Joint MAP for Blind Deconvolution: When Does it Work?

Renu M. Rameshan,

Subhasis Chaudhuri, Rajbabu Velmurugan

Indian Institute of Technology Bombay, India.

Indian Conference on Vision Graphics and Image Processing 2012

Outline

- Introduction
- Joint estimation for blind deconvolution
- Motivation for using PSF prior
- Choice of regularization factor
- Conclusions

Introduction

$$y = k \otimes x + n$$

y Blurred, noisy observation

x Original sharp image

k Point spread function - PSF

n Additive white Gaussian noise

Estimate x *and* k *given only* y *and the noise statistics.*

Bilinear ill-posed problem

- x and k are both unknowns.
- Keeping one of the variables fixed, the problem is linear, ill-posed.
- Blind deconvolution is a bilinear, ill-posed problem.
- Using regularizers and a quadratic data fitting term leads to the cost function:

Cost function

$$C(\underline{x}, \underline{k}) = \| \underline{y} - K\underline{x} \|^2 + \lambda_x R_x(x) + \lambda_k R_k(k)$$

$R_x(x)$ - Image regularizer

$R_k(k)$ - PSF regularizer

λ_x - Image regularization factor

λ_k - PSF regularization factor

Use *alternate minimization* to estimate x and k.

MAP estimation (1of 2)

$$p(x, k|y) \propto p(y|x, k)p(x)p(k)$$

$p(y|x, k)$ – likelihood

$p(x)$ – image prior $\quad$ $p(k)$ – blur prior

x and k are assumed to be independent

$$C(\underline{x}, \underline{k}) = \| \underbrace{\underline{y} - K\underline{x}}_{\substack{\text{DF} \\ \text{(data fitting term)}}} \|^2 + \lambda_x \underbrace{R_x(x)}_{\propto p(x)} + \lambda_k \underbrace{R_k(k)}_{\propto p(k)}$$

MAP estimation (2 of 2)

$$R_x(x) = -\log p(x)$$

$$R_k(k) = -\log p(k)$$

A commonly used image prior is

$$\log p(x) = -\sum_i (|g_{h,i}(x)|^\alpha + |g_{v,i}(x)|^\alpha)$$

$g_{h,i},\ g_{v,i} -$ first order horizontal, vertical difference at location ***i***.

MAP – reported failures and solutions

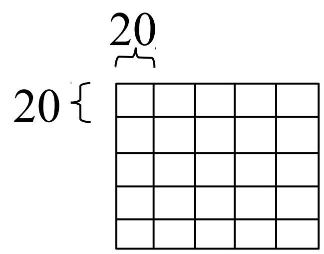
[Fergus 2006]

- A gradient based image prior minimizes all the gradients, whereas natural images do have some strong gradients.
- Estimate PSF by maximizing the marginal probability and use non-blind deconvolution.

[Shan 2008]

- Uses a modified likelihood function and an image prior which use the global and local properties of images.

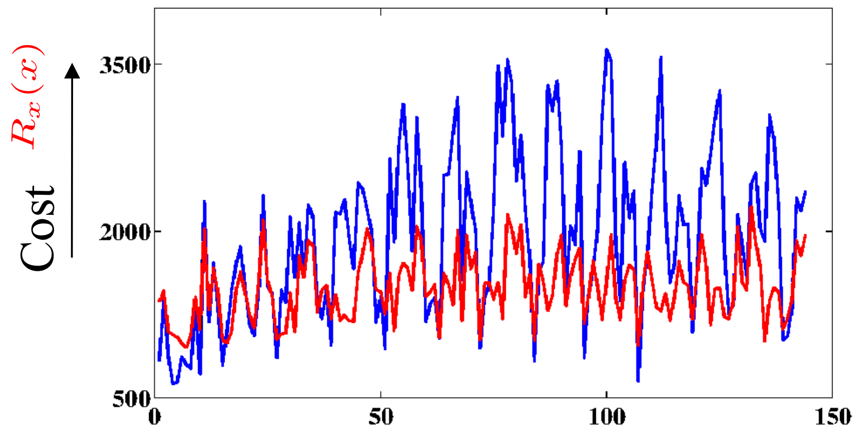
Behaviour of prior w.r.t. blurring



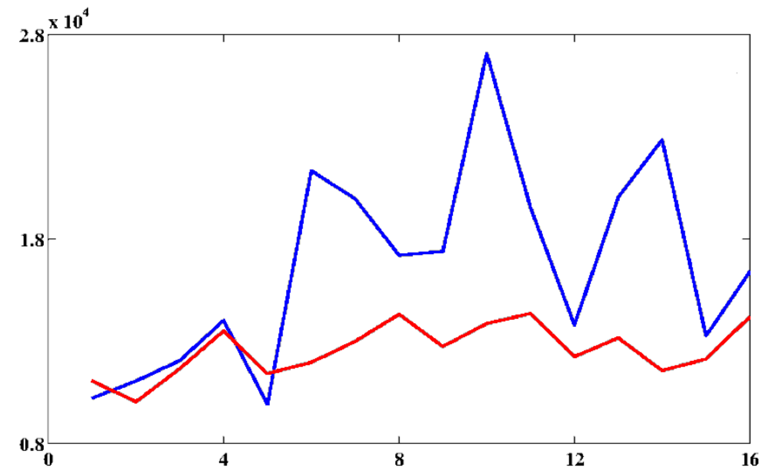
Original

Blurred

Block size: 20



Block size: 60



Block number

Why joint estimation fails? [Levin 2009] (1 of 2)

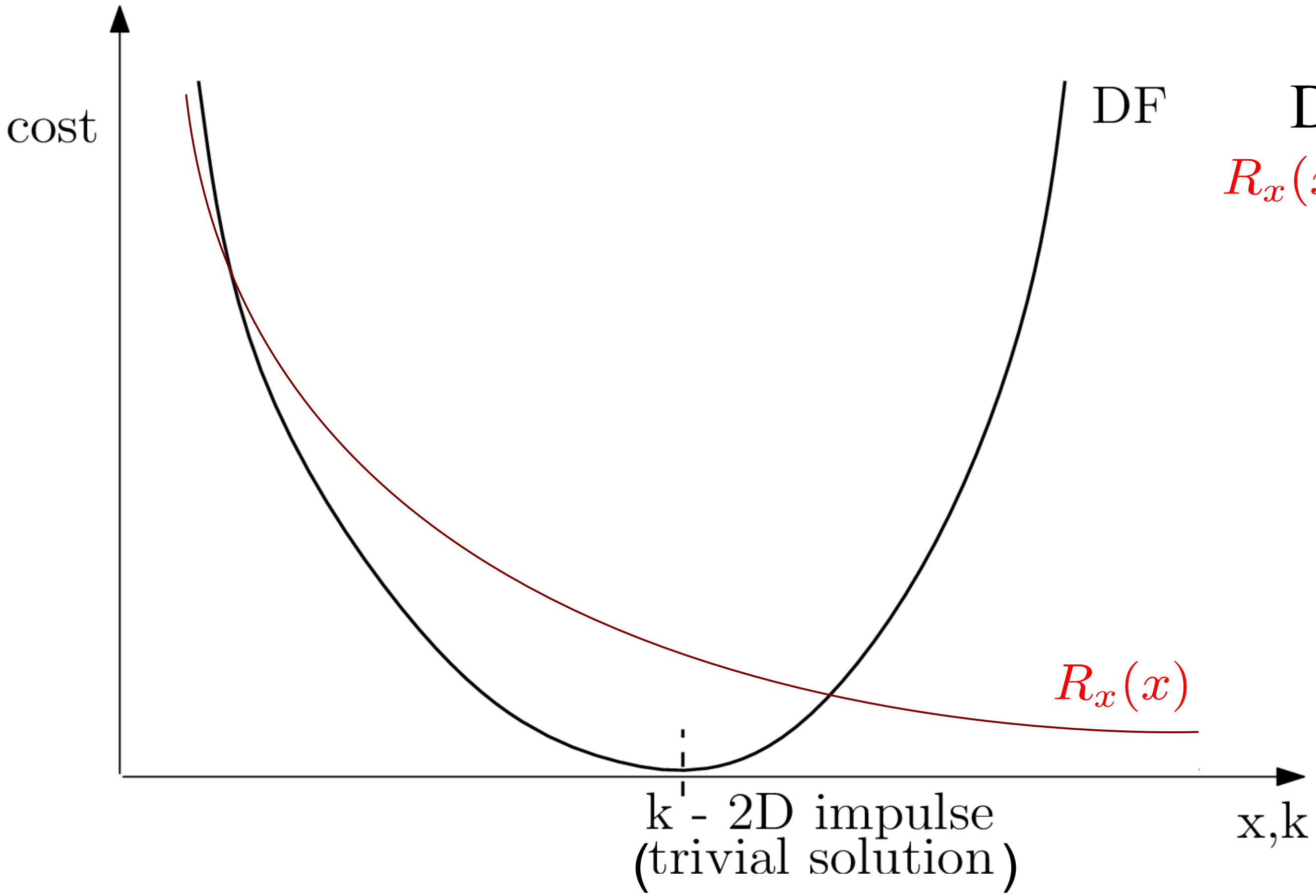
With $R_x(x)$ as the image prior and PSF prioir assumed to be uniform, the estimate is

$$(\hat{x}, \hat{k}) = \arg\min_{x,k} DF(x,k) + \lambda_x R_x(x)$$

Trivial solution: k an impulse and $x = y$.

[Levin 2009] estimates k by marginalization and uses a non-blind method for deconvolution.

Why joint estimation fails? (2 of 2)



DF – Data fitting term

$R_x(x)$ – Image prior

When joint estimation works? (1 of 2)

Use a PSF prior which prevents the trivial solution of the PSF becoming an impulse.

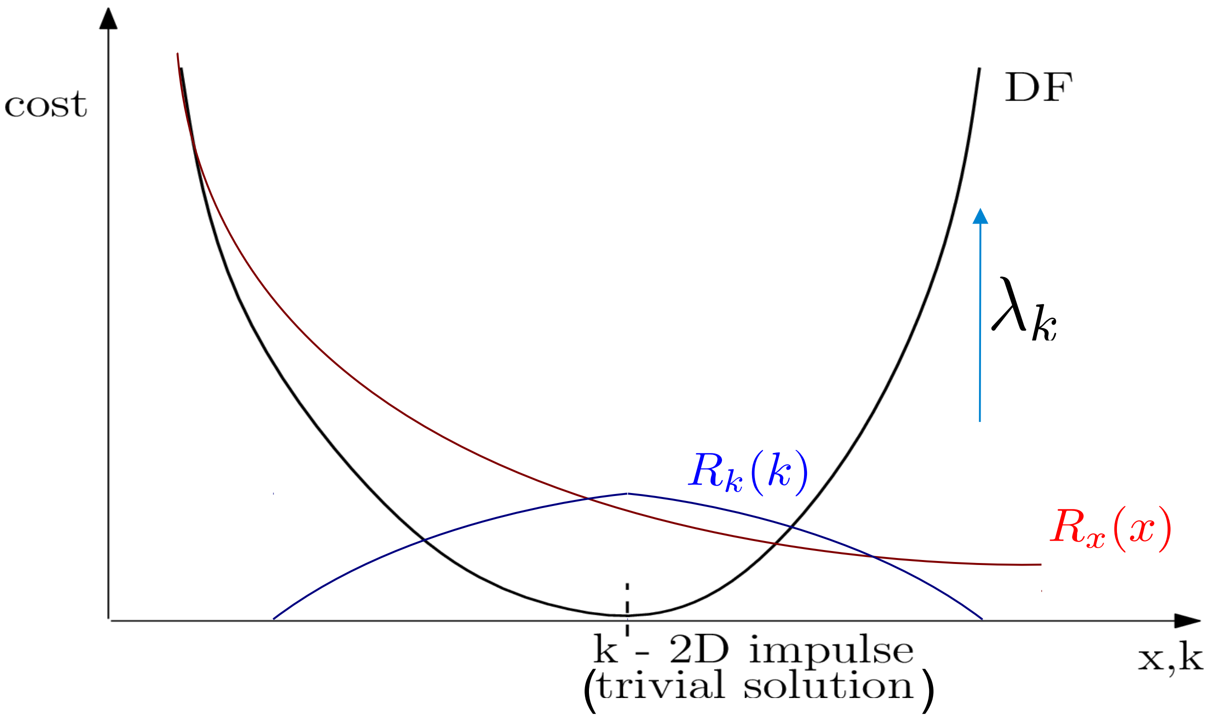
With this, the estimate becomes,

$$(\hat{x}, \hat{k}) = \arg\min_{x,k} DF(x,k) + \lambda_x R_x(x) + \lambda_k R_k(k),$$

where $R_x(x)$ is defined in a manner similar to $R_k(k)$, *i.e.*

$$R_k(k) = -\sum_i (|g_{h,i}(k)|^\alpha + |g_{v,i}(k)|^\alpha)$$

When joint estimation works? (2 of 2)



DF – Data fitting term

$R_x(x)$ – Image prior

$R_k(k)$ – PSF prior

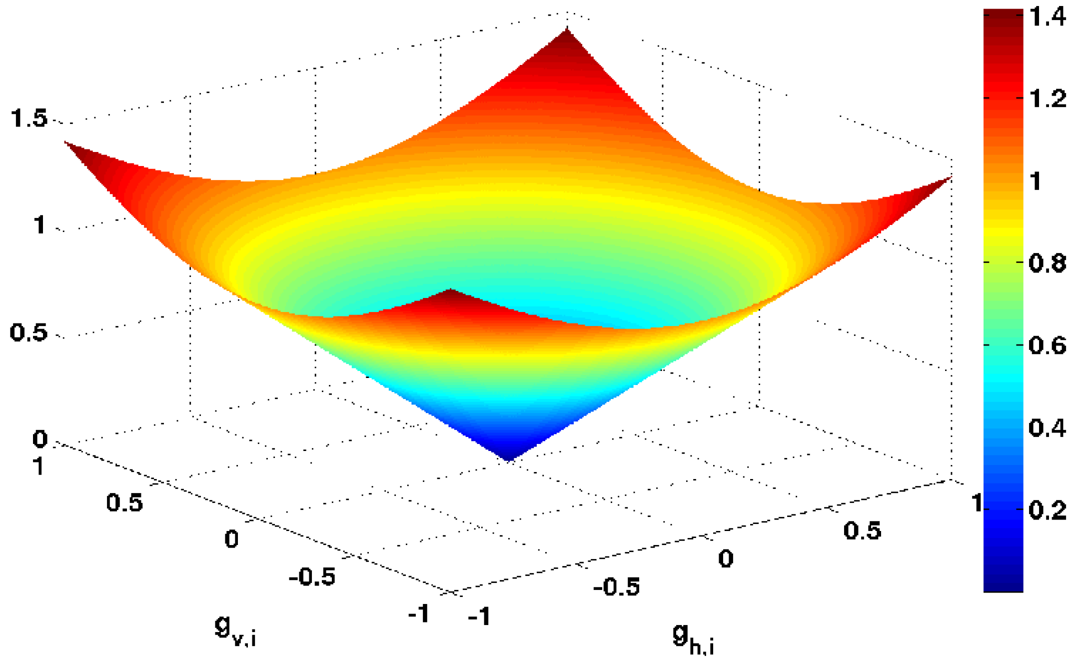
PSF regularizer prevents trivial solutions

Claim1. *The function* $R_k(k) = \sum_i |g_{h,i}(k)|^\alpha + |g_{v,i}(k)|^\alpha$ *attains the maximum when* k *is the impulse function, with* k *constrained to satisfy* $\sum_m \sum_n k(m,n) = 1$.

Complete proof: Please refer paper.

Sketch of the proof for total variation regularizer

$$TV(k) = \sum_i \sqrt{g_{h,i}^2(k) + g_{v,i}^2(k)}$$



Monotonic increasing function with minimum at zero.

$$|g_{h,i}|,\ |g_{v,i}| \in [0,\ 1]$$

$$R_{k_{max}} = 2 + \sqrt{2}$$

Choice of regularization factor (1 of 2)

Using a PSF regularizer alone is not sufficient, it is necessary to select an appropriate regularization factor too.

Claim2. *The lower bound of the PSF regularization factor,* $\lambda_{k_{min}}$ *is given by*

$$\lambda_{k_{min}} = \frac{\lambda_x E(R_x)}{R_{k_{max}}}.$$

where $E(R_x)$ *is the expectation of the image regularizer (* R_x*) and* $R_{k_{max}}$ *is the maximum of* $R_k(k)$*.*

Complete proof: Please refer paper.

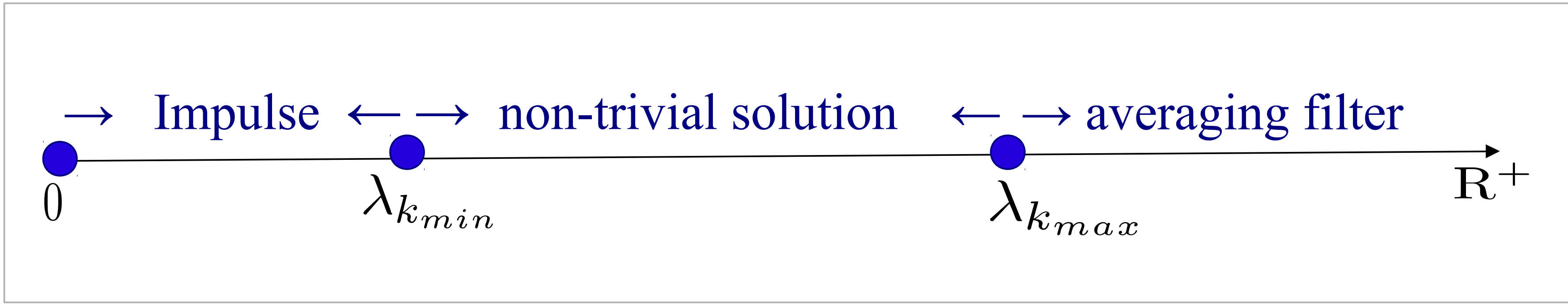
Choice of regularization factor (2 of 2)

Since the minimum of the function $R_k(k)$ is zero, it cannot be used for upper bounding λ_k.

$$\frac{\lambda_x E(R_x)}{R_{k_{max}}} < \lambda_k < \beta \frac{\lambda_x E(R_x)}{R_{k_{max}}},$$

$\beta > 1$ is an empirically chosen factor.

Estimated PSF behavior



$\lambda_k \in [0,\ \lambda_{k_{min}}]$ - Estimated PSF tends to the trivial impulse

$\lambda_k \in (\lambda_{k_{min}},\ \lambda_{k_{max}})$ - Non-trivial solution

$\lambda_k > \lambda_{k_{max}}$ - Averaging filter – some amount of deblurring is there

Results

- We show the results for two image priors
  - TV prior
  - Wavelet prior
- TV was chosen as the PSF prior in both the cases.
- For $\lambda_k << \lambda_{k_{min}}$ there is no deblurring – the trivial solution.
- For $\lambda_k >> \lambda_{k_{max}}$ there is deblurring with artifacts.

Noisy

TV (proper λ_k)

Wavelet (proper λ_k)

$\lambda_k << \lambda_{k_{min}}$

$\lambda_k >> \lambda_{k_{max}}$

Noisy

TV (proper λ_k)

Wavelet (proper λ_k)

$\lambda_k << \lambda_{k_{min}}$

$\lambda_k >> \lambda_{k_{max}}$

Conclusions

Joint estimation of the image prior gives non-trivial results provided

- an appropriate PSF prior is used
- PSF regularization factor is chosen properly.

Thank you