CS208 Quiz I (Spring 2014)

Time: 1 hour **Date:** Feb 12, 2014

- **Be brief, complete, and stick to what has been asked.**
- **You may cite results/proofs covered in the class without reproducing them.**
- **Do not copy solutions form others.**
- **Penalty for copying: FR grade.**

1. *[20 marks]* Give DFAs accepting the following languages over the alphabet $\Sigma = \{0, 1\}$:
   (a) The set of all strings that, when interpreted as binary integers, are even and multiple of 3.
   (b) The set of all strings that, when interpreted as binary integers, are multiple of 6.
   (c) The set of all strings that, when interpreted as binary integers, are even or multiple of 3.
   (d) The set of all strings that, when interpreted as binary integers, are even and not a multiple of 3.

2. *[5+5+5 marks]* Let $\Sigma = \{0, 1\}$.
   (a) Give an NFA accepting the set of strings $w \in \Sigma^*$ that can be written as concatenation $u.v$ of two strings $u, v \in \Sigma^*$, where u when interpreted as binary is even, while v when interpreted as binary is a multiple of 3.
   (b) Give a regular expression corresponding to the NFA constructed in 2.(a).
   (c) Using the subset-construction give at least 10 states of the DFA corresponding to the NFA in the 2.(a).

3. *[5+5+5 marks]* Let $\Sigma = \{0, 1\}$. For a fixed number $k \in \mathbb{N}$, let $\mathcal{L}_k$ be the set of all languages of strings of length at most k, let $\mathcal{L}_*$ be the union of all such languages, and let $\mathcal{L}$ be the set of all languages over Σ. Formally,

$$\begin{aligned}
\mathcal{L}_k &= \{L \subseteq \Sigma^* \, : \, |w| \leq k \text{ for all } w \in L\}\,, \\
\mathcal{L}_* &= \bigcup_{k \geq 0} \mathcal{L}_k, \text{ and} \\
\mathcal{L} &= \{L \subseteq \Sigma^*\}\,.
\end{aligned}$$

Decide which of the following conjectures are true and which are false. Justify your answer.

- For a fixed $k \in \mathbb{N}$ every language in the set $\mathcal{L}_k$ is regular.
- Every language in the set $\mathcal{L}_*$ is regular.
- Every language in the set $\mathcal{L}$ is regular.