0.26
City	0.52	0.65	0.52	0.63

- Can rural folks estimate their α correctly?
- If they cannot, would migrating to a larger city help?
- Should they drop out of coaching classes after the first year?

NSSO sample: Expenditures

Table 5. Average household spending on education by families having one studying member (our analysis of 68th round, NSSO, 2012).

		Andhra Pradesh Urban	Andhra Pradesh Rural	Rajasthan Urban	Rajasthan Rural	Odisha Urban	Odisha Rural	Tamil Nadu Urban	Tamil Nadu Rural
Households with one studying male	Mean (Rs.)	9919	5706	19096	4362	5765	1787	11046	8493
	Number of Samples	365	373	235	263	143	291	373	293
	Gini	0.61	0.58	0.56	0.64	0.65	0.70	0.64	0.67
Households with one studying female	Mean (Rs.)	9233	3752	9369	3431	4278	2292	12653	6949
	Number of samples	281	245	98	126	94	191	321	259
	Gini	0.61	0.55	0.60	0.56	0.82	0.76	0.65	0.69

JEE:Students with household income over Rs. 6 lakhs are 5 times more likely to suceed than students with income less than Rs. 1 lakh.

What has been achieved

Provided a framework for analysing and comparing efforts and outcomes in a competitive exams.

Limitations:

- The same currency for α, β and x. How good is that?
- What is being tested, say α' is a proxy for what is desired to be tested, i.e., α. Example: Aptitude for science is being tested through a particular exam.

Way Out:

- multi-attribute $\alpha = (\alpha^1, \alpha^2)$ and $x = (x^1, x^2)$. Ordering as per $\alpha^1 + \alpha^2 + x^1 + x^2$.
- Correlations between α and α' and errors in estimation of α.

Other issues

Do preparing for competitive exams help you learn Physics?

- Which physics? What use? Little evidence of side benefits.

Multiple-Choice Exams. What is being tested? Antlers?

- Highly coachable, esp. at the high end.

Therefore...

Competitive Exams as they are: All sorts of problems.

- Implementing a flawed meritocracy.
- Not selecting the right people, not allocating them correctly.
- Too little analysis of side-effects on society.

Happy alternatives exist.

- Fix placements.
- Keep pass fraction high. Have a battery of testing points. Bring state merit lists.
- More subjective and conceptual questions. Use letters of references, essays, project work.
- Allow for regional subjects, the arts and culture.

Broader Questions

National-level Competitive Exams

Broader Policy

- Centralizes the conduct and measurement of Science and Higher Education.
- Robs the region-specifity of the professions.
- CBSE curriculum: 50 pages on Structure of Atom, of which 20 on nucleus. Zero pages on water. *Chemistry in everyday lives*, last chapter of 16 pages in a curriculum of 700 pages.
  - Promotes a weird science of cute universal laws.
  - State curricula mimicking CBSE: increase α!
  - Is this good? Will we ever solve the water problem?
- Hinders the constitutional right of the States.
- Impacts students and their self-esteem.
- Reinforces existing inequality of opportunities.

Even Broader Questions

- Will the girl from Buldhana ever become a doctor? And if she does, will she serve Buldhana?
- Is the top 2% even well defined? Is it right to concentrate so much power and prestige in the top 2%?
- What if these, the top 2%, utilize their branding for other ends?

- Is not the "global community" the free rider?
- Does the Indian CEG generate value? Or is it a transfer of value?

The real crux

Demand side: Scientific and Engineering Services.

The real crux

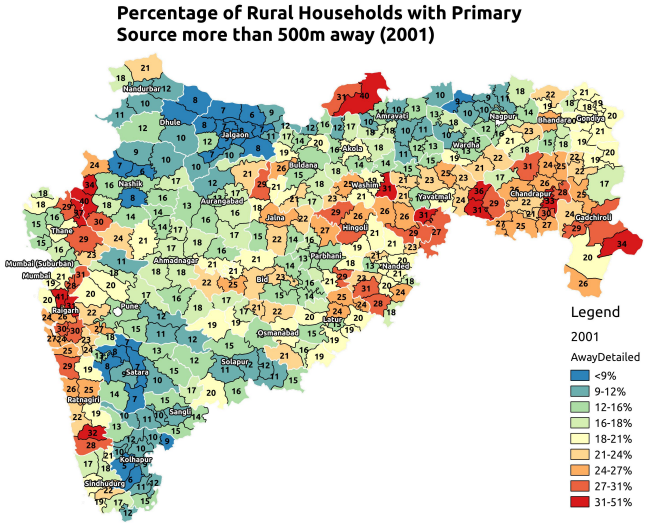
Demand side: Scientific and Engineering Services.

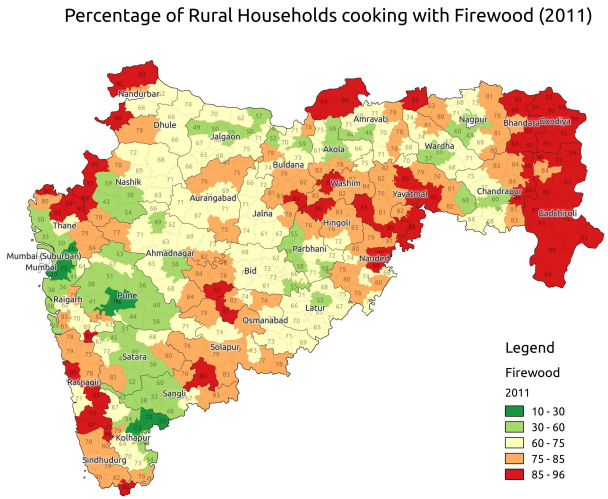
This is how we are...

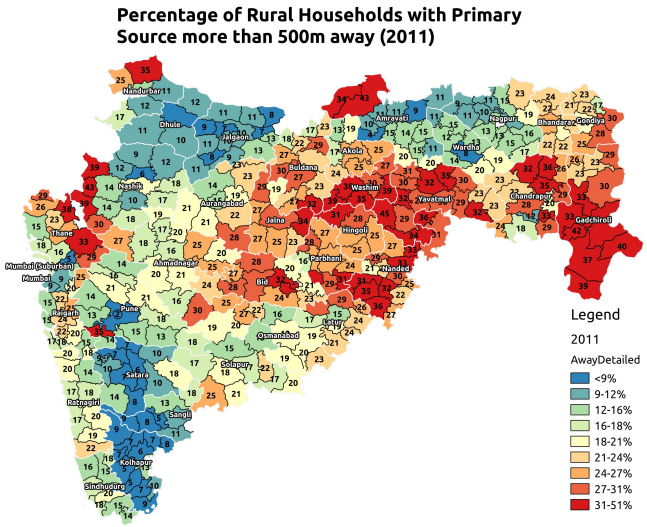
Scientific and Engineering Services

This is what the census says...

- Household cooking with firewood.
- Source of drinking water more than 500m away.

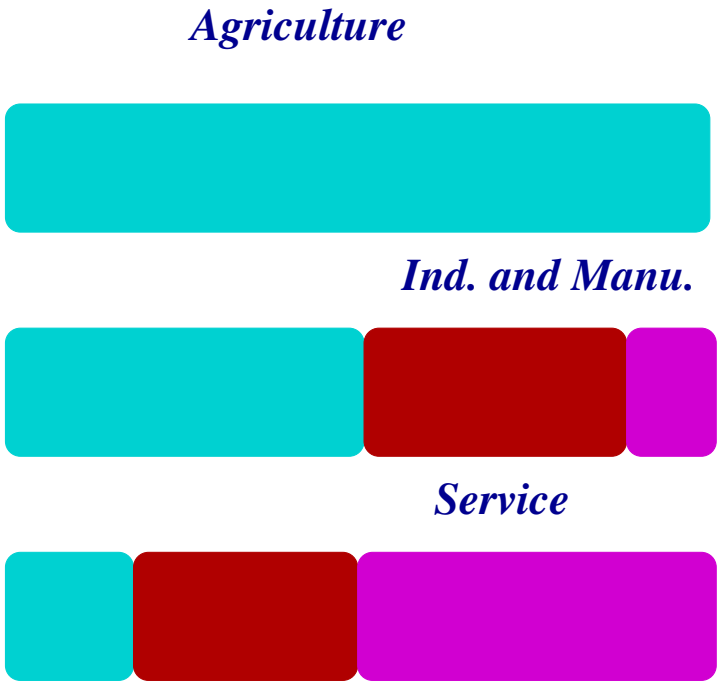






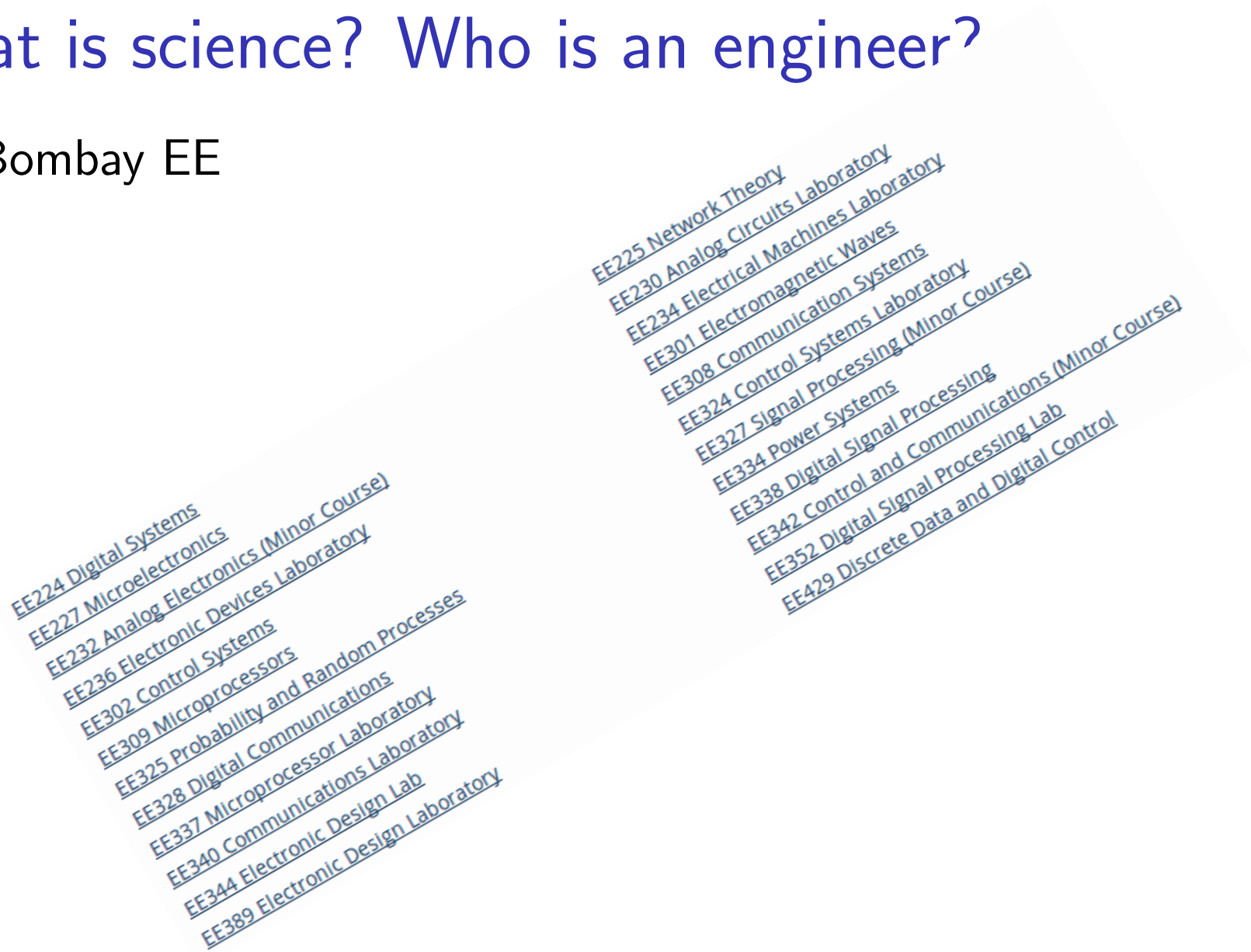
The usual trajectory of a society

- Farmers-Engineers-Scientists-Authors.
- Close engagement and parity.
- Growth through formalization and higher productivity.
- Growth through cutural production.



What is science? Who is an engineer?

IIT Bombay EE



What is science? Who is an engineer?

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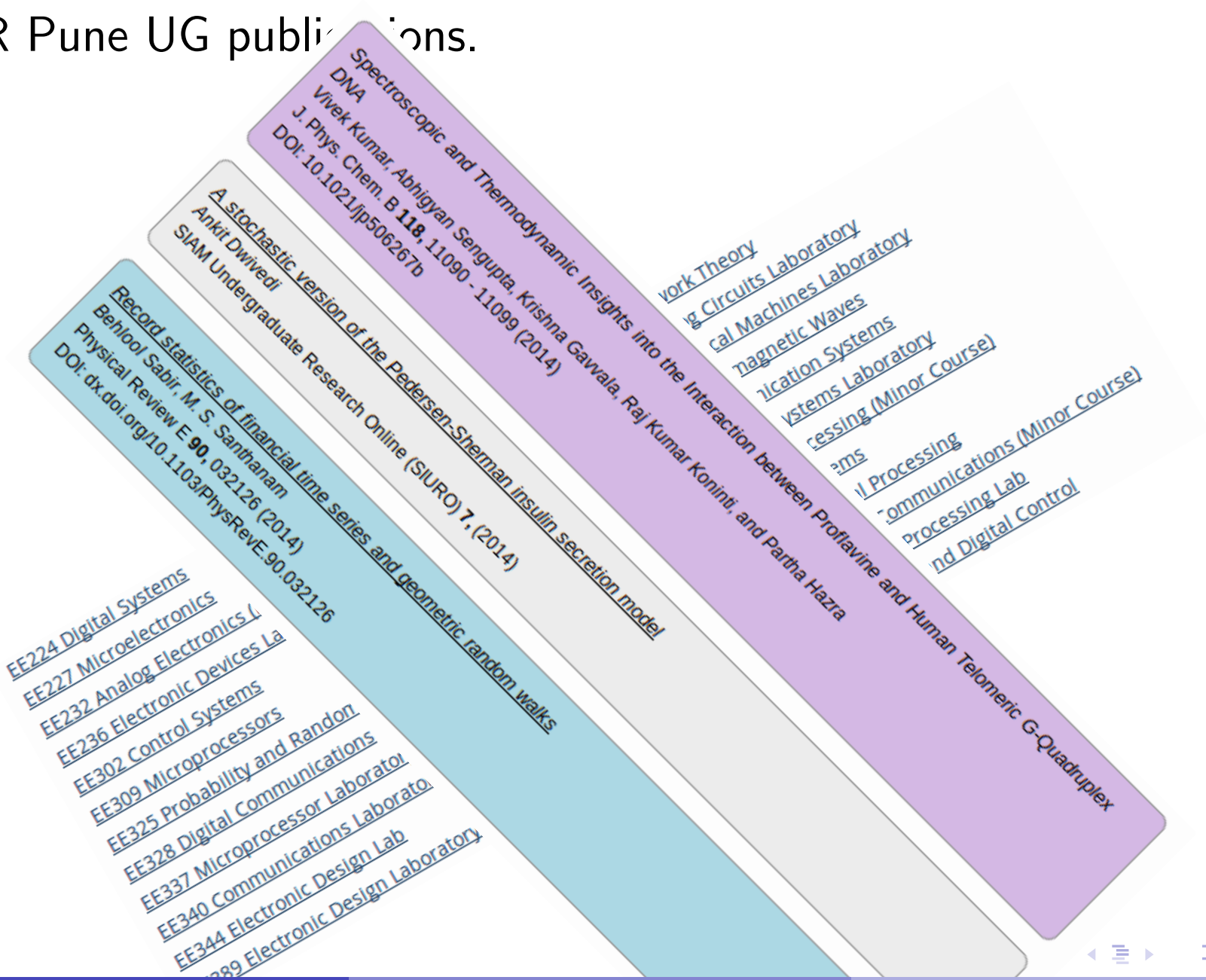


What is research?

Table 3. Number of papers with phrase in the title and with at least one author from India (Scopus)

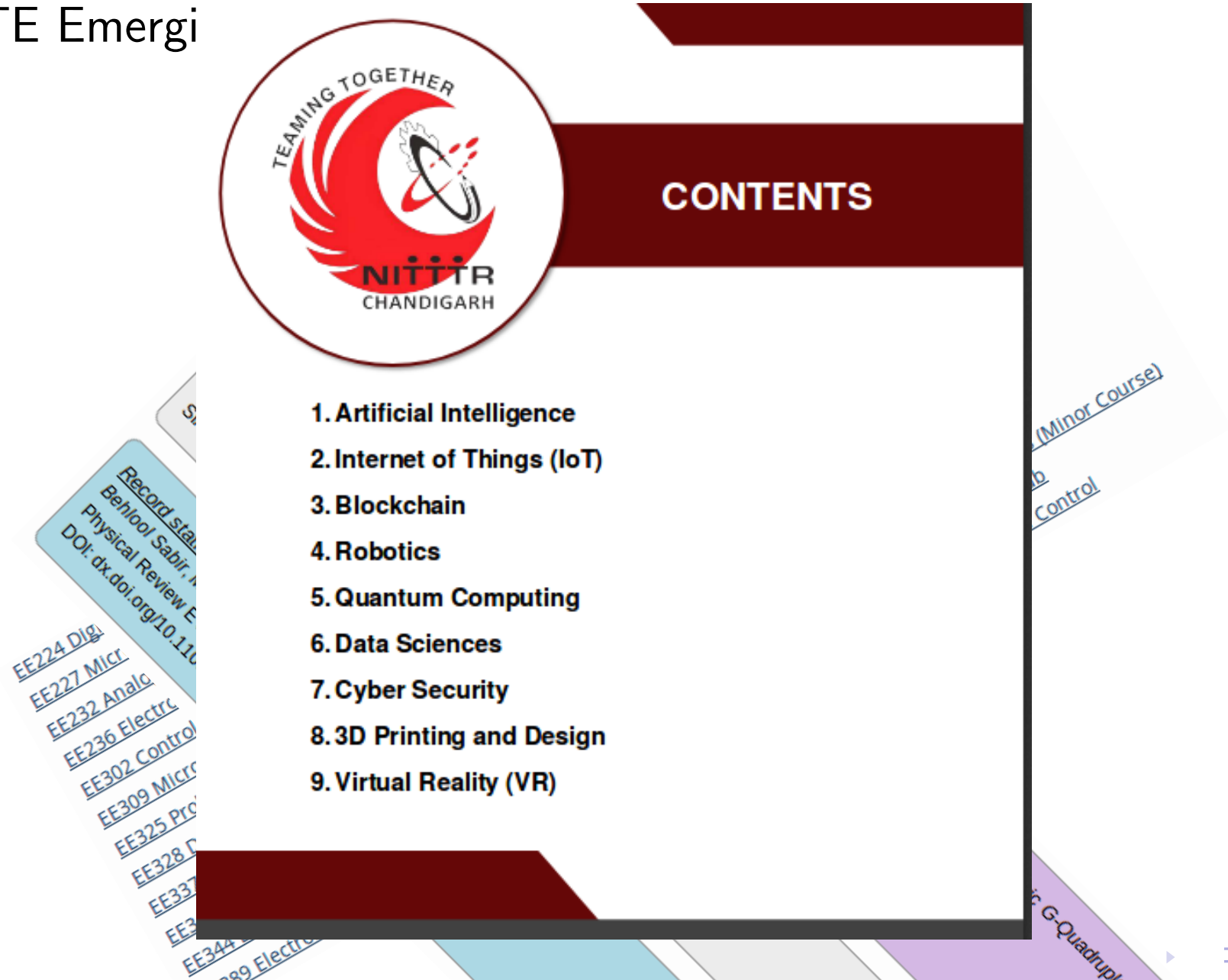
Topic (phrase)	All years preceding 2003	2003–09 (TEQIP I)	2010 onwards (TEQIP II)
Neural network	692	1818	2467
Fuzzy logic	110	327	759
Wavelets	96	905	1846
Genetic algortihms	262	989	1373

Table 4. Number of papers with phrase in the title and with at least one author from India (Scopus)

Topic (phrase)	All years preceding 2003	2003–09 (TEQIP I)	2010 onwards (TEQIP II)
Water supply	84	74	87
Sanitation	30	51	63
Groundwater models	11	29	70
Public transport	5	15	25
Power grid	12	56	288

What is the future of engineering?

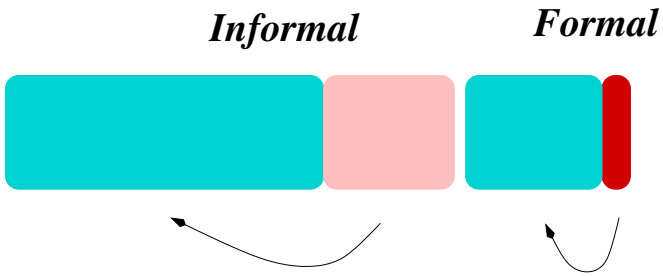
AICTE Emergi



What is happening?

- Extremely narrow and elite definition of engineering and science. Especially for a developing society.
  - At the school level too!
  - More suited for first-world society, their concerns and their companies.
- Largely un-teachable at the ordinary mortal college.
  - Severe problem of branding and *standards*.
  - Very few *certified* scientists and engineers. Very few things that they can and are willing to do.

The Indian Society

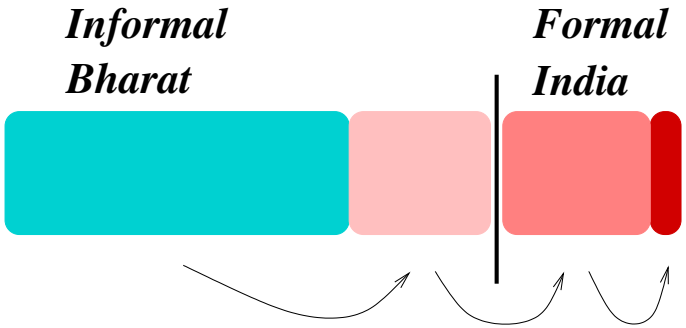


- A dichotomy of the formal and the informal.
- A small region of influence of elite science. Others served by informal agents.

Broken Water Supply	$\Rightarrow$	Tankers RO Plant	$\Rightarrow$	Rs. 1/liter
Smoky Houses	$\Rightarrow$	Illness	$\Rightarrow$	Expensive Healthcare

- **Huge opportunity for Rents**.

The Indian Society



- India: The formal urban english-speaking. Basking in the gentle glow of global knowledge systems.
  - Hunger Attack ⇒ Delivery App., Public Transport ⇒ Cab aggregator app.
- Bharat: The informal, rural and urban under-belly. The under-served bottom 80%. Resorting to NGOs and *abhiyan*s. Enormous opportunity for rents.
- *Elite education*: Life-boat. *To escape and not to serve.*
- *Aspirational dysfunction*. A society of rent-seekers.

The Indian Economy

India	Agriculture	Industry	Services	Per capita (in USD)
GDP (2012) (%)	17.4	25.8	56.9	1.5K
Employment (%)	51.1	22.4	26.6	-
GDP China	10	44	46	6.8K
GDP S. Korea	3	40	57	25K
GDP Germany	1	28	71	43K

Steel consumption. Points to few buisness models, e.g., *Railways.*

India	57	China	477
Other Asia	69	Japan	506
Egypt	95	USA	306
UK	145	Netherlands	200

A premature de-industrialization. An over-sized largely informal service sector.

What is the future?

The way the world of science is going...

- Increasing experto-cracy. Big business aligned global science.
- Brandedness of knowledge, products and culture.

Far from sustainable.

- What about *falsifiability* and *repeatability*? Extremely important that the University acts as a trusted 3rd party.

And in India:

- People as beneficiaries of Science - smart-phone dreams of delivery agents and sales-girls.
  - Our weakness for certainty, however degrading.
  - Collapse of the rural.
- Collapse of social comprehension⇒ Extreme Rents. Environmental Collapse.
- India is the leading edge, the *vishwa-guru* in this.

On the other hand...

Let alone orange juice, provisioning of basic services to the whole population itself limited by carrying capacity.
A good life for all within planetary boundaries, D.W.ONeill et. al, Nature Sustainability, volume 1, pages 88-95 (2018).

The difficulty of this transition.

$\Rightarrow$

Domestic-use water through-out the year at 80 liters per day $\Rightarrow$ rivers must flow all the year or large dams must be built.

Sustainability and Vernacular Science

- New way of *provisioning* ⇒ new way of doing science, of choosing scientists and laboratories.
- A different social comprehension of science. A new role for culture.

- Perhaps "why is my bus late?" is the new particle physics, and "where is my groundwater" the new artificial intelligence.
- Questions requiring an interdisciplinary and creative thinking of the highest order.

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In other words

A change in both the subject areas and methodology of science.

Thanks