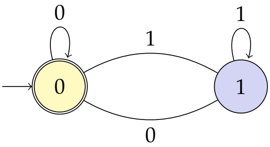
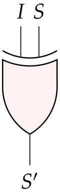
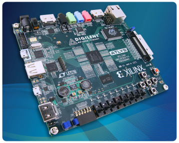
CS 226: Digital Logic Design

Lecture 6: Karnaugh Maps

Ashutosh Trivedi







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Logical functions

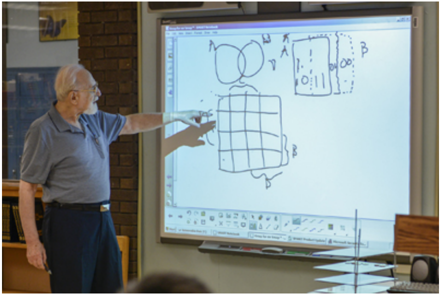
How to represent logical functions:

1. Truth-tables
2. Algebraic expressions
3. Venn diagrams

Logical functions

How to represent logical functions:

1. Truth-tables
2. Algebraic expressions
3. Venn diagrams
4. Karnaugh maps



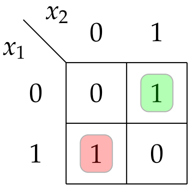
Dr. Maurice Karnaugh, developer of Karnaugh maps or K-maps (1954), explaining K-maps

Karnaugh-maps

- A clever way to represent a truth-table
- It represent the truth-table in such a visual manner to help identifying manipulations of the following nature:
  - $x_1 \cdot x_2 + x_1 \cdot \overline{x_2} = x_1$
  - $x_1 \cdot x_2 \cdot x_3 + x_1 \cdot \overline{x_2} \cdot x_3 + x_1 \cdot x_2 \cdot \overline{x_3} + x_1 \cdot \overline{x_2} \cdot \overline{x_3} = x_1$.
- An intuitive visual way to simplify logical expressions involving $2, 3, 4, 5$, and 6 variables
- The minimization process gives insight to the working of CAD tools.
- We will begin by introducing K-maps of 2 variables, and continue with K-maps of higher complexity.

Two variable Karnaugh maps

x_1	x_2	$f(x_1, x_2)$
0	0	0
0	1	1
1	0	1
1	1	0



$f(x_1, x_2) = \overline{x_1}x_2 + x_1\overline{x_2}$

- A box or a cell corresponding to each (minterm) line of truth-table
- The placement 1′s in corresponding cell identifies the minterms representing a function

Two variable Karnaugh maps

x_1	x_2	$f(x_1, x_2)$
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0	1	1
1	0	1
1	1	0

x_1 \ x_2	0	1
0	0	1
1	1	0

$$f(x_1, x_2) = \overline{x_1}x_2 + x_1\overline{x_2}$$

- A box or a cell corresponding to each (minterm) line of truth-table
- The placement 1's in corresponding cell identifies the minterms representing a function
- Nice graphics, Ashutosh! but what's the point?

Two variable Karnaugh maps

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1	0	1
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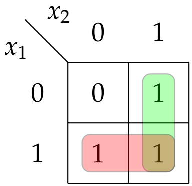

$f(x_1, x_2) = x_1 + x_2$

– Did you see what we did here?

Two variable Karnaugh maps

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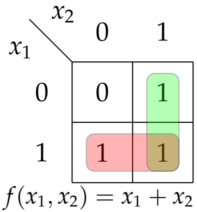

- Did you see what we did here?
- green group $= \overline{x_1}x_2 + x_1x_2 = x_1$
- red group $= x_1\overline{x_2} + x_1x_2 = x_2$
- These groupings of 1's helped us to simplify the logical expressions.

Two variable Karnaugh maps

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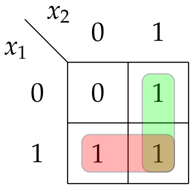

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- We are allowed to make (down and across) groups of 1 2, 4 cells.
- Try to minimize number of groupings while covering all the 1 cells.
- Do not introduce redundant groupings.

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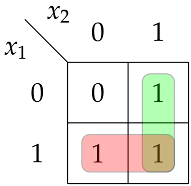

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- We are allowed to make (down and across) groups of 1 2, 4 cells.
- Try to minimize number of groupings while covering all the 1 cells.
- Do not introduce redundant groupings.
- Let's work out some more examples.

Three variable Karnaugh maps

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

x_1 \ x_2x_3	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$$f(x_1, x_2, x_3) = x_1\overline{x_2} + \overline{x_1}x_2$$

- Alert: Did you notice the Gray code ordering?
- Groupings of $1, 2, 4,$ and 8 cells possible

Three variable Karnaugh maps

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

x_1 \ x_2x_3	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$$f(x_1, x_2, x_3) = x_1\overline{x_2} + \overline{x_1}x_2$$

- Alert: Did you notice the Gray code ordering?
- Groupings of $1, 2, 4$, and 8 cells possible
- "Ends are connected"
- Let's work out some examples!

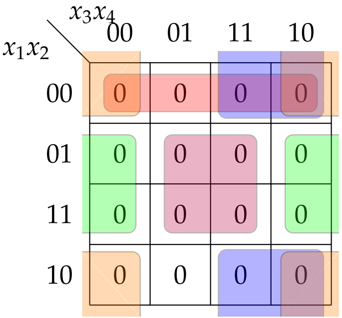
Three variable Karnaugh maps: Example 1

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

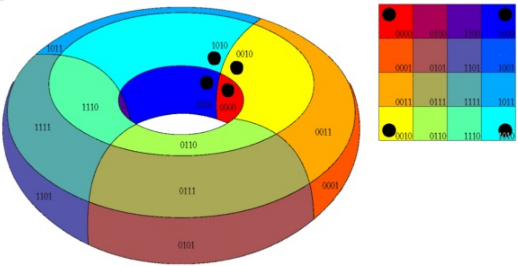
x_1 \ x_2x_3	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$$f(x_1, x_2, x_3) = x_2$$

Four variable Karnaugh maps



Four dimension K-map connected as a Torus



Thumb Rules

1. No zeroes allowed.
2. No diagonal groupings
3. Only power of 2 cells in each grouping
4. Every one must be at least in one grouping
5. Overlapping is allowed
6. Groups may “Wrap-around”
7. Fewest number of groups possible

Additional Topics

– Five and Six variable Karnaugh maps
– Incompletely specified functions
– Generating product-of-sum representation using K-maps