Graph Clustering for Keyword Search

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Introduction

- Keyword searching -important paradigm of searching.
- Keyword search on external memory datagraphs could perform better if the nodes that are connected to each other are retrieved together.
- **Clustering**: finding a grouping of graph nodes such that, connections within it are dense; inter-cluster edges are low.
- **Community**: set of real-world entities that form a closely knit group
- **Objective function**: distance-based measures, cut-size, community-related measures: modularity, conductance
- **Graph Conductance**:

$$\Phi(S) = \frac{|\partial(S)|}{min(Vol(S), Vol(\bar{S}))}$$

For $S \subseteq V$:

* $Vol(S)$: sum of node-degrees in S
* $\partial(S)$: edges from S to $\bar{S}$

Clustering by Graph Partitioning

input: `k` - desired number of partitions

Objective

- group the nodes into `k` clusters, such that, all clusters are of roughly the same size.
- minimize the number of cut edges.

Metis

1. Coarsen the graph, by collapsing edges and grouping nodes.
2. Create a good partition on the smallest graph.
3. Project this partition back onto the original graph, by refining the partition in the intermediate levels.
4. Recursively partition the two clusters obtained, to get `k` partitions.

Shortcomings:

- Cannot find communities of varying sizes.
- Since it creates multiple versions of the graph, requires lot of memory.

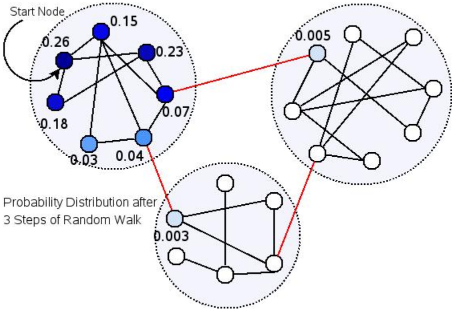
Finding Communities using Random Walks on Graphs

Random walks:

- a graph traversal technique.
- Probability distribution of a walk: probability of a random walk of k steps, started at a particular *startNode*, to be at a particular node at the instant/step of inspection (*nodeProbability*).

Clustering using Random walks:

- Objective: find the cluster to which a particular node belongs, or the enclosing cluster of a seed set.
- Intuition:
  - Walk started from a node in the cluster will remain within it, with a large probability.
  - Probability distribution of the random walk gives a rough ranking of the nodes of the graph.
  - A good cluster can be obtained by considering the highest ranking nodes, and by using conductance to choose the best.

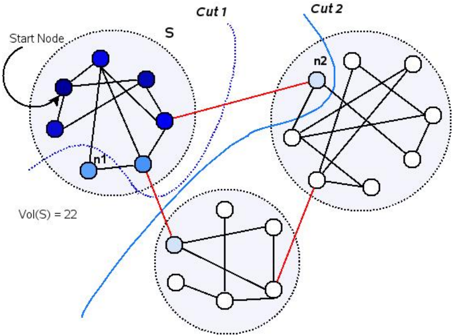


- Sudden drop in probability, outside the cluster boundary

$$\Phi(S) = \frac{2}{22} = 0.09$$

Cut 1: $\Phi(S - n_1) = \frac{4}{22-2} = 0.2$

Cut 2: $\Phi(S + n_2) = \frac{3}{22+3} = 0.12$



- Dip in conductance at cluster boundary

Clustering using Nibble Algorithm [ST04]

Objective: find the cluster to which the seed node belongs

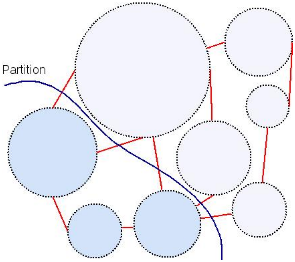
Nibble Algorithm:

input: Start node v, Graph G, Max Conductance θ_0

1. Compute the bound on maxIterations, t_0, and threshold, ϵ.
2. Start spreading probabilities from v.
3. Truncate the walk by setting *nodeProbability* to 0 where it is $< \epsilon$
4. Sort the nodes in the decreasing order of their probabilities.
5. Check if a j exists such that:
   - Conductance of the first j nodes $\leq \theta_0$
   - The above set of nodes satisfy predefined requirements on its volume.
6. If a j was found, then return the first j nodes of the sorted set.
7. Otherwise, do the next step of spreading probabilities and repeat from Step (3).

Partitioning using Nibble:

1. Merge the clusters returned by `Nibble`.
2. Stop merging when the volume exceeds a predetermined fraction of G.
3. Shortcoming: processes the graph in top-down manner - difficult for large graphs.



Clustering using Nibble with seed set [AL06]

- Objective: find the enclosing community for a 'seed set' of nodes
- Modification to Nibble: assign equal probabilities to all nodes in the seed set, and spread from all seed nodes.
- Shortcoming: Seed set is chosen manually.

Shortcomings of the Nibble algorithm

- Specify the conductance of the clusters, apriori.
- May terminate at larger conductance, before finding the best.
- User cannot control the cluster size.
- No control over the spread of the walk.

Clustering using `Modified-Nibble` algorithm : Outline

Overall clustering algorithm

1. Choose a start node.
2. Nibble out a cluster for the start node, and remove it from the graph.
3. Repeat from step (1), until the entire graph is processed.

- Proceed by removing one cluster at a time, rather than processing the entire graph at once.
- Beneficial for clustering massive graphs

`Modified Nibble` Algorithm

1. Set the initial probability of the start node to 1 and start spreading probability from it, for a specific number of steps (batch).
2. Find the best cluster for the currently active nodes, using `Find Best Cluster` algorithm.
3. If the cluster obtained has same or higher conductance than the best cluster of the previous iteration, stop and return the latter.
4. Else, if the conductance has reduced, continue spreading of probabilities from all the active nodes (next batch), and repeat from step (2).

- The conductance of clusters are not taken as input from the user.
- The algorithm finds the cluster of best conductance.

`Find Best Cluster` Algorithm

1. Consider the nodes in the decreasing order of probabilities.
2. The candidate clusters C^i contain nodes from 1 to i of the sorted set.
3. Compute the conductance of all the candidates.
4. Return the one with smallest conductance as the best cluster.

- The algorithm always finds a cluster, unlike the `Nibble` algorithm, which will return a cluster only if it satisfies some specific requirements.

Sample execution of Modified Nibble clustering algorithm

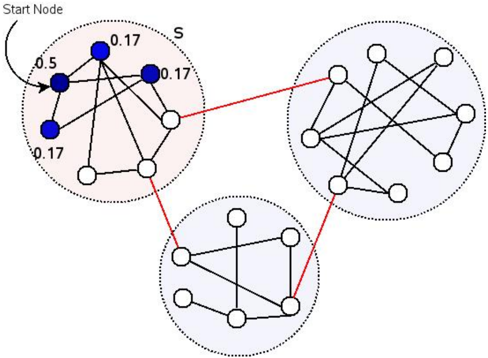


Fig: Prob. distrn. after 1 step

Batch 1

$\Phi(\textit{best cluster}) = \frac{4}{12} = 0.33$

Preferred cluster S, not found yet.

Batch 2

$\Phi(S) = \frac{2}{22} = 0.09$

$\Phi(Cut1) = \frac{4}{22-2} = 0.2$

$\Phi(Cut2) = \frac{3}{22+3} = 0.12$

Best Cluster $= S$

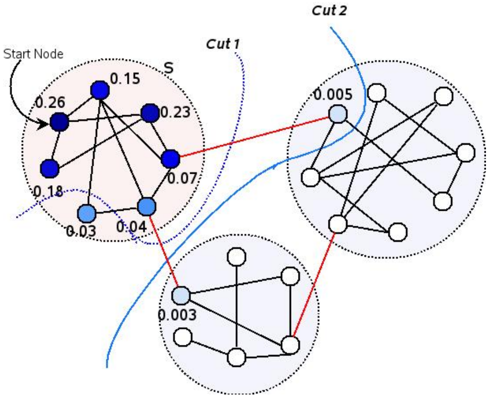


Fig: Prob. distrn. after 3 steps

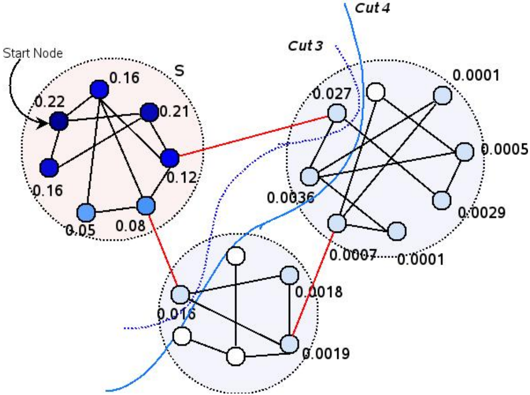


Fig: Prob. distrn. after 5 steps

Batch 3

$\Phi(Cut3) = \frac{4}{28} = 0.14$

$\Phi(Cut4) = \frac{6}{32} = 0.18$

Best Cluster $= S$

H1. Start node

Ideal setting (communities are known beforehand): choose the node which is most '*central*' to the cluster.

(a) Max degree

(b) Min degree

- High-degree nodes are mostly hub nodes.
- Could create many short-cut paths; random walk could spread to a large proportion of the graph, in a few steps.
- Nodes with lower out-degree are usually towards the periphery of the graph.
- Removing clusters from the periphery could make the processing of the core, easier.

Parameters and Heuristics

H2. Nodes spreading in each step

(a) Spread from all active nodes

(b) Only a single node spreads in each step

- δ: amount of probability received by a node, which is yet to be spread to its neighbors.
- A node spreads `spreadProbability` fraction of only its δ; remaining gets added to its `nodeProbability` (not transferable).
- Node to spread next in each step, is the one with largest value for δ.
- Number of iterations in a batch: $\mathtt{m} \times \mathtt{maxClusterSize}$.
- `m` controls the amount of spreading in the graph, prior to testing for best cluster.

Parameters and Heuristics

H3. Self-transition probability of a random walk

- Determined by `spreadProbability`.
- Lower values tend to over-emphasize proximity to the start node.
- Higher values can blur the cluster boundary rapidly.
- `spreadProbability` set to 0.5 for most experiments.

Parameters and Heuristics

H4. Number of iterations in a Batch

- Each invocation of `FindBestCluster` involves sorting - slow down the clustering process considerably.
- Concept of `Batch` of random walks:
  - Use a series to decide the number of steps in a batch.
  - Invoke `FindBestCluster` only after the batch of steps.

Arithmetic Plus Geometric Progression (APGP)

$$t_i^{apgp} = (a + id) + (a\,r^i),\ \ i = 0, 1, 2, \ldots$$

- Choose smaller values for `r` and larger values for `d`.
- For larger values of i, terms of GP will surpass those of AP.
- Number of times sorting is done: $O(log\ totalNumSteps)$

H5. Upper bound on total number of random walk steps

- If the conductance of the best cluster found in a batch has lowered, the spreading of probabilities is continued.
- Upper bound: `maxClusterSize`
  - Ensures that, all nodes of a cluster whose diameter is `maxClusterSize`, are touched before spreading of probabilities is discontinued.

Parameters and Heuristics

H6. Upper bound on number of active nodes

- The random walk can spread to the entire graph, if left checked.
- Intuition for random walk based clustering - it is possible to extract a cluster by exploring only a local neighborhood of the start node.
- Restrict the size of this neighborhood to `maxActiveNodeBound`.

$$\texttt{maxActiveNodeBound} = \texttt{f} \times \texttt{maxClusterSize}$$

Parameters and Heuristics

H7. Behavior on `maxActiveNodeBound`

If the number of active nodes is restricted, options when the number of active nodes reach the bound:

(a) Stop processing and output the best cluster obtained so far.

(b) Continue with spreading, but propagate to only those nodes that are already active.

- Bound might be reached rapidly, due to hub nodes.
- Identifying a good cluster in a very few steps of the walk, becomes difficult.
- Terminating the walk as soon as the bound is reached (option (a)) can hurt the overall quality of the clustering.
- Disadvantage: increases the processing time.

H8. Compaction procedure

- `Modified Nibble` procedure may return clusters of sizes much smaller than `MaxClusterSize`.
- Large number of supernodes in the graph .
- Bundle together, multiple clusters.

CP1. Blind and greedy compaction of all clusters

CP2. Edge aware compaction of all clusters

CP3. Naïve compaction of tiny clusters

  - Both CP1 and CP2 improve edge compression, but create dense graphs.
  - Combine only tiny clusters that don't have any cut edges.
  - Applying CP3 compaction will not make the supernode graph denser.

Co-citation heuristic

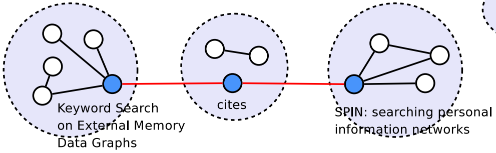
- Co-citation of A_1 and A_2 occurs, when C links to both A_1 and A_2.
- If all co-cited nodes were in a single cluster, all edges to them will be condensed to a very few superedges.

H9. Remove hub nodes

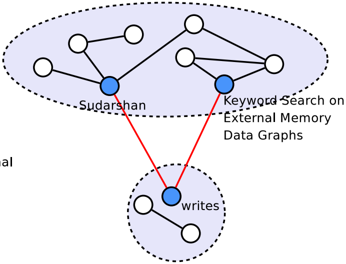
- Select nodes of indegree at least `maxClusterSize`.
- Choose the top $\mathtt{t} \times \mathtt{maxClusterSize}$ and create `t` clusters of size, `maxClusterSize`.
- Execute the clustering procedure on the remainder graph.

Graph formations

- In `FindBestCluster`, candidate clusters were generated by considering the graph nodes in the order of their increasing probabilities.
- Straightforward implementation leads to some interesting formations in the supernode graph - observed from experiments conducted on sample datasets.

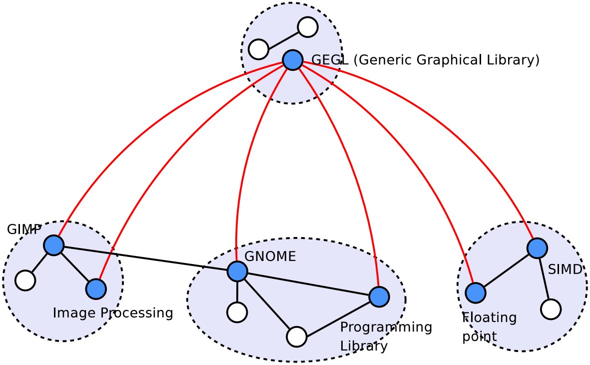


Bridge formation



V formation

Umbrella formation



- *abandoned* nodes: nodes that are separated from all its neighbors.
- Many reasons for occurrence of formations:
  e.g.n_c is a hub which connects to many authoritative nodes. Each neighbor gets absorbed into the cluster for its domain, leaving out n_c.
- Results in more cache misses during search.

Graph formation heuristics

H10. Graph formation heuristic

(a) Post-process
  - After the best cluster is found, add the abandoned nodes to it.
  - Can increase the size of the cluster beyond `maxClusterSize`.

(b) Abandoned node awareness
  - Prevent the occurrence of formations right from the creation of candidate clusters.
  - Add all abandoned nodes to the candidate clusters.
  - Discard candidates whose size goes beyond `maxClusterSize`.

Final `Modified-Nibble` algorithm

input: Graph G, `maxClusterSize`

Overall clustering algorithm

1. If **H9** (co-citation) is used, remove hub nodes from graph.
2. Choose a start node, using **H1**.
3. Nibble out a cluster for the start node, and remove it from the graph.
4. Repeat from step (2), until the entire graph is processed.
5. Use **H8** to compact the clusters obtained.

`Modified Nibble` Algorithm

1. Set the initial probability of the start node to 1.
2. Batch `i`:
   - spread probabilities from all active nodes or a single node (**H2**).
   - amount spread is decided by **H3**.
   - number of iterations in this batch is decided by **H4**.
   - if `maxActiveNodeBound` is used (**H6**), according to **H7**:
     (a) stop this batch and proceed to step 6
     (b) continue, but spread only to already active nodes.
3. Find the best cluster C_i for Batch `i`, using Modified FindBestCluster algorithm.
4. If C_i has same or higher conductance than C_{i-1}, stop and set C_{best} as C_{i-1}, and go to step 6.
5. Else, C_{best} is C_i and start next batch. But, if number of iterations have reached the bound set using **H5**, then go to step 6.
6. If graph heuristic **H10** is used and is set to (a)-post process, add the abandoned nodes of C_{best} to it.
7. Return C_{best} as the best cluster of start node.

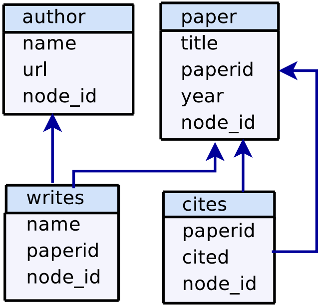
Modified FindBestCluster Algorithm

1. Consider the nodes in the decreasing order of probabilities.
2. The candidate clusters C^i contain nodes from 1 to i of the sorted set.
3. If graph heuristic **H10** is used, and is set to (b) - abandoned node awareness, for all candidates, add the abandoned nodes; and discard larger ones.
4. Compute the conductance of all remaining candidates.
5. Return the one with smallest conductance as the best cluster.

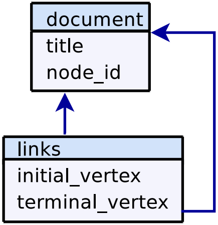
Experiments and Analysis

`Digital Bibliography Library Project (dblp)` (2003 version)

- Tables: `author`, `cites`, `paper`, `writes`
- Number of nodes: 1,771,381
- Number of edges: 2,124,938
- max degree = 784



`Wikipedia` (2008 version)



- Tables: `document`, `links`
- Number of nodes: 2,648,581
- Number of undirected edges: 39,864,569
- max degree = 267,884

Base implementation of Modified Nibble clustering

Heuristic / Parameter		Choice / Value
H1 - start node	:	max degree
H2 - nodes spreading in each step	:	all active nodes
H3 - self-transition probability	:	0.5
H4 - number of steps in a batch	:	APGP ($\mathtt{a}=2$, $\mathtt{d}=7$, $\mathtt{r}=1.5$)
H5 - maximum number of steps	:	`maxClusterSize`
H6 - `maxActiveNodeBound`	:	$\mathtt{f} = 500$
H7 - behavior on H6	:	stop on `maxActiveNodeBound`
H8 - compaction	:	CP1 - blind & greedy compaction
H9 - co-citation	:	no

- Doesn't take care of the graph formations.

BI - for short

Node and edge compression

$$\text{Node Compression} = \frac{\text{number of nodes in the original graph}}{\text{number of clusters}}$$

$$\text{Edge Compression} = \frac{\text{number of edges in the original graph}}{\text{number of inter-cluster edges}}$$

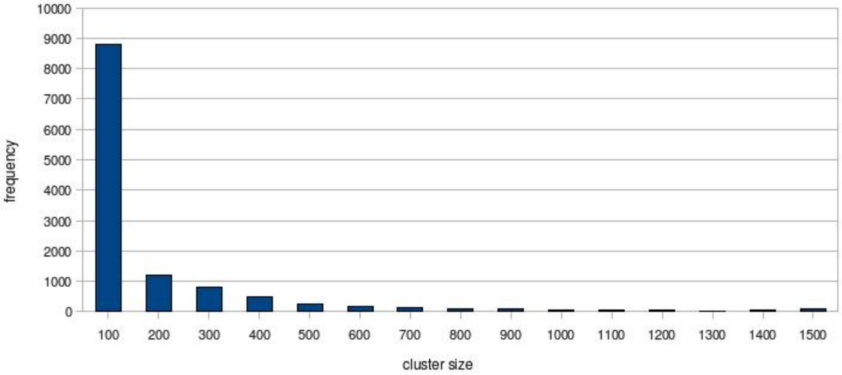
- Node compression is easier to obtain.
- Edge compression - main indicator of quality of clustering.
- Higher the edge compression, better the clustering.

Compression on dblp3

maxClusterSize	# clusters	edge compression
100	24,113	10.31
200	12,698	12.78
400	6,709	15.53
800	3,505	18.55
1500	1,909	23.46

- By increasing `maxClusterSize` from 100 to 1500, compression improves 2 times.

Chart of cluster size vs. frequency of `dblp3`



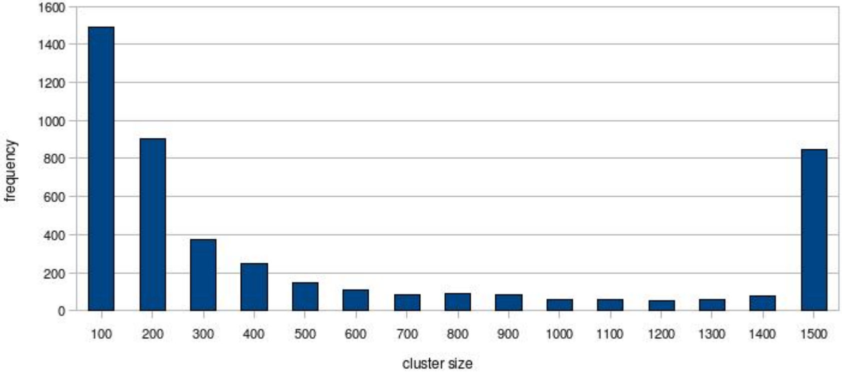
- Indicates that the inherent clusters of `dblp3`, are mostly of size 100 to 400.

Compression on wiki

maxClusterSize	# clusters	edge compression
200	16,208	3.203
400	8,052	5.031
1500	2,205	21.299

- By increasing `maxClusterSize` from 200 to 1500, compression improves by more than 6 times.

Chart of cluster size vs. frequency of `wiki`



- There are many communities in wikipedia of large size.
- The last entry indicates that there are communities of even larger size.

Analysis of the effect of heuristics and parameters on compression

H2 - nodes spreading in each step

(a) all active nodes spread in each step of the walk

(b) only a single node spreads in each step

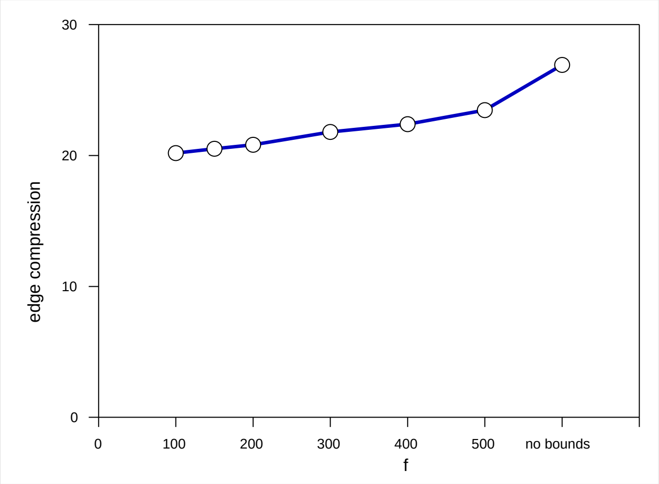
H2	# clusters	# inter-cluster edges	edge compression
(a)	61,633	96,101	22.115
(b)	73,839	118,406	17.946

Edge compression on `dblp3`. (*settings: maxClusterSize = 1500, no compaction*)

- Higher compression with H2(a).

H6 - upper bound on active nodes I

$$\texttt{maxActiveNodeBound} = \texttt{f} \times \texttt{maxClusterSize}$$



Effect of `f` on edge compression in `dblp3` *(mcs = 1500)*

H6 - upper bound on active nodes II

- Edge compression improves with increase in `f`.
- Compression improves to about 27 when number of active nodes are not bound.
- With $\mathtt{f} = 500$, compression obtained is 23.4.
- For an improvement in compression by a factor of 1.14, we incur 2.5 times the processing cost.
- We upper bound the number of active nodes, with $\mathtt{f} = 500$.

H7 - behavior on maxActiveNodeBound

Following options when the number of active nodes reach the bound:

(a) terminate the search

(b) continue spreading, but only to current active nodes

	# clusters	**edge compression**	**time**
H7(a)	77,462	14.39	1.5 hrs
H7(b)	65,883	16.54	4 days

Edge compression on `dblp3` (*settings: startnode - minDegree, no compaction*)

- Edge compression improves when the search for clusters is continued on reaching the bound.
- But, processing time shoots up, to 4 days.
- We use option H7(a) - stop on `maxActiveNodeBound`.

H9 - co-citation heuristic for wikipedia

- H9 heuristic - remove hub nodes, prior to clustering.
- Number of hub nodes removed = $\mathtt{t} \times \mathtt{maxClusterSize}$.

t	# clusters	edge compression
0	2,350	22.431
1	2,294	29.867
2	2,290	30.554

Edge compression on `wiki`. (*settings: minDegree start, H7(b)-continue on maxActiveNodeBound*)

- When top indegree nodes are removed, edge compression increases from 22.4 (t=0) to 29.8 (t=1) .
- Degree of co-citation of these nodes are high.
- But, by removing twice the number of top indegree nodes, improvement is negligible - co-citation drops with decreasing degree.
- H9 could create many short-cut paths in the supernode graph.

H10 - heuristics for graph formations

Heuristic	maxClusterSize	Bridge	V	Umbrella
BI	200	480	148	3,466
BI	400	412	126	3,014
BI + H1(b)	400	584	95	4,588
BI + H1(b) + H7(b)	400	327	22	1,058

Graph formations on `dblp3` (*settings: no compaction*)

Heuristic	maxClusterSize	Umbrella
BI	1500	180,725
BI + H1(b) + H7(b)	1500	291,068
BI + H1(b) + H7(b) + H9	1500	246,864

Graph formations on `wiki` (*settings: no compaction*)

- Graph formations are prevalent.

H10 - heuristics for graph formations

(a) Post-process

Dataset	maxClusterSize	Final maxClusterSize
dblp3	200	323
wiki	1500	5627

Increase in the final cluster size using H10(a)

- Using H10(a), increase in the final cluster size for `wiki` is not within acceptable limits.
- H10(b) : Abandoned node awareness - will produce formation-free clusters of size within the `maxClusterSize` parameter.
- We will use H10(b).

Final settings for Modified Nibble clustering

Heuristic / Parameter		Choice / Value
H1 - start node	:	max degree
H2 - nodes spreading	:	all active nodes
H3 - self-transition probability	:	0.5
H4 - number of batch steps	:	APGP with `a`=2, `d`=7, `r`=1.5
H5 - max number of steps	:	`maxClusterSize`
H6 - `maxActiveNodeBound`	:	`f` = 500
H7 - behavior on H6	:	stop on `maxActiveNodeBound`
H8 - compaction	:	CP3-naïve compaction of tiny clusters
H9 - co-citation	:	no
H10 - graph formation	:	abandoned node awareness

Comparison with Other Clustering Algorithms

Final Implementation of `Modified Nibble` clustering algorithm (FI), compared with:

- EBFS
- Metis

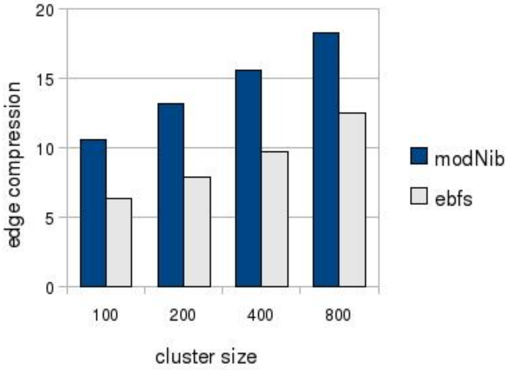
Comparison metrics:

- edge compression on `dblp3` and `wiki` datasets.
- connection query performance, using the Incremental Expansion Backward search algorithm on `dblp3`

  e.g. `krishnamurthy parametric query optimization`
- near query performance on `dblp3`

  e.g. `author (near data mining)`
- time and space requirements for clustering.

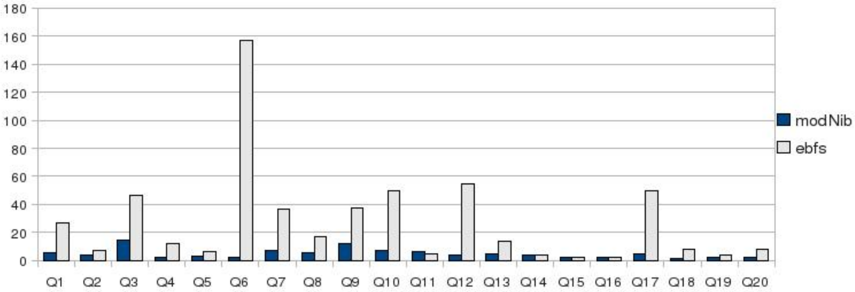
EBFS: edge compression



Edge compression on `dblp3` of FI and EBFS

- FI is able to achieve better edge compression than EBFS, on the `dblp3` dataset.

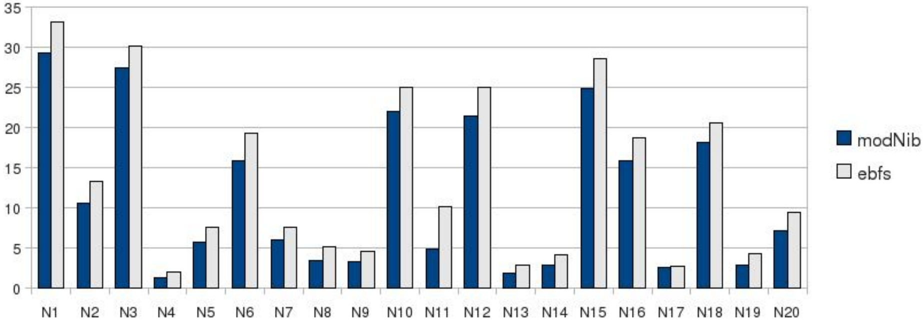
EBFS: performance on connection queries



CPU + IO time (sec) : connection query on `dblp3`

- Final implementation of modified nibble is out-performing ebfs by a very large margin.

EBFS: performance on near queries



CPU + IO time (sec) : near queries on `dblp3`

- FI is able to beat EBFS on all queries considered.

Metis

- Difficulty in comparing FI with Metis: parameters and objectives are much different.
- For comparison purposes, we use clusterings whose `maxClusterSize` and average cluster sizes are comparable.

FI clustering used for `dblp3`

- `maxClusterSize` = 400
- number of clusters = 31,215

Metis clustering used for `dblp3`

- `k` (number of clusters) = 30,000
- maximum cluster size = 335

Metis: Edge compression

Edge compression on dblp3

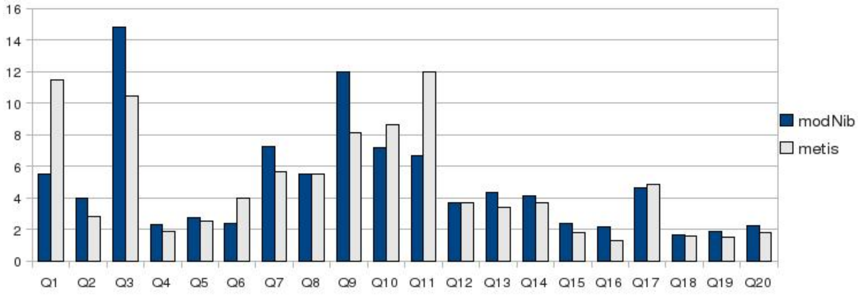
	#clusters	maxClusterSize	edge compression
FI	31,215	400	15.6
Metis	30,000	335	9.616

Edge compression on wiki

	#clusters	maxClusterSize	edge compression
FI	11,305	1600	17.3
Metis	3,000	1,096	15.7
Metis	4,000	16,353	9.13

- Modified Nibble is able to achieve better edge compression than Metis.

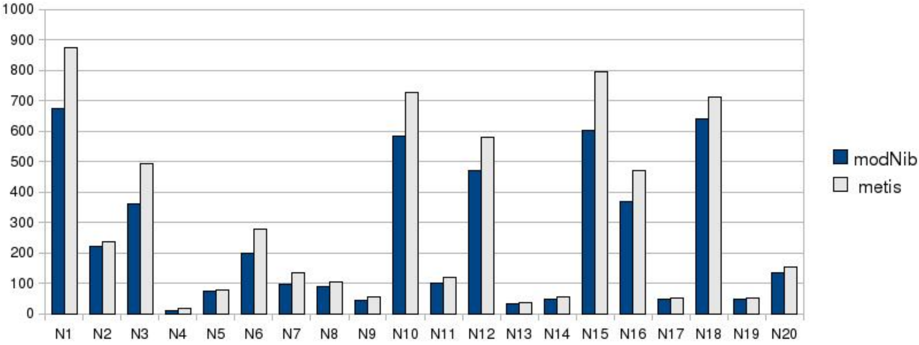
Metis: performance on connection queries



CPU + IO time (sec) : connection query on `dblp3`

- Metis performs really well on some keyword queries, while FI outperforms Metis on some others.
- Difference in performance can also be caused by the queries under consideration.

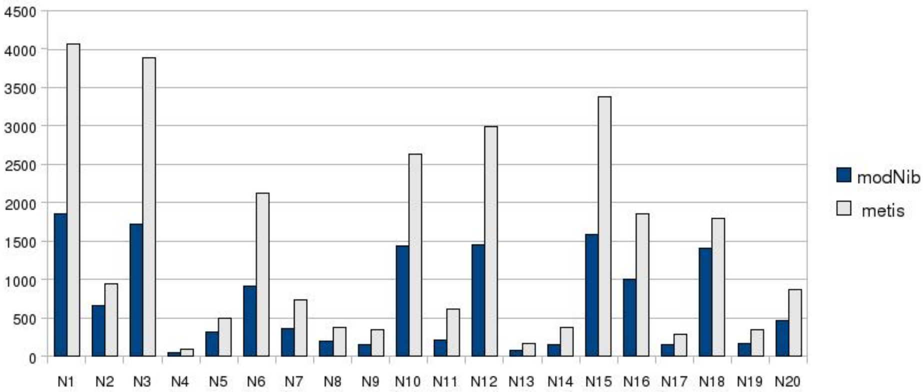
Metis: performance on near queries



number of supernodes with near keywords match : near queries on `dblp3`

- In all cases, number of supernodes with near keywords match, for FI is lesser than Metis.
- Clusters produced by FI, also groups the paper titles in dblp3.

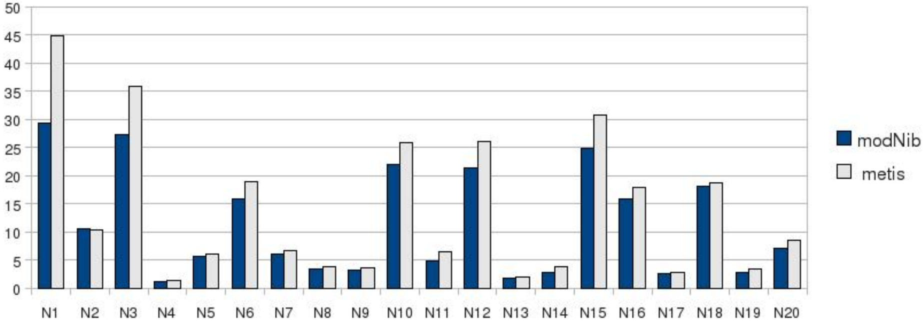
Metis: performance on near queries



cache misses : near queries on `dblp3`

- FI has significanly lesser cache misses than Metis.

Metis: performance on near queries



CPU + IO time (sec) : near queries on `dblp3`

- FI outperforms Metis on almost all queries considered.

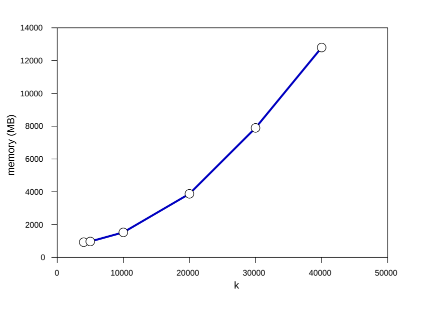
Time and space required for clustering

Modified Nibble Clustering algorithm:

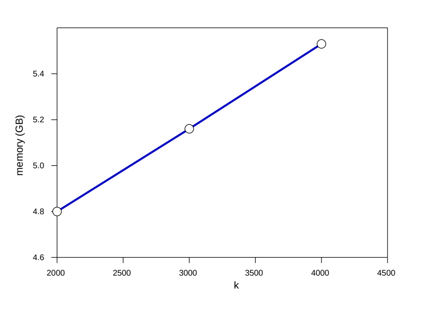
dataset	**time**	**space**
dblp3 (132 MB)	$\sim$ 1.5 hrs	190 MB
wiki (1.9 GB)	$\sim$ 1.5 days	2 GB

- Space requirements of FI - very close to the size of the graph.
- It was found that difference in time and space required, for different `maxClusterSize` is negligible.

Time and space required for clustering : Metis



k vs memory for dblp3



k vs memory for wiki

- Space required grows almost linearly with k.
- Constants are very high (e.g. for k = 40,000 on dblp3, memory required is 12.8 GB).
- Time taken: dblp3 - 5 mins, wiki - 1.5 hrs.
- But, since clustering is done offline, time may not be an issue, but space may be.

Conclusions

- We proposed an algorithm called Modified Nibble Clustering algorithm, for clustering data represented as graphs, using the technique of random walks. It improved upon the earlier Nibble algorithm.
- Outlined several heuristics that improved its performance.
- Compared our algorithm with EBFS and Metis, where the metrics used were edge compression, keyword search performance, time & space requirements for clustering, on sample graphs.
- Results showed that Modified Nibble clustering outperformed EBFS uniformly, and Metis, for some metrics.

Future Work

- Formulating a clustering objective for getting good connection query performance, on external memory search systems.
- Test the effect of combinations of heuristics.
- Test the performance of Modified Nibble clustering algorithm on larger graphs, that fit in memory.
- Modifying the algorithm to run in a distributed environment, so that massive graphs can be handled.
- Improve the speed of clustering process, by nibbling out multiple clusters in parallel.

References

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[AL06] Reid Andersen and Kevin J. Lang. Communities from Seed Sets. *Proceedings of the 15th international conference on World Wide Web*, pages 223-232, 2006.

[KK98] George Karypis and Vipin Kumar. Multilevel k-way Partitioning Scheme for Irregular Graphs. *Journal of Parallel and Distributed Computing 48*, pages 96-129, 1998.

[Sav09] Amita Savagaonkar. Distributed Keyword Search. *MTech. Project Stage 3 Report, Indian Institute of Technology, Bombay*, 2009.

[ST04] Daniel A. Spielman and Shang-Hua Teng. Nearly-Linear Time Algorithms for Graph Partitioning, Graph Sparsification, and Solving Linear Systems. *ACM STOC-04*, pages 81-90, 2004.

Extra Slides

Detailed pseudocode

Overall clustering algorithm

input: Graph G

1. Set $G' = G$.
   If co-citation heuristic H9 is used, set G' to the remainder graph, after removing hub nodes.
2. Choose start node n_s according to H1.
3. Obtain cluster $C_s = \texttt{ModifiedNibble}(n_s, G')$
4. Set $G' = G' - C_s$, and save C_s.
5. Repeat from step (2), until G' is null.
6. Compact the clusters obtained, using H8 procedure.

ModifiedNibble I

input: start node n_s, Graph G'

1. initialization:
   - set `nodeProbability` of n_s to 1 and add it to the `activeNodes` set.
   - set `maxSteps` according to H5.
   - if number of active nodes are bounded, calculate `maxActiveNodeBound` using H6.
   - set `totalSteps` to 0.
2. `Batch i`:
   initialization:
   - get term t_i from the series chosen using H4.
   - set `batchSteps` to (t_i - `totalSteps`).
   - but, if t_i exceeds `maxSteps`, set `batchSteps` to (`maxSteps` - `totalSteps`).

ModifiedNibble II

do the following for `batchSteps` number of times:

1. spread from all nodes in `activeNodes` or a single node, according to H2.
2. the amount of spreading is determined by `spreadProbability` as chosen in H3.
3. update `nodeProbability` of all nodes, with the probabilities accumulated from their neighbors.
4. update `activeNodes` set to contain all nodes with positive values for their `nodeProbabilities`.
5. if number of active nodes are bounded, check if `maxActiveNodeBound` has been reached. If yes, then, according to the choice of H7, do as below:
    - H7(a) : stop this batch, and proceed directly to step 3.
    - H7(b) : continue this batch, but spreading is done to only those nodes, which are already in `activeNodes`.

ModifiedNibble III

3. obtain cluster $C_i = \texttt{ModifiedFindBestCluster(activeNodes, } G')$.
4. find conductance of C_i w.r.t the current graph G', $\Phi_{G'}(C_i)$.
   - if $\Phi_{G'}(C_i) \geq \Phi_{G'}(C_{i-1})$, set C_{best} to C_{i-1}, and go to step 6.
   - else, set C_{best} to C_i
5. do the following and repeat from step 2 onwards (`Batch i+1`).
   - if t_i exceeds `maxSteps`, go to step 6.
   - else, set `totalSteps` to t_i.
6. if graph heuristic H10 is being used, and is set to H10(a), set C_{best} to $C_{best} \cup \{n_c \mid n_c \textit{ is abandoned by } C_{best}\}$
7. return C_{best} as the best cluster of n_s.

ModifiedFindBestCluster

input: set `activeNodes`, *graph* G'

1. normalize the `nodeProbability` of all nodes in `activeNodes`
2. sort the nodes in `activeNodes` set, in the decreasing order of their degree-normalized `nodeProbabilities`.
3. candidate clusters C^j - set of nodes from 1 to j, in sorted order, where $j = min(\texttt{maxClusterSize}, |\texttt{activeNodes}|)$.
4. if the graph heuristic H10 is used, and is set to H10(b), then do the following:
   - set each C^j to $C^j \cup \{n_c \mid n_c \textit{ is abandoned by } C^j\}$
   - if for any j, $|C^j|$ exceeds `maxClusterSize`, discard C^j.
5. for all remaining candidate clusters, compute $\Phi_{G'}$.
6. return that candidate, which has the smallest conductance.

Clustering using Nibble Algorithm

Objective: find the cluster to which seed node belongs

Nibble Algorithm:

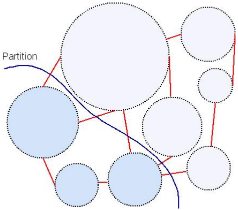
input: Start Vertex v, Graph G, Conductance θ_0, a positive integer b

1. Compute t_0 ($\propto ln(m)/\theta_0^2$), γ ($\propto \theta_0/ln(m)$), ϵ_b ($\propto \theta_0/ln(m)t_0 2^b$)
2. Start a lazy random walk from v
3. At each step: (until t_0)
    - Do the Truncation Operation with threshold $= \epsilon_b$
    - Sort the nodes in the decreasing order of their probabilities
    - Check if a $\tilde{j}$ exists such that:
        - $\Phi(\{1, ..., \tilde{j}\}) \leq \theta_0$
        - $Pr(\tilde{j}) \geq \gamma/Vol(\{1, ..., \tilde{j}\})$
        - $Vol(\{1, ..., \tilde{j}\}) \leq \frac{5}{6}Vol(V)$, then, output $C = \{1, ..., \tilde{j}\}$
4. Do the next step of random walk and repeat from Step (3)

Random Nibble Algorithm:

input: G, θ_0

1. Set v to be the largest degree vertex of G
2. Choose b in $1, \ldots, \lceil log(m) \rceil$ according to
$$Pr[b = i] \propto 2^{-i}$$
3. Call $\texttt{Nibble}(G, v, \theta_0, b)$



Partition Algorithm:

input: G, θ_0, $p \in (0, 1)$

1. Compute number of iterations j ($\propto m\lceil lg(1/p) \rceil$
2. Start with the entire graph, i.e., set W to V
3. Call $\texttt{RandomNibble}(G(W), \theta_0)$
4. Add the cluster nodes returned by `RandomNibble` to the answer
5. Now, remove these nodes from W
6. If $Vol(W) \leq \frac{5}{6} Vol(V)$, then stop
7. Else, repeat from Step (3)

Multiway Partition Algorithm:

input: G, θ, p

1. Set θ_0 to $(5/36)\theta$
2. Compute number of iterations t ($\propto (lg\ m)^2$)
3. Start with the entire vertex set, i.e, set $\mathcal{C}_1$ to V
4. In each step: For each component $C \in \mathcal{C}_t$,
   Call `Partition`$(G(C), \theta_0, p/m)$
5. Add the two partitions returned to $\mathcal{C}_{t+1}$ and repeat from Step 4
6. Final clustering is given by $\mathcal{C}_{t+1}$

Running Time:

`Nibble` : $O(2^b ln^4(m)/\theta_0^5$

`Multiway Partition` : $O(m\ (lg(1/p)\ lg^{O(1)}(m))/\theta^5)$

Clustering using Seed Sets [AL06]

Objective: find the enclosing community of a "seed set" of nodes

Algorithm:

1. Assign equal probabilities to all nodes in the seed set, and start spreading probabilities.
2. Sort the vertices in descending order of their degree-normalized probabilities.
3. Truncate the walk for nodes with probabilites lesser than a predefined threshold.
4. Find a j such that the set of first j nodes, C, satisfy the test for a good community: the probability outside C is lesser than a predetermined fraction of $\Phi(C) \times \#numSteps$
5. If a j is found, stop and return that set as the community.
6. Else, continue the random walk from step (2) onwards.

Shortcoming:

The seed set is chosen manually.

H1 - start node

	edge compression		
	maxClusterSize		
start node	200	400	800
min degree	11.81	14.39	16.95
max degree	12.78	15.53	18.55

Table: Edge compression on `dblp3`

- Compression obtained maxDegree start is always higher than that of minDegree.

H3 - spread probability

spreadProbability	# clusters	edge compression
25	79,065	16.052
50	78,435	16.163
75	74,356	17.495
85	71,364	18.371
95	65,616	19.367

Edge compression on `dblp3`. (*settings: H2(b), mcs = 1500, no compaction.*)

- Edge compression increases with `spreadProbability`.
- Number of clusters reduces by about 13,500 - clusters found are of larger size.
- With higher `spreadProbability`, larger fraction of total probability can escape the cluster boundary.
- Larger clusters could be merging together multiple smaller ones.
- To avoid such effects, we use 0.5 for all the experiments.

H8 - compaction techniques

Following compaction techniques tried:

CP1 Blind and greedy compaction of all clusters

CP2 Edge aware compaction of all clusters

CP3 Naïve compaction of tiny clusters

- CP1 and CP2 improves edge compression, since they combine clusters which may have edges across them.
- But, applying CP1 and CP2, made the supernode graph, denser.
- Searching in a dense supernode graph, quickly spreads to a very large fraction of it, and can incur more cache misses.
- CP3 doesn't affect edge compression and does make the supernode graph denser.

We choose CP3, since we want to strike a balance between the following:

- number of supernodes in the graph
- denseness of the supernode graph

Q1	sudarshan soumen
Q2	vapnik support vector
Q3	divesh jignesh jagadish timber querying XML
Q4	sudarshan widom
Q5	giora fernandez
Q6	david fernandez parametric
Q7	chaudhuri agrawal
Q8	widom database
Q9	raghu deductive databases
Q10	"prabhakar raghavan" "raghu ramakrishnan"
Q11	rozenberg "petri nets"
Q12	rozenberg janssens "graph grammars"
Q13	silberschatz "disk arrays"
Q14	ramamritham "real time"
Q15	"howard siegel" SIMD
Q16	frieze "random graphs"
Q17	romanski ada
Q18	banerjee "distributed memory" multicomputers
Q19	didier "possibilistic logic"
Q20	tamassia "graph drawing"

connection queries for `dblp3` dataset

N1	author (near "data mining")
N2	paper (near christos faloutsos nick roussopoulos)
N3	author (near "query processing")
N4	author (near "possibilistic logic")
N5	paper (near chaudhuri agrawal)
N6	paper (near "deductive databases")
N7	paper (near "random graphs")
N8	author (near "handwriting recognition" "subgraph isomorphism")
N9	paper (near "branching programs")
N10	paper (near "petri nets" "context free grammars")
N11	author (near "graph grammars")
N12	author (near "load balancing")
N13	author (near "scan circuits")
N14	author (near "kolmogorov complexity" "match making")
N15	author (near "distributed memory" multicomputers)
N16	author (near "image retrieval")
N17	author (near "reliability performance")
N18	paper (near smith siegel McMillen)
N19	author (near "maximum matchings" "game trees")
N20	author (near "NP complete")

near queries for `dblp3` dataset