TD 603
Water Resources

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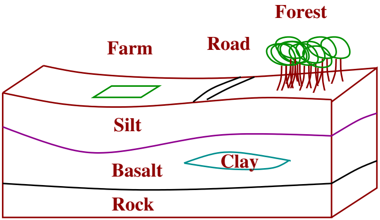
Lecture 5: Aquifer

Recap



A typical region has many features, both above and below the ground, which affect the water balance.

- surface features affect infiltration.
- underground features affect the accumulation and movement of groundwater.

Also recall that soil has many parameters related to water:

- Porosity, specific yield n, S_y: the maximum volume fraction of water, and that which is available.
- Conductivity K: The ability of the soil to allow the movement of water.

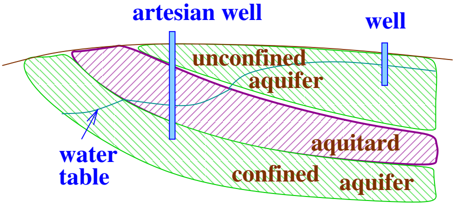
Aquifer

An aquifer is an underground soil-strata which allows the storage and movement of water.

- $K > 0.1 cm/s$ and $S_y > 0.1$
- Roughly coarse silts and sands.

Aquifers

- Materials which are poor in conductivity or storage are called aquitards.
- Example: Base Rock, Clays.
- Unconfined aquifer: accessible from the surface.
  - also *replenishable*: maximum sustainable pumping rate is recharge rate.
- Confined or partially confined: access blocked or limited by aquitard.
  - also *fossil*: depletion is almost permanent.



- The water table itself may cross many layers.
- Extraction of water from confined and unconfined layers cause different changes.

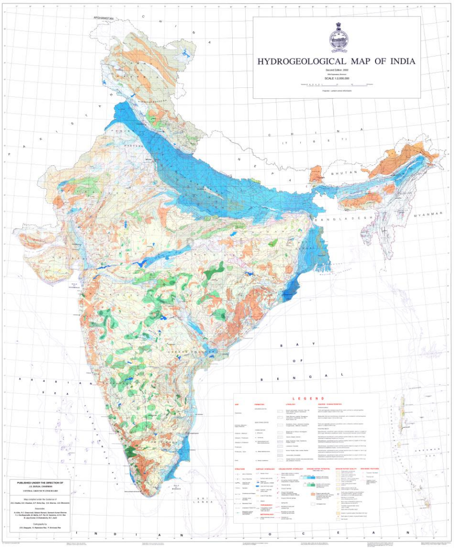
Aquifers

blue	high-porosity
green	porosity due to fractures
beige	little/no porosity

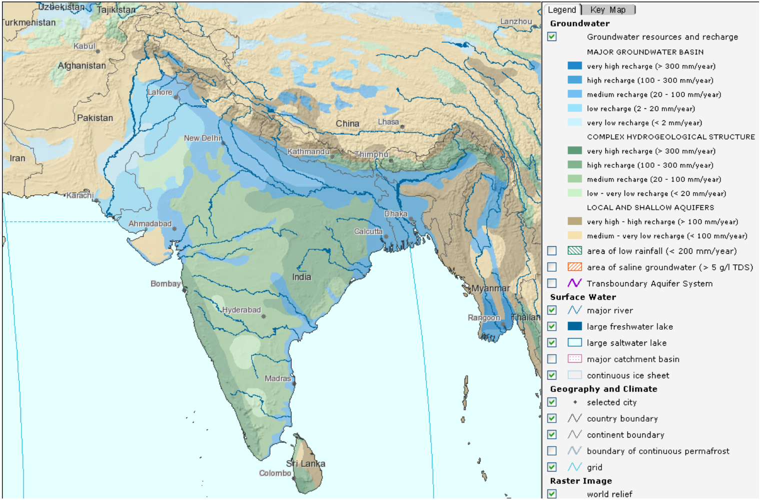
Indian aquifers:

- The Gangetic Plain: porous, shallow aquifer.
- The Deccan Trap: moderately deep and fractured.
- The Kutch: Silt/Clay shallow.

Mostly unconfined



Groundwater and Recharge source: UNESCO and whymap.org (BGR)



Aquifer Characteristics

Iso-Unisotropic: Variation of conductivity with direction of flow.

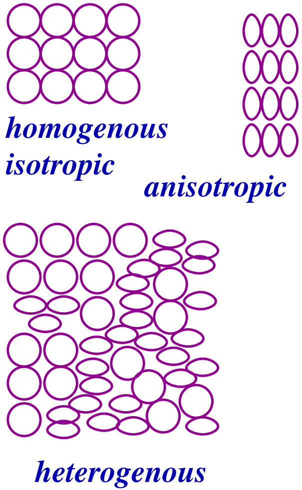
Homo./Heterogeneous: Variation of conductivity with location.

Aquifer thickness: The depth to which the aquifer extends.

Transmissivity T: The product $K \times L$, where K is conductance and L is depth. In general

$$T = \int K(l)dl$$

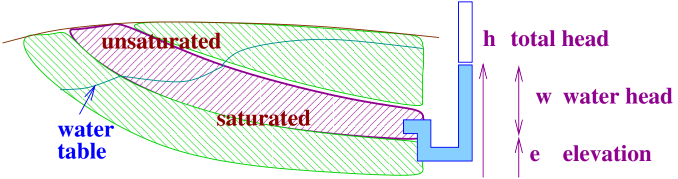
the depth-integral.
Storativity: Aquifer *elasticity* (skipped here).



The basic structure

- The ground is itself divided into two parts, the saturated and the unsaturated.
- Moisture equals porosity in the saturated, but diminishes rapidly as we go up.
- **The porosity θ is a smooth function over the terrain**

- The total head is a sum of the water-head and the elevation
- $w \geq 0$ iff the point is saturated.
- The water table is precisely when $w = 0$.
- **The heads h, e are smooth functions over the terrain**



Differential form of Darcy

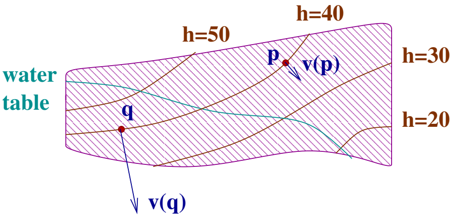
- The functions $w = h - e$ is the water head.
- The surface $w = 0$ is the water table.
- Let us plot surface $X_c = \{p | h(p) = c\}$. These are the equi-potential surfaces. Examples are shown.
- At any point p, there is a water velocity vector $v(p)$.

Darcy's Law

The velocity vector $v(p)$ is given by:

$$v(p) = \left[K \cdot \frac{\partial h}{\partial x}, \ K \cdot \frac{\partial h}{\partial y}, \ K \cdot \frac{\partial h}{\partial z} \right]$$

where $K = K(p)$ is the conductance of the soil at that point.



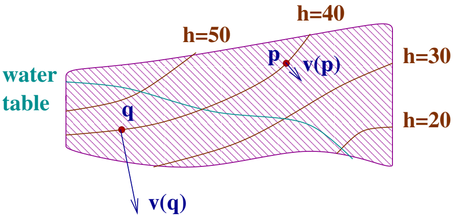
Velocity vectors

- The velocity vector $v(p)$

$$v(p) = \left[K \cdot \frac{\partial h}{\partial x}, \; K \cdot \frac{\partial h}{\partial y}, \; K \cdot \frac{\partial h}{\partial z} \right]$$

thus equals $K \cdot grad(h)$ where $grad(h)$ is the *gradient* of h.

- This implies that $v(p)$ is always perpendicular to the equipotential surfaces.



- The Darcy Law holds for the unsaturated regions as well, except that conductances in this region are very small.
- Thus, in the figure $K(p) << K(q)$.
- Conductance may be directional, i.e., (K_x, K_y, K_z), then

$$v = \left[K_x \frac{\partial h}{\partial x}, \; K_y \frac{\partial h}{\partial y}, \; K_z \frac{\partial h}{\partial z} \right]$$

Velocity vectors are almost true!

Our Objective: Real life scenarios

Inputs

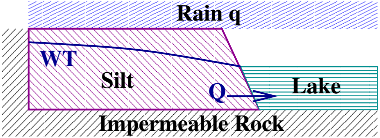
- Terrain data:
  - conductances and other data for each point.
- Rainfall and withdrawal data
  - infiltration, wells

Outputs

- Heads at all points.
  - water table, velocities
- Moistures.

Additional Inputs

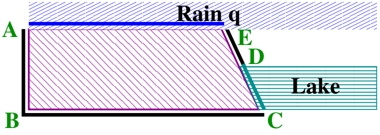
- Transient vs. Steady state
- Boundary conditions



A lake and its watershed

- The background is hard-rock.
- Rainfall rate q is knowm.
- All terrain data is known.
- What is the discharge Q from the banks into the lake?
- What is the water-table WT in the terrain?

The Lake Problem



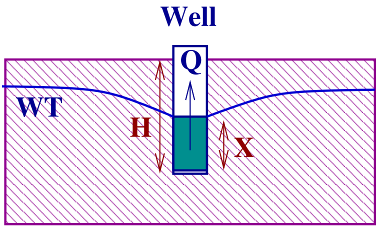
Boundary Conditions

- AB,BC is no flow.
- AE is with water influx q mm/day.
- CD is with a knwon constant head $h = H$, the height of the water in the lake.

- Simplifying assumption: ED does not have rain.
- Lake level does not increase!
    - Actually, because of the discharge Q, lake level increases.
- We want the steady state.
    - Actually, there are two seasons. So a periodic solution is desirable.
- Furthermore, it is clear that $Q = q \times area(AE)$. So its really a summer-monsoon problem.

Even under these assumptions, the problem is NOT easy.

The Well Problem



A well is 10m (H) deep and $8m$ in diameter and is situated in a farm. The farmer would like to withdraw Q liters/day. Please advice if this is sustainable.

- Measure/acquire Terrain conductance, porosity.
- Equilibrium desired so a steady state problem.
- Far-field Boundary: What to put far away?
  - No flow. No steady state.
  - Constant Head Realistic?
- Well Boundary: Seepage from aquifer to well in region X.
  - constant head X in region X.
  - No flow above that.
  - But X is unknown.

So, this problem is also not very easy!

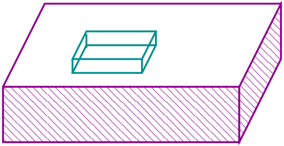
Farm-pond

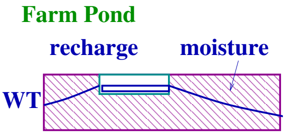
A farmer is considering a farmpond of size $10m \times 30m \times 2m$, of about Rs. 10,000 in direct and indirect costs. The objectives are:

- Recharge for better moisture in the second crop.
- Use for paddy crop during lull-periods in monsoons.

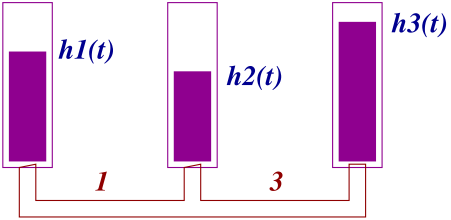
Please advise.

- A real-life techno-economic problem.
- Mainly unsaturated flows (moisture) and transient analysis.
- Crop related information: wilt-points.
- Evaporation-Transpiration rates and Infiltration.
- Monsoon behaviour.





Numerical 1 with $S_y = 1$



$$\begin{bmatrix} dh_1/dt \\ dh_2/dt \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -2 & -7 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3H \end{bmatrix}$$

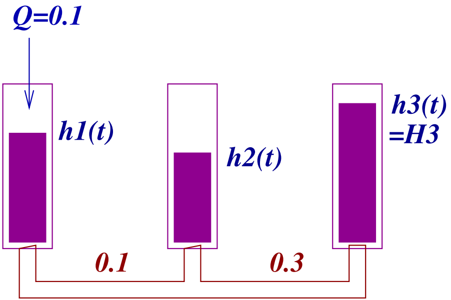
$$\begin{aligned} S_y dh &= (Ah + b) * dt \\ h &= h + dh \end{aligned}$$

With $h_i(0) = H_i$.
We have:

$$\begin{aligned} dh_1/dt &= (h_2 - h_1) \\ dh_2/dt &= (h_1 - h_2) + 3 \cdot (h_3 - h_2) \\ h_3 &= H - h_1 - h_2 \\ dh_2/dt &= (h_1 - h_2) + 3 \cdot (H - h_1 - 2h_2) \end{aligned}$$

t	h_1	h_2	h_3
0	3	2	5
0.01	2.99	2.10	4.91
0.02	2.98	2.19	4.83
0.03	2.97	2.28	4.75

Numerical 2 with $S_y = 0.1$



$$\begin{bmatrix} dh_1/dt \\ dh_2/dt \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3H_3 \end{bmatrix}$$

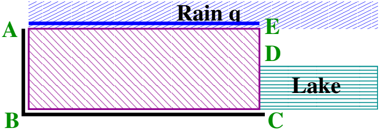
$$\begin{aligned} S_y dh &= (Ah + b) * dt \\ h &= h + dh \end{aligned}$$

With $h_i(0) = H_i$ but h_3 fixed.
We have:

$$\begin{aligned} S_y dh_1/dt &= 0.1 * (h_2 - h_1) + 0.1 \\ S_y dh_2/dt &= 0.1 * (h_1 - h_2) + 0.3 \cdot (H_3 - h_2) \end{aligned}$$

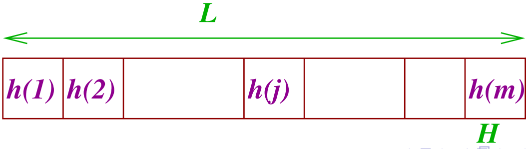
t	h_1	h_2	h_3
0	3	2	5
0.01	2.98	2.37	5
0.02	2.98	2.45	5
0.10	3.04	3.02	5
0.20	3.14	3.48	5

Lake Problem



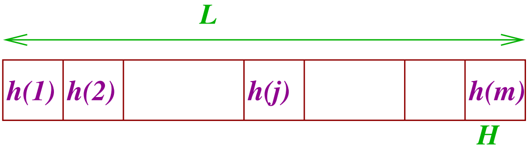
function [hh,x,nn,deltaj]=lakesim(H,q,L,m,K,Sy,dt,n,eps,h)

```
// H is the lake height
// q is the rainfall, L is the length of the land
// m is the number of cells
// K is conductance, Sy is the specific yield

// n is the number of time steps, dt is the time step
// eps is the error bound, h is the starting heads.

// hh:  output, x:the X axis points, nn:  iter, deltaj:  errors
```



The Key update:

```
dd=L/m
for j=2:m-1
      vin=(h(j+1)-h(j))*K*h(j+1)/dd;
      vin=vin+(h(j-1)-h(j))*K*h(j)/dd +q*dd;
      deltaj(j)=vin/Sy/dd;
      hn(j)=h(j)+vin*dt/Sy/dd;
end;
hn(m)=H;
vin=vin+(h(2)-h(1))*K*h(2)/dd +q*dd;
hn(1)=h(1)+vin*dt/Sy/dd;
```



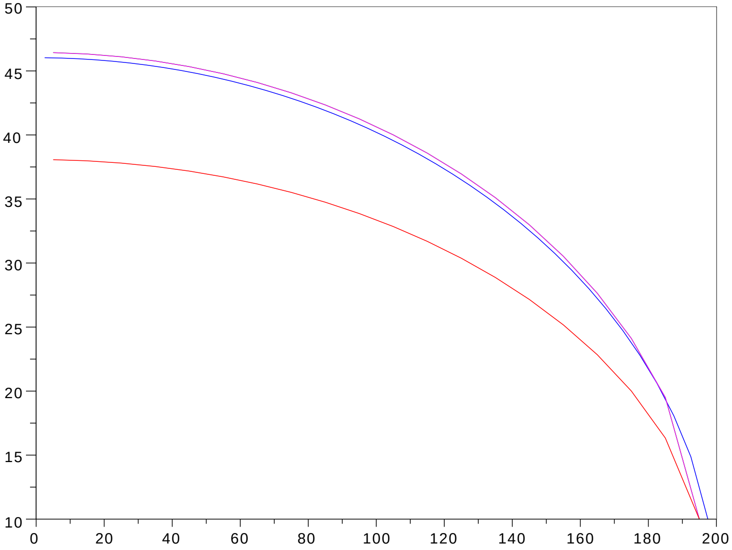
Various Plots

H=10, q=0.1, L=200, K=0.2,Sy=0.3, n=80000, eps=0.00001

```
[hh,xx,nn,ddel]=lakesim(H,q,L,20,0.2,0.3,0.5,n,eps,h')
plot(xx,hh,'g')
[hh,xx,nn,ddel]=lakesim(H,q,L,40,0.2,0.3,0.5,n,eps,[h
h]')
```

didnt work

```
[hh,xx,nn,ddel]=lakesim(H,q,L,40,0.2,0.3,0.2,n,eps,[h
h]')
plot(xx,hh,'b')
[hh,xx,nn,ddel]=lakesim(H,q,L,20,0.2,0.5,0.5,n,eps,h')
plot(xx,hh,'m')
[hh,xx,nn,ddel]=lakesim(H,q,L,20,0.3,0.3,0.5,n,eps,h')
plot(xx,hh,'r')
```


50
45
40
35
30
25
20
15
10
0
20
40
60
80
100
120
140
160
180
200

Discussion

1. How do you think agriculture affect infiltration? And Forests? Why?
2. Groundwater typically moves a few meters vertically and a few hundreds of meters horizontally, per day. What can be inferred from this data?
3. Where in the world should you find confined aquifers?
4. What geological events should influence aquifer thickness and quality?
5. What does negative water head really signify?
6. How would you observe velocity vectors in the lab?
7. Discuss another real-life scenario not covered in class.
8. Is our conservation law of the last slide, good enough?