Tweening Boundary Curves of Non-simple Immersions of a Disk

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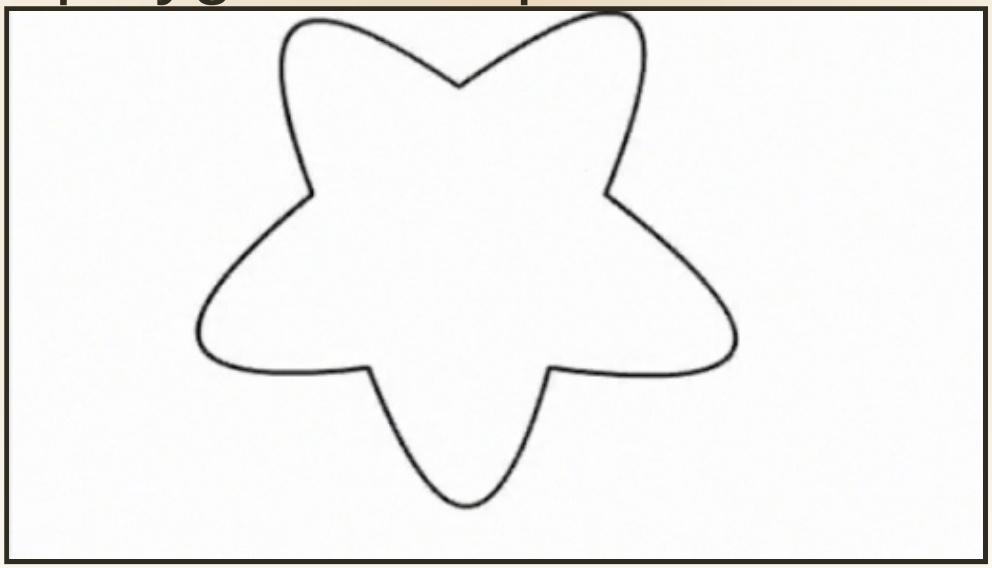
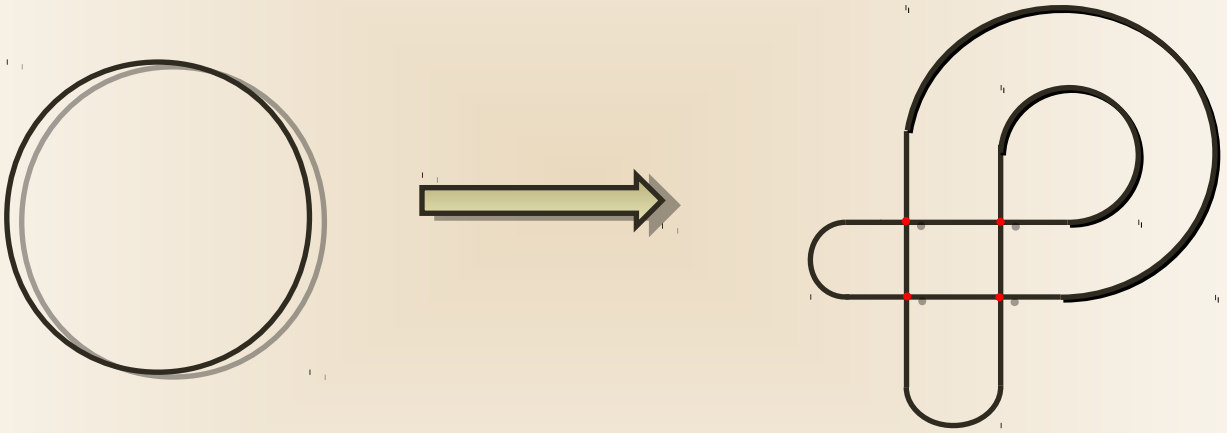
Tweening (Shape morphing)

- Smoothly transforming one geometric shape *(source)* into another *(target)*
- Most of the work in 2D shape morphing confined to simple polygonal shapes



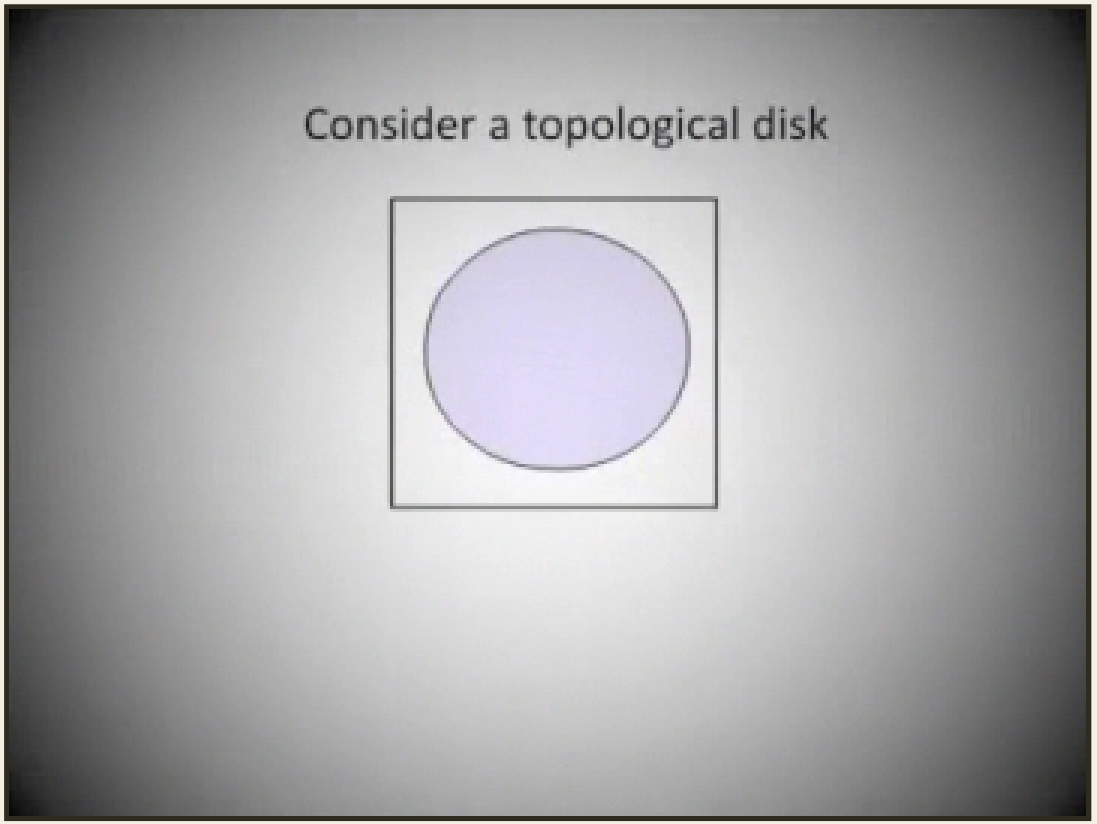
Tweening non-simple shapes

- No clear metric to evaluate the morphing if the source/target is *self-intersecting*

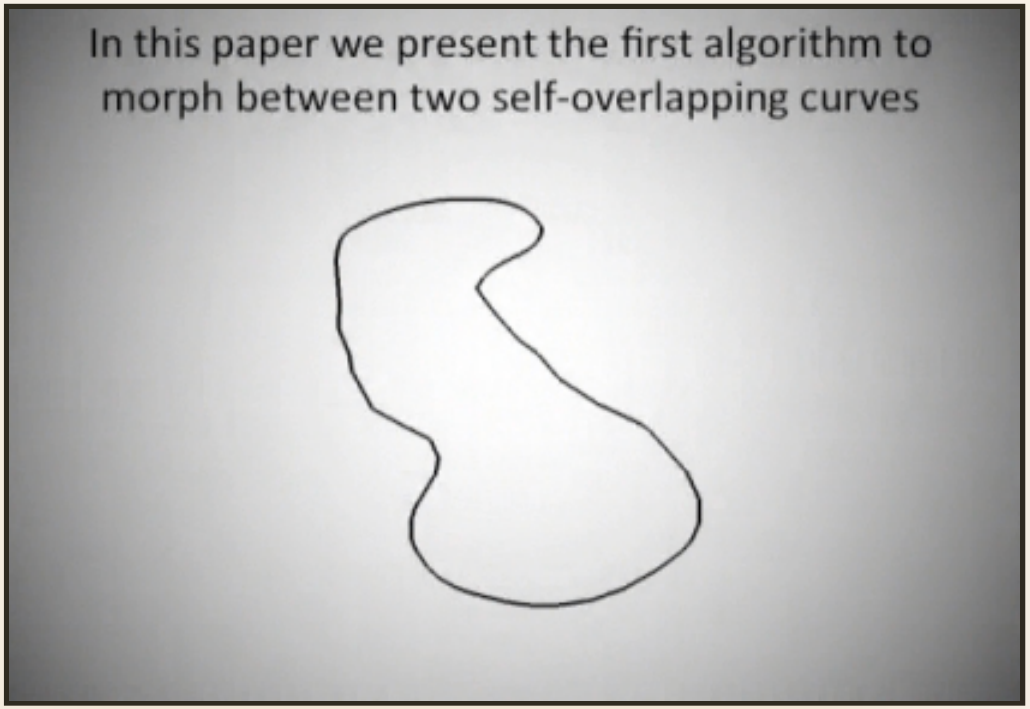


- However, there exists an interesting class of self-intersecting curves, which can be theoretically morphed into one another

Self-overlapping curves



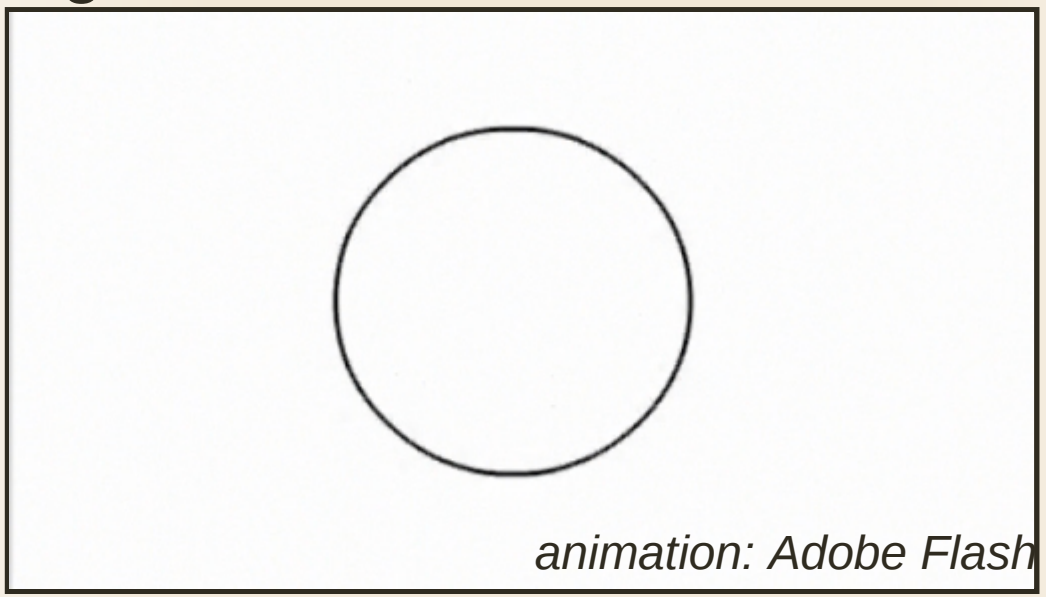
Morphing between Self-overlapping curves



Constraint: All the intermediate curves are self-overlapping (with no twists)

Approach

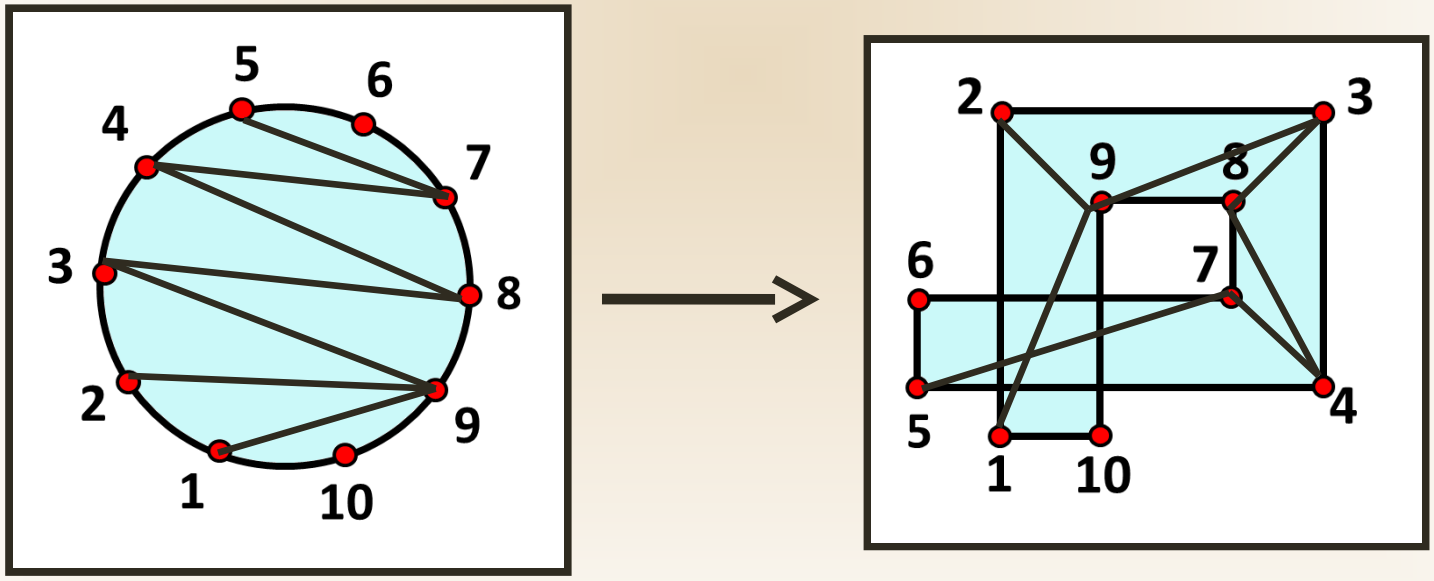
- Transforming curve boundaries not enough for morphing



- So, interior of the source should be transformed to the interior of the target

Approach

- Interior morphing requires a one-to-one mapping between the interiors of the source and target
- A compatible triangulation is one such mapping

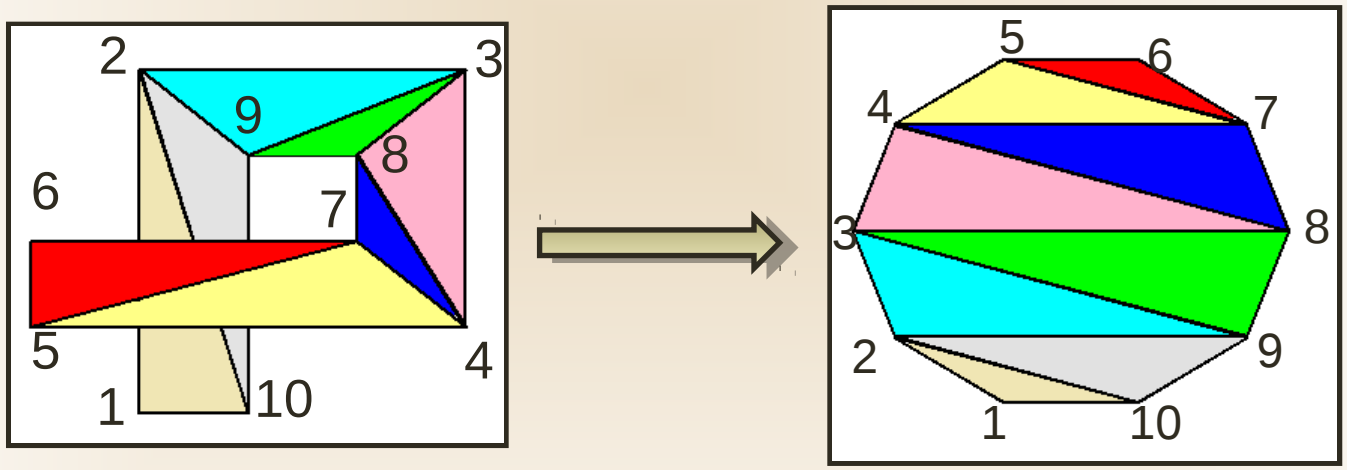


Finding a triangulation of a self-overlapping curve

- Shor's [1992] algorithm triangulates a self-overlapping curve
- Dynamic programming method
- Time complexity: $O(n^3)$, n: number of vertices

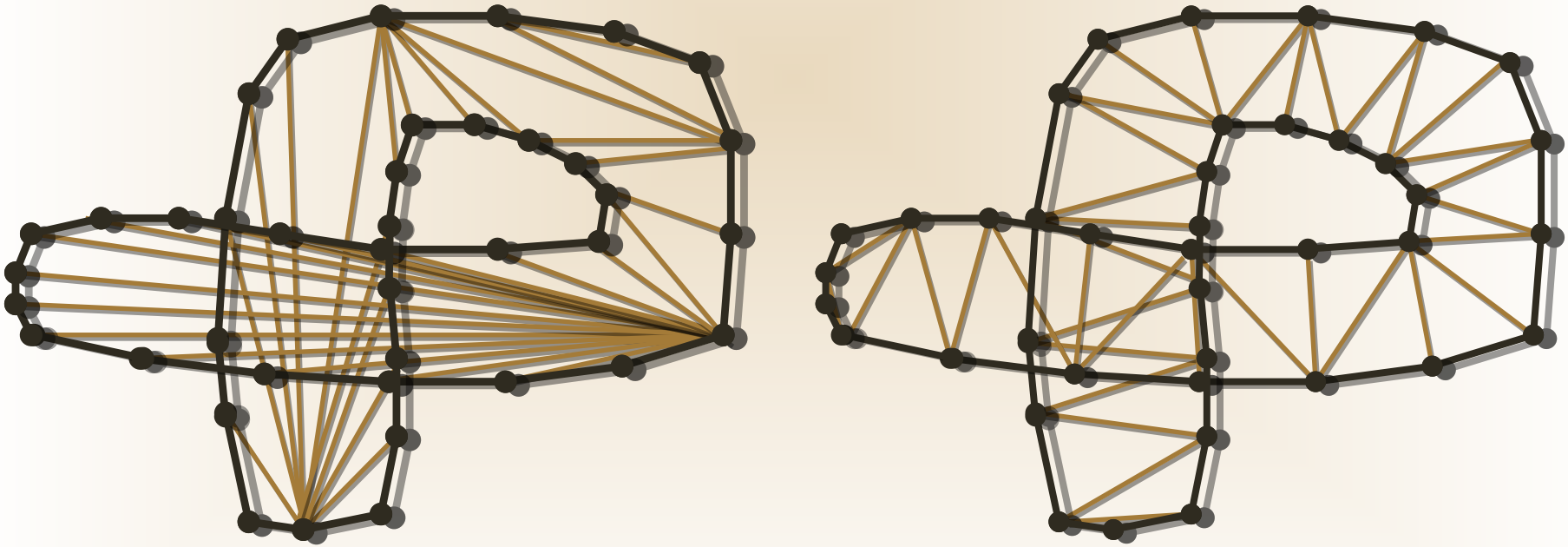
Valid Triangulation

Mapping of vertices of the curve to a circle, and imposing the given triangulation of the curve to the circle produces non-self-intersecting triangulation on the circle.

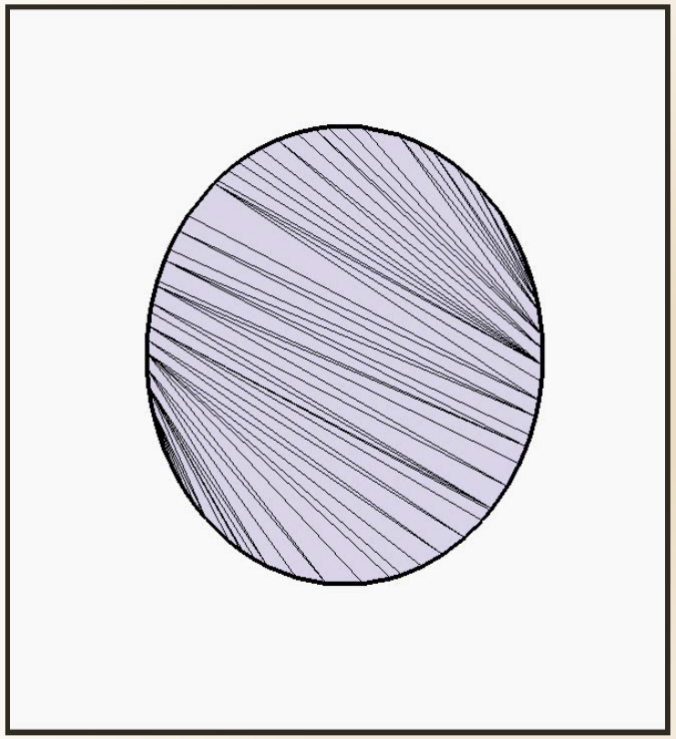


Valid Triangulation

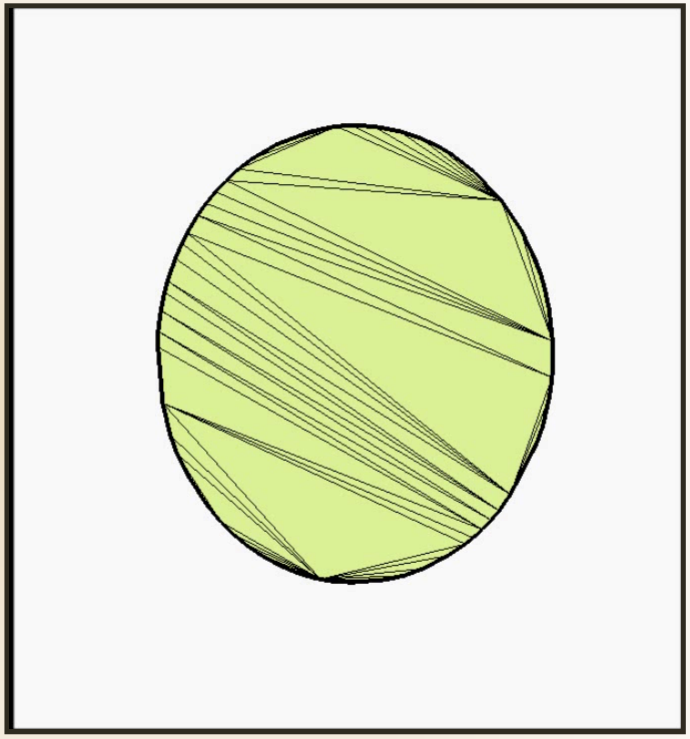
- There are many valid triangulations
- Picking a triangulation that is suitable for morphing application is expensive



Effect of Triangulation On Morphing



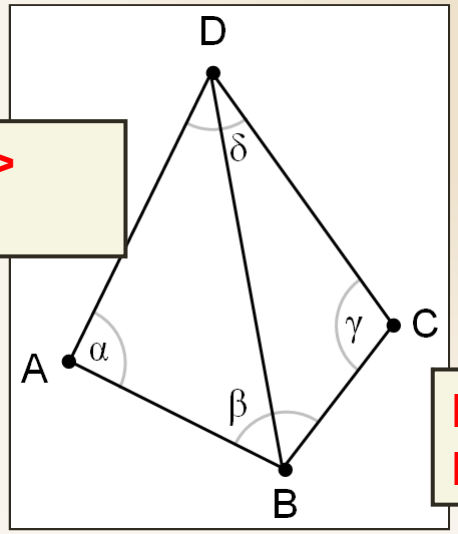
Triangulation 1



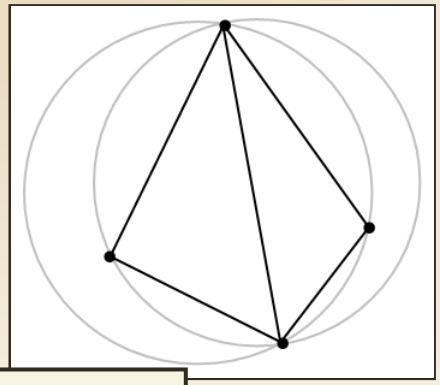
Triangulation 2

Which Triangulation is Good for Morphing?

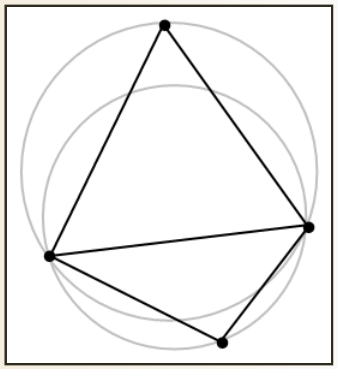
- It appears to be the one that has “fat” triangles ***- Delaunay property***
- What is a Delaunay property?



α + γ > 180°

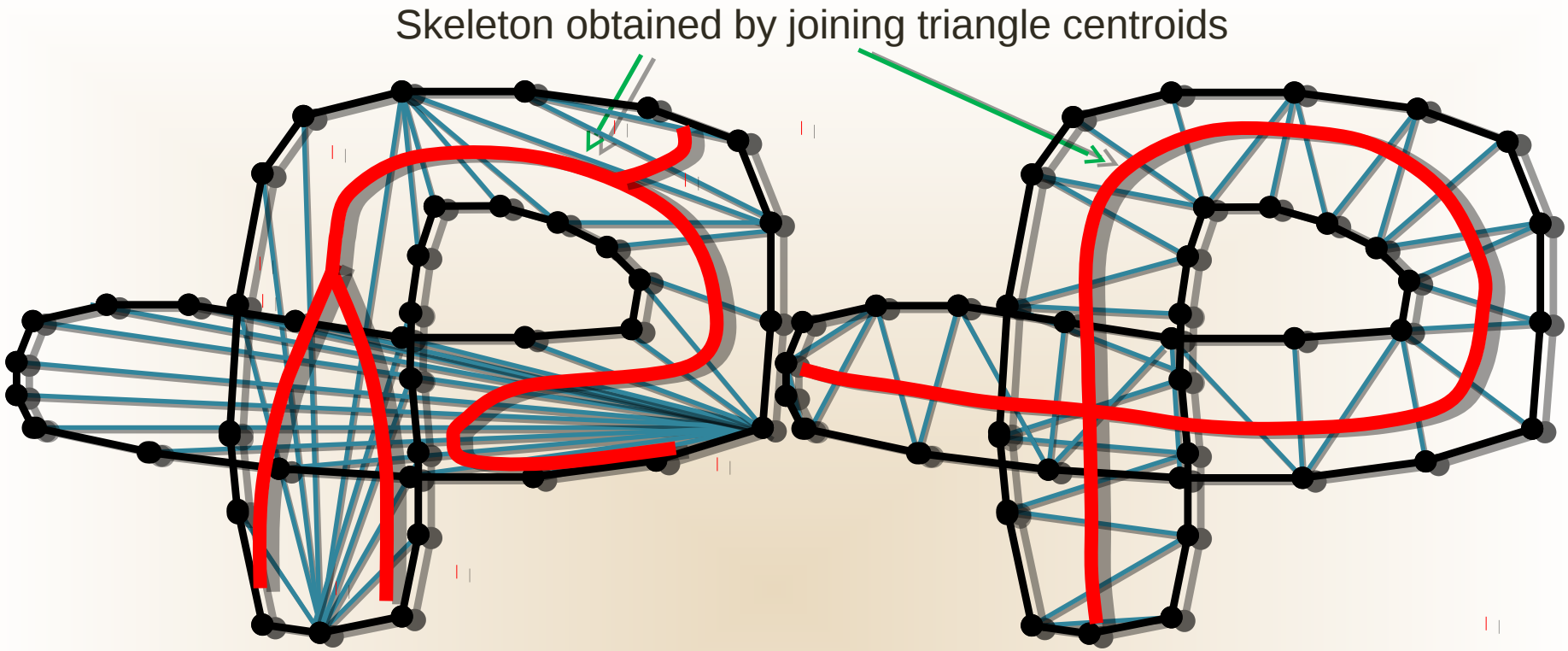


Non Delaunay



Flipping common edge produces Delaunay

Pictures : Wikipedia



Random triangulation does not show true disk deformation, hence not suitable for morphing

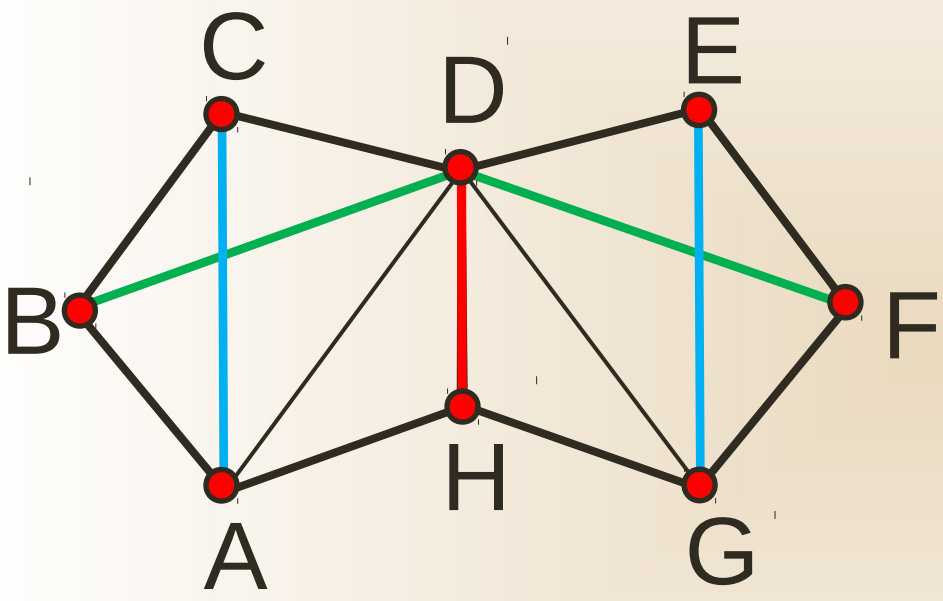
CDT brings out the disk deformation, eventually producing visually better morphs

How to get a triangulation that satisfies Delaunay Property?

- Directly computing such a triangulation using Shor's algorithm is expensive

- So we find "a" triangulation using Shor's method and modify it to satisfy Delaunay property

- Technique : Series of Edge-Flipping

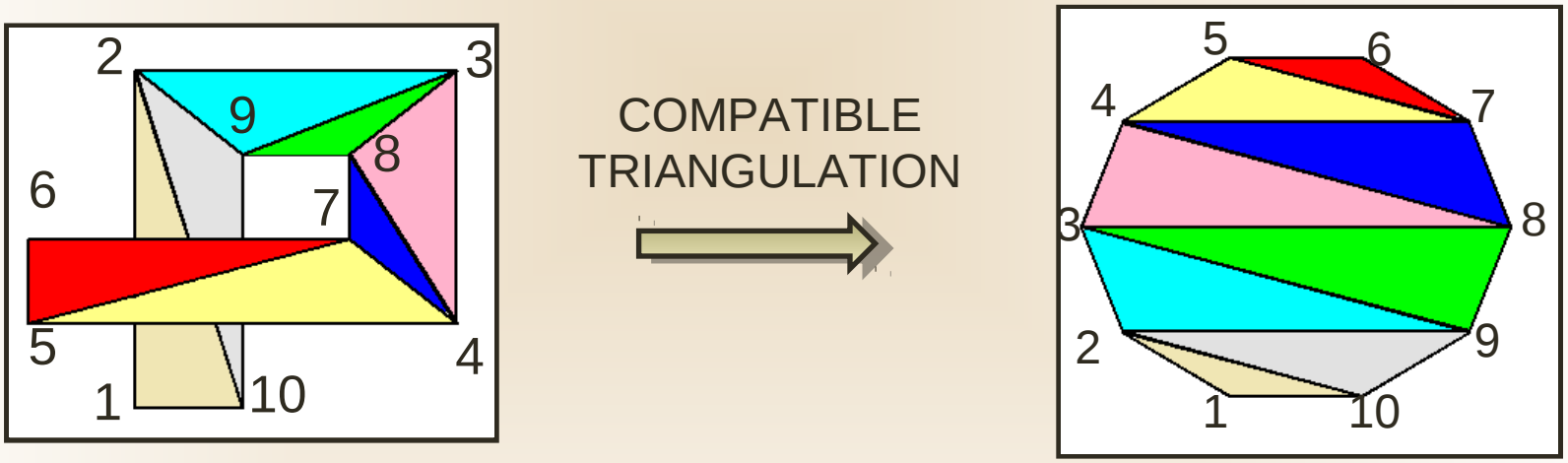


Re-meshing by a series of edge flips maintaining Delaunay Criterion

The red edge DH resting on a concave vertex cannot be flipped

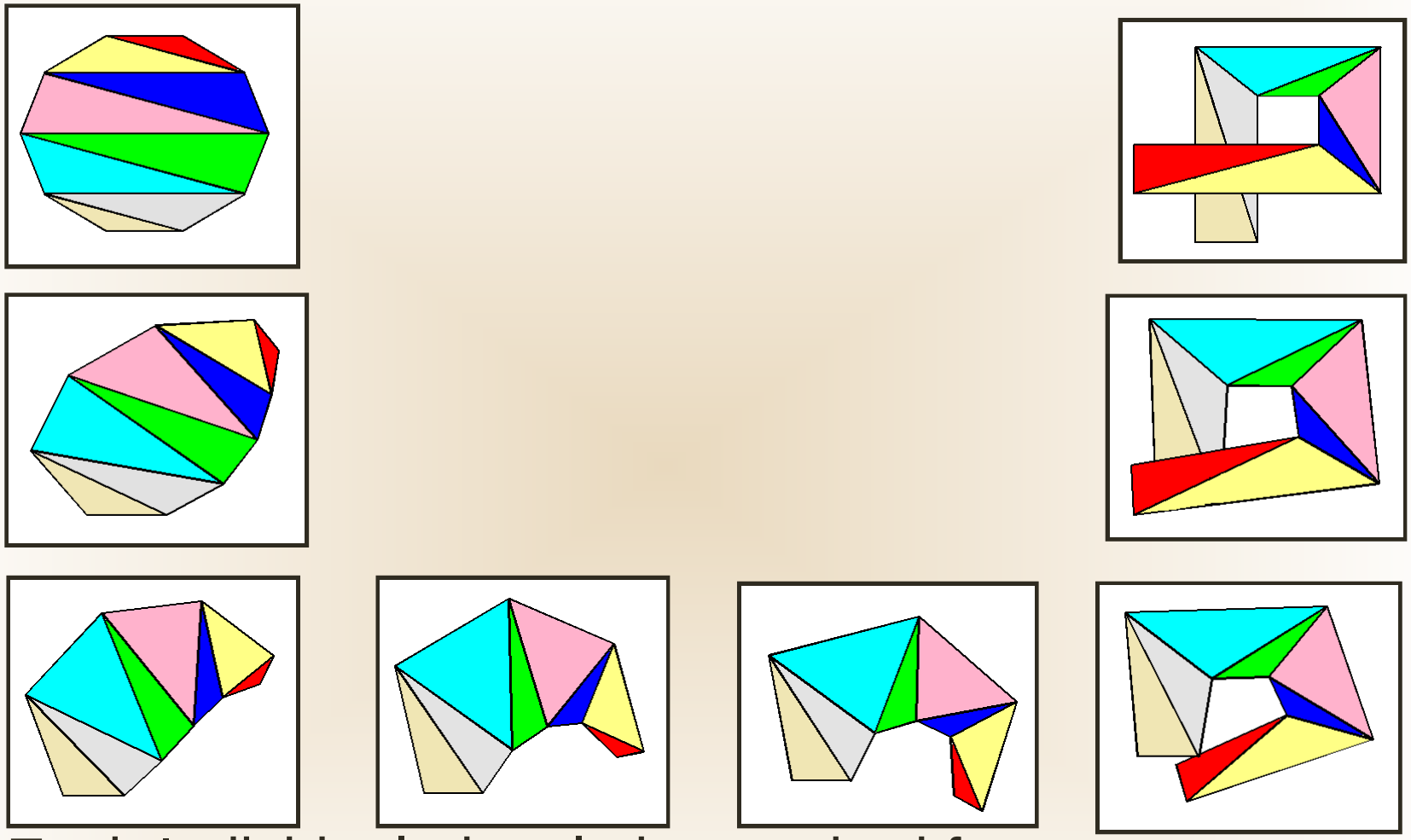
Morphing a Curve to a Circle

- Given a high quality valid triangulation of a self-overlapping curve, this triangulation is imposed on the circle, which is a convex shape

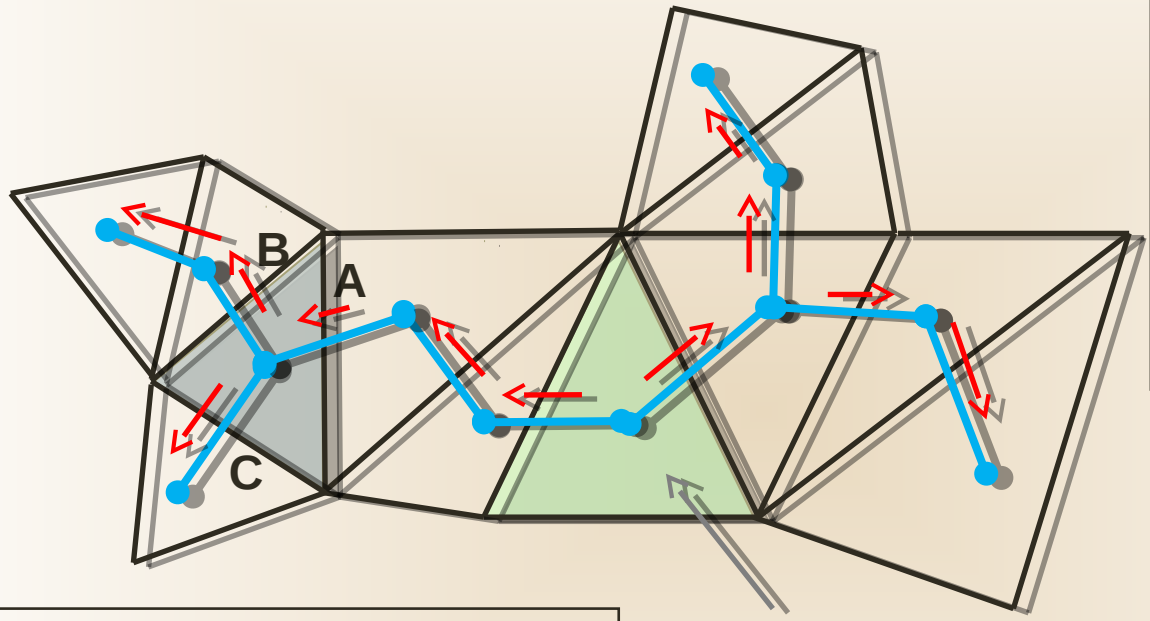


- Each triangle is then morphed individually

Morphing Compatible Triangulations



Each Individual triangle is morphed from source to target



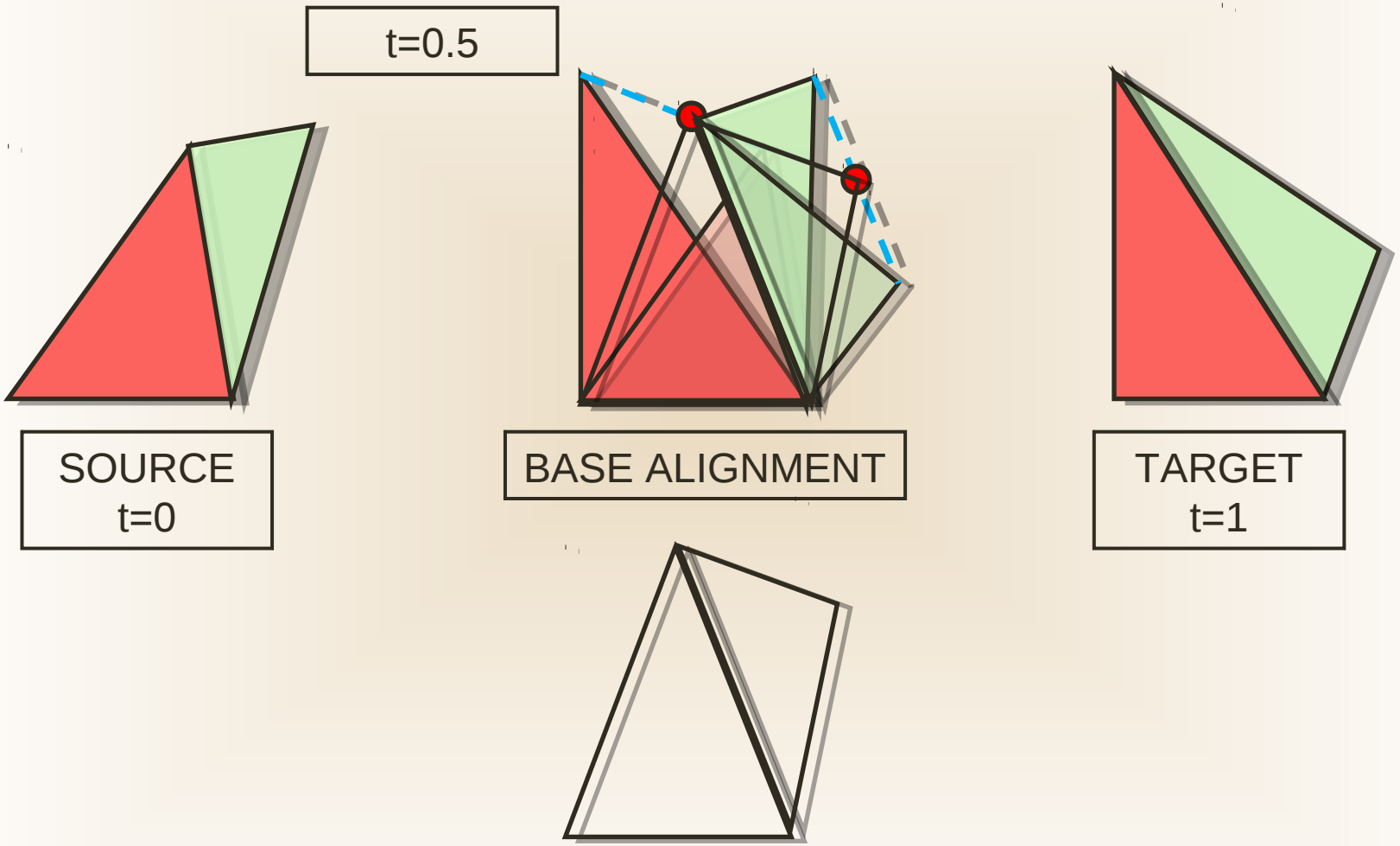
Triangulation and **dual Tree**
Compatible triangulations have the same tree

Direction of propagation shown with red arrows

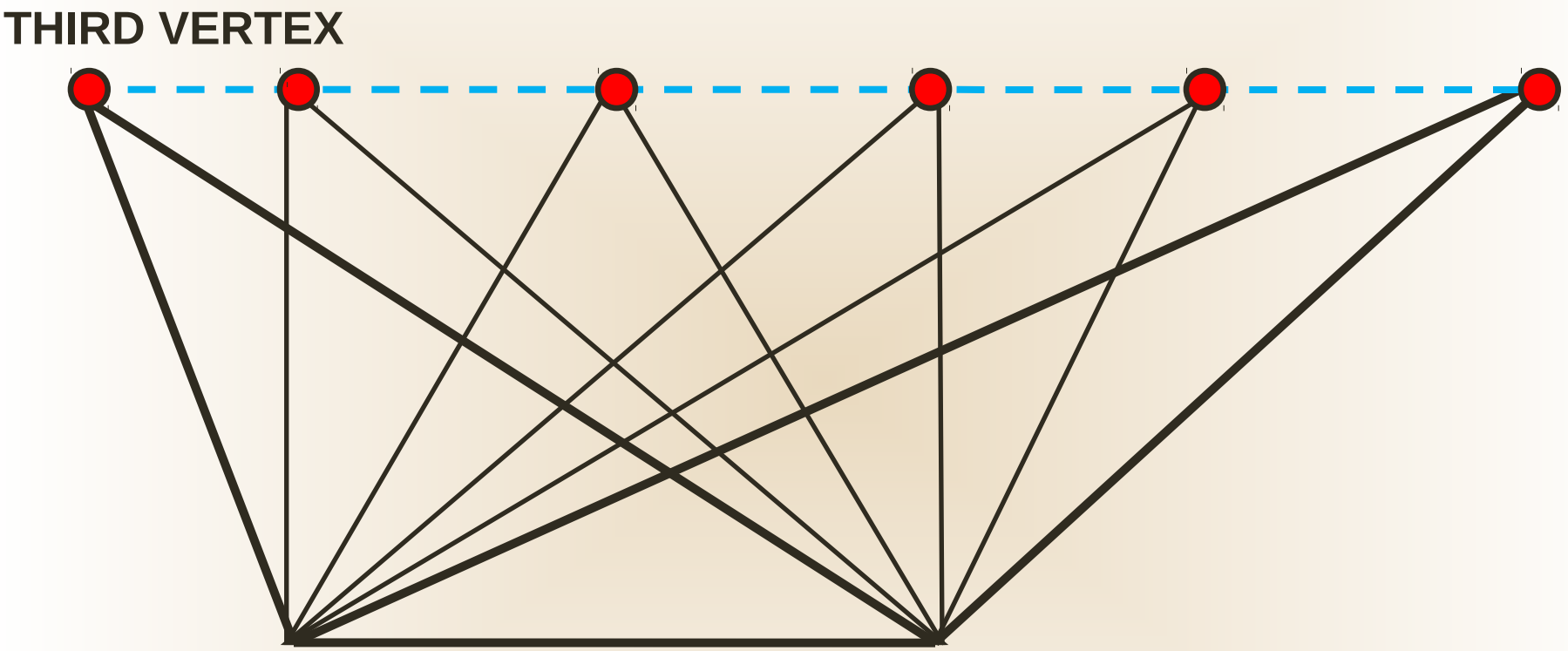
A – base edge (incoming)
B, C – propagation edges (outgoing)
Tree structure allows 1 base edge and 0,1, or 2 propagation edges

Arbitrary triangle chosen as starting triangle (root of the tree)

Morphing Compatible Triangulations



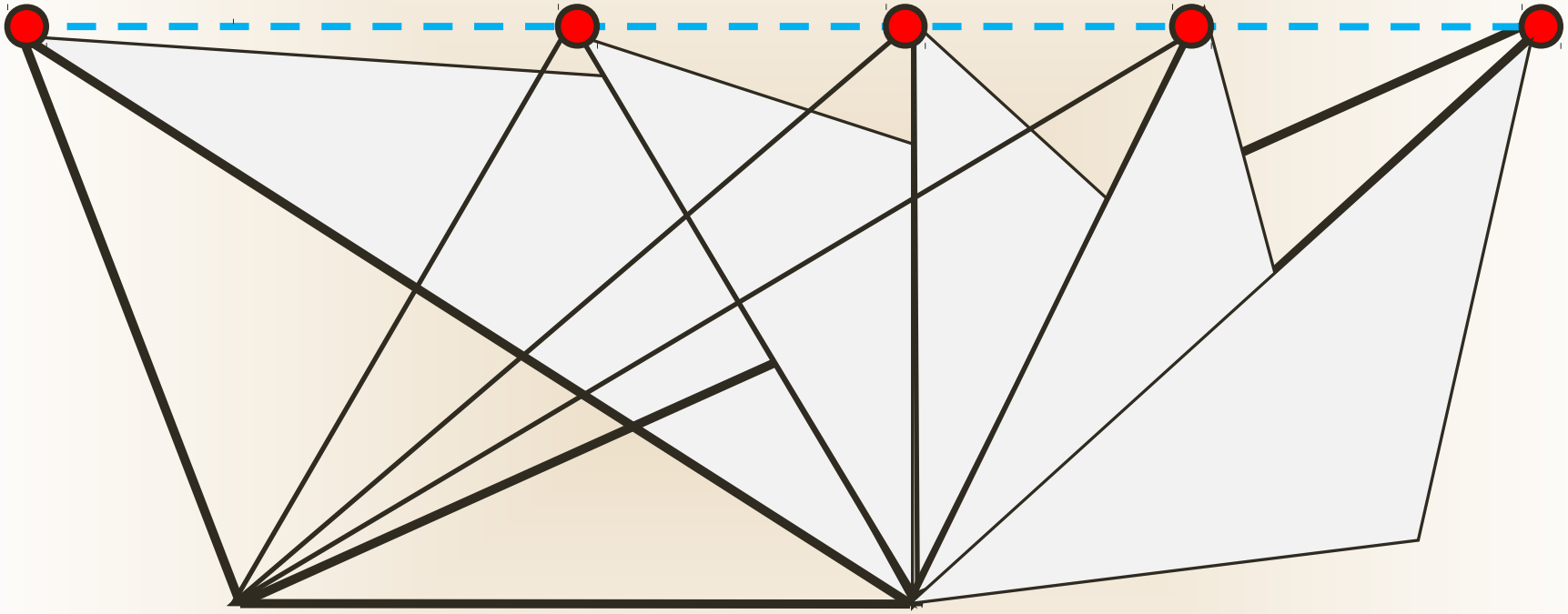
Morphing Compatible Triangulations



The transformed source triangle at any intermediate position can be obtained by linearly interpolating the position of the third vertex

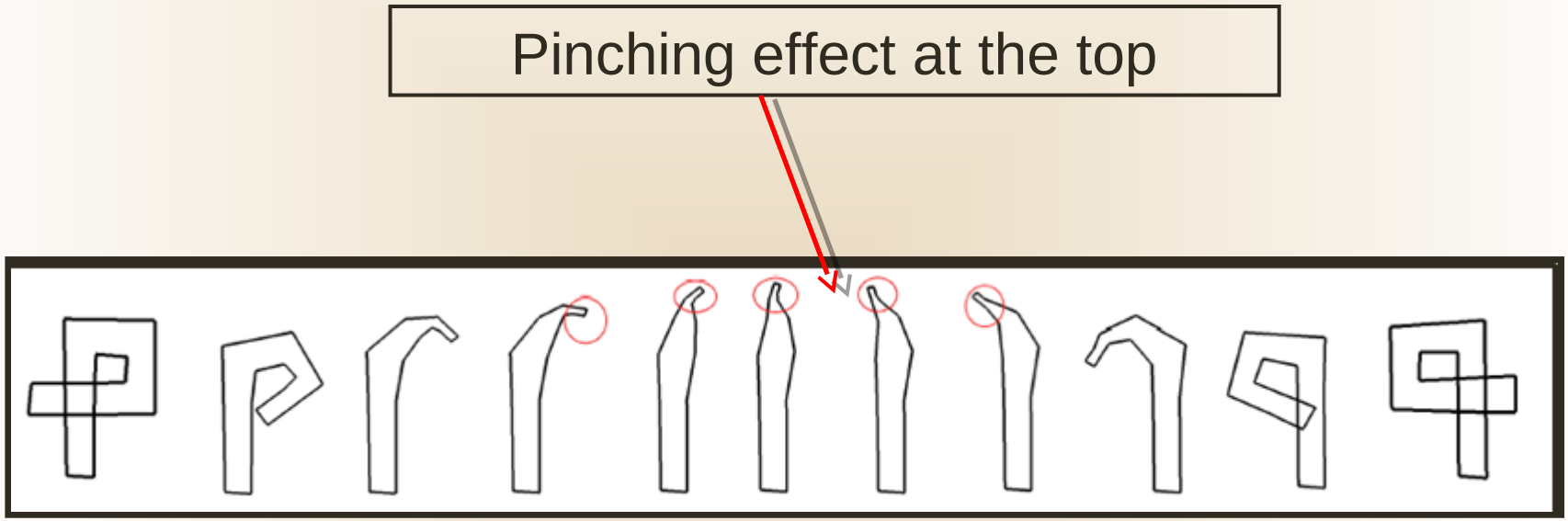
Morphing Compatible Triangulations

THIRD VERTEX



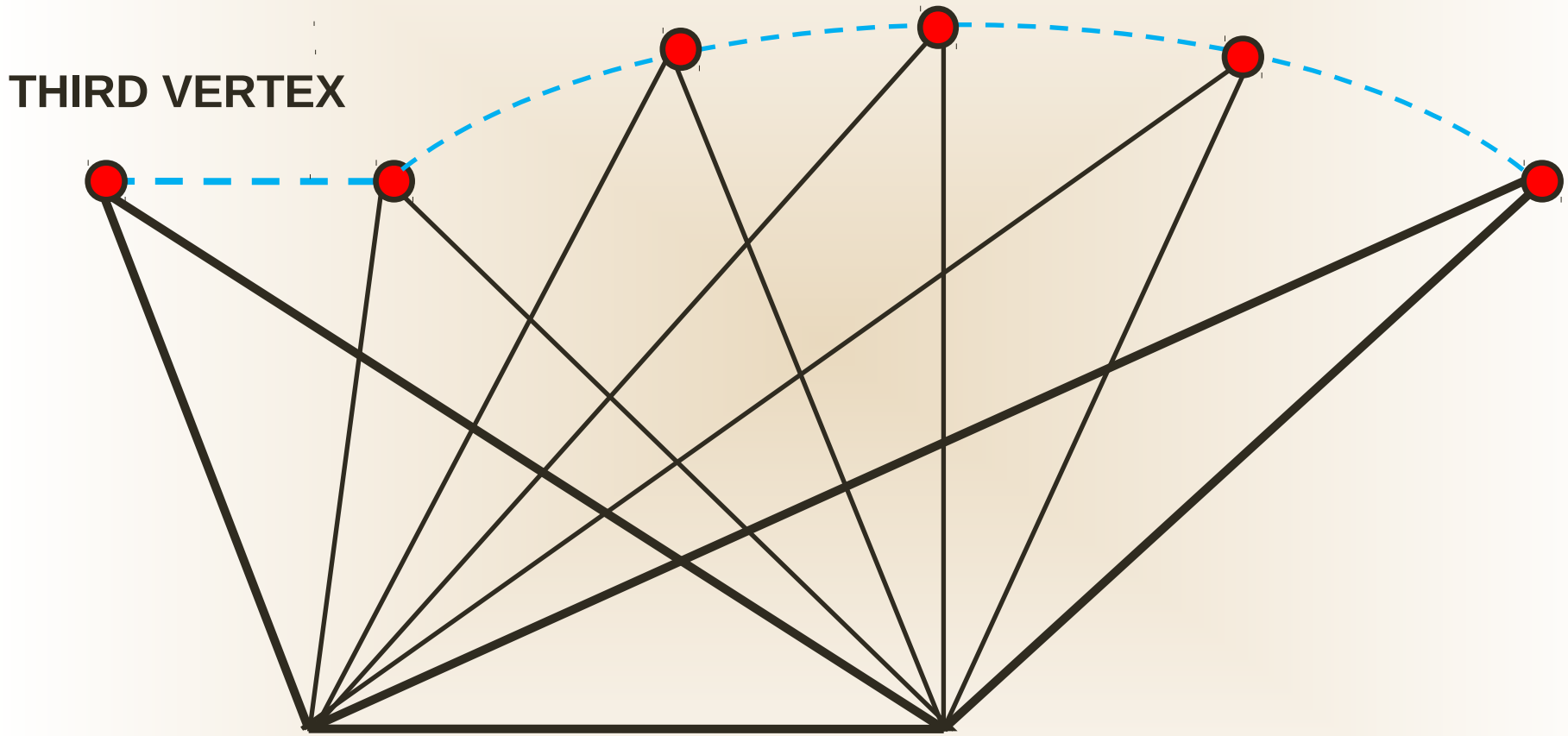
- Area of a triangle should vary monotonically for a smooth morph
  - Area of the child triangle varies non-monotonically with linear interpolation of the third vertex

Morphing Compatible Triangulations



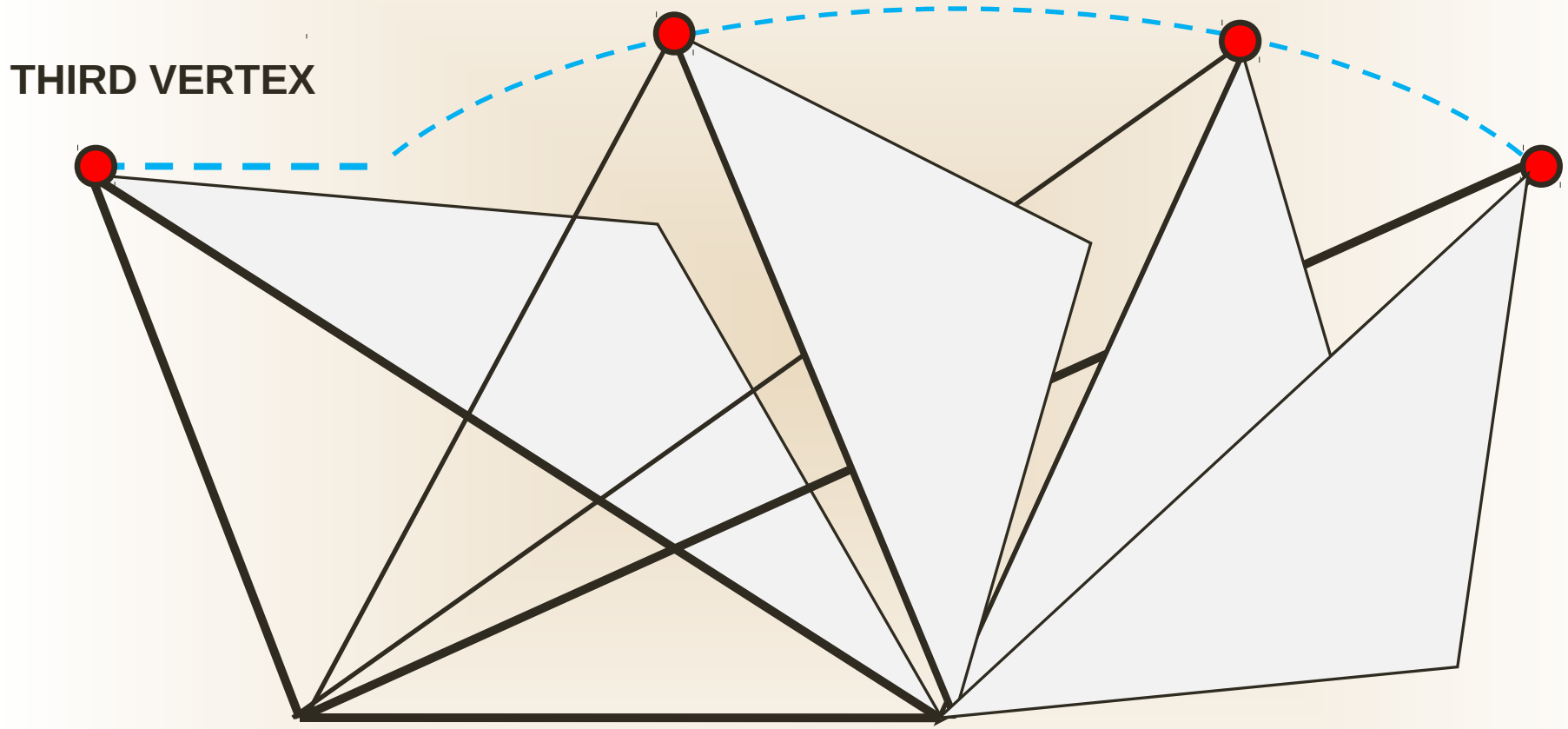
Morphing with linear interpolation

Morphing Compatible Triangulations



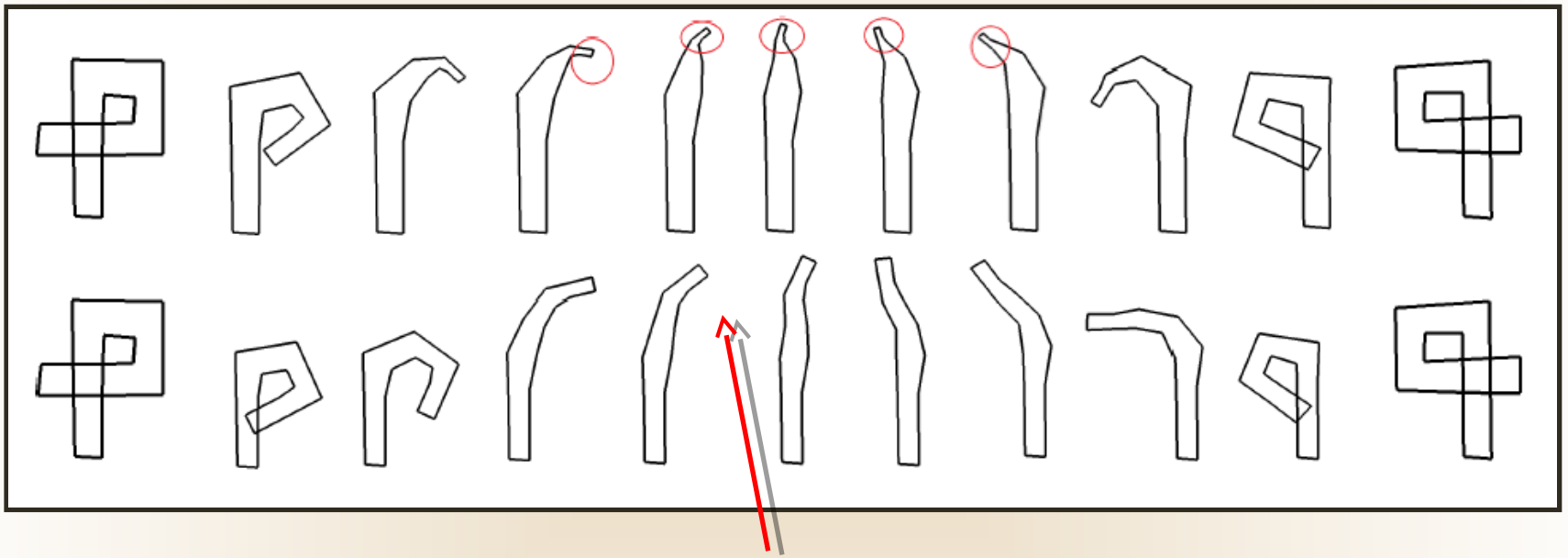
A slide and rotate mechanism used to preserve the propagation edge length

Morphing Compatible Triangulations



The area of the child triangle also varies monotonically

Morphing Compatible Triangulations



No pinching effect

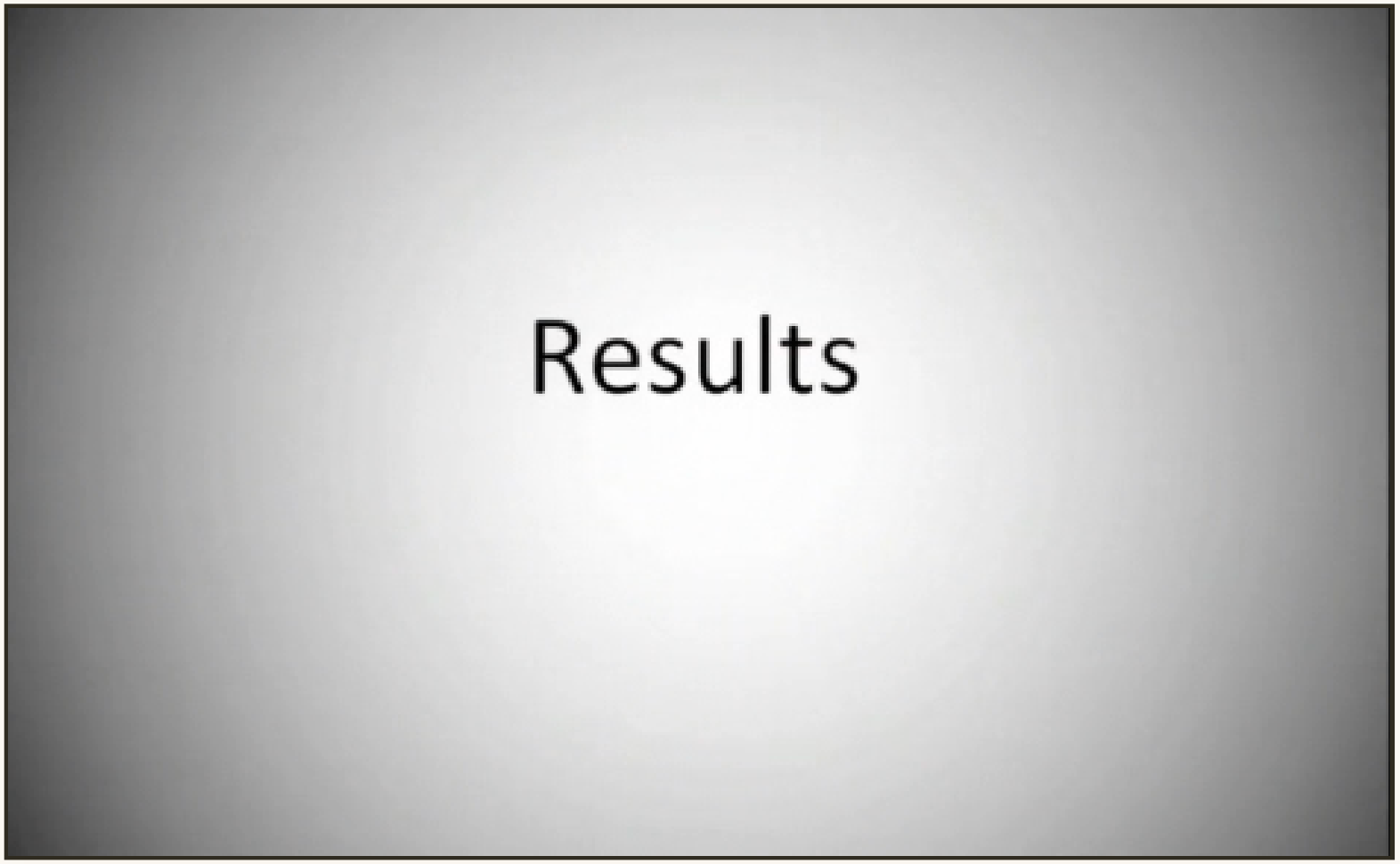
Top: Morphing with linear interpolation
Bottom: Morphing with rotation and sliding

Morphing Compatible Triangulations

- The same technique can be used to morph between any two self-overlapping curves having compatible triangulations

- We also introduce a technique for morphing incompatible triangulations (please see paper for details)

Results



Conclusion

- We have introduced the first algorithm to produce high quality morphs between self-overlapping curves
- Our algorithm can handle curves with incompatible triangulations

Thank You