

Workshop on Essential Abstractions in GCC

GDFA: Generic Data Flow Analyser for GCC

GCC Resource Center
(www.cse.iitb.ac.in/grc)

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Outline

- Motivation
- Common abstractions in data flow analysis
- Implementing data flow analysis using *gdfa*
- Design and Implementation of *gdfa*



Motivation behind gdfa

- Specification Vs. implementation
- Orthogonality of specification of data flow analysis and the process of performing data flow analysis
- Practical significance of generalizations
- Ease of extending data flow analysers



Part 2

*Common Abstractions in Data
Flow Analysis*

Common Form of Data Flow Equations

$$X_i = f(Y_i)$$

$$Y_i = \sqcap X_j$$



Common Form of Data Flow Equations

Data Flow Information

So far we have seen sets (or bit vectors).
Could be entities other than sets for
non-bit vector frameworks.

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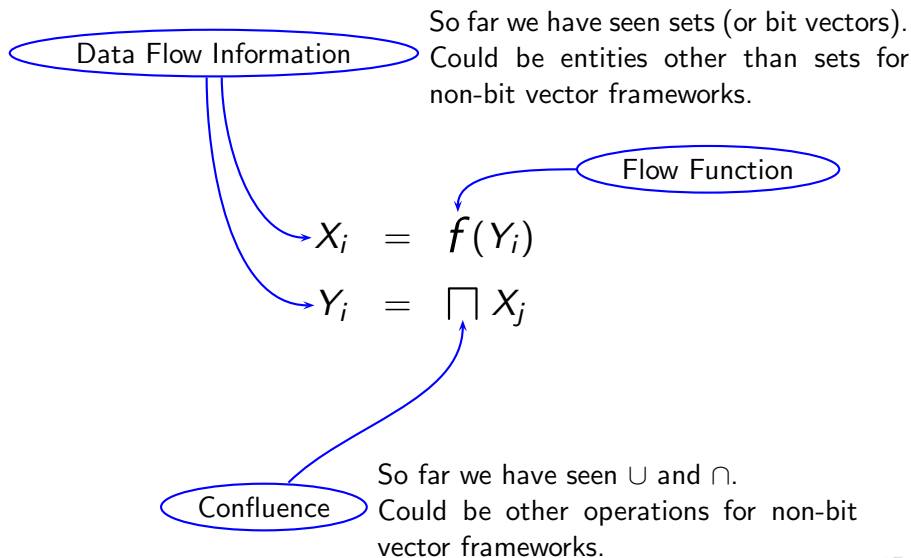
Flow Function

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Common Form of Data Flow Equations



A Taxonomy of Bit Vector Data Flow Frameworks

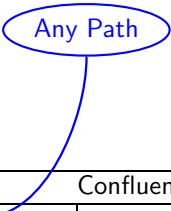
	Confluence	
	Union	Intersection
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Anticipable Expressions
Bidirectional (limited)		Partial Redundancy Elimination (Original M-R Formulation)



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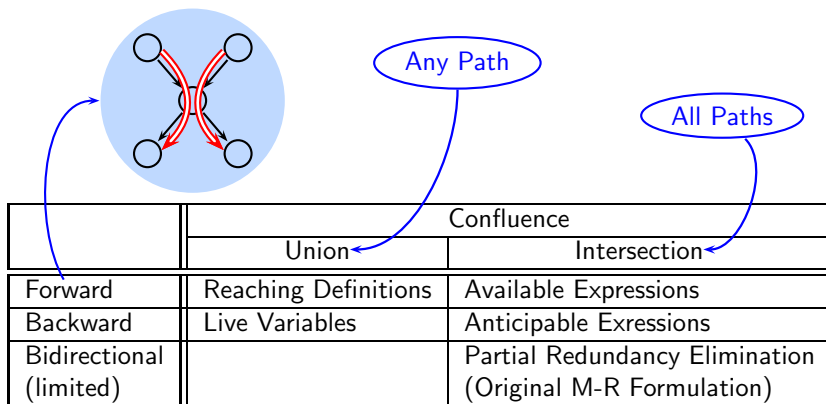
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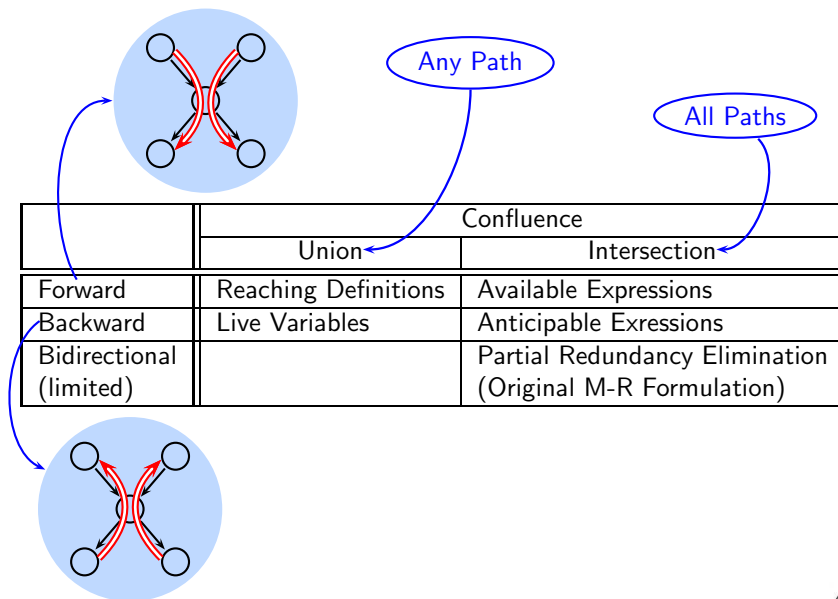
All Paths



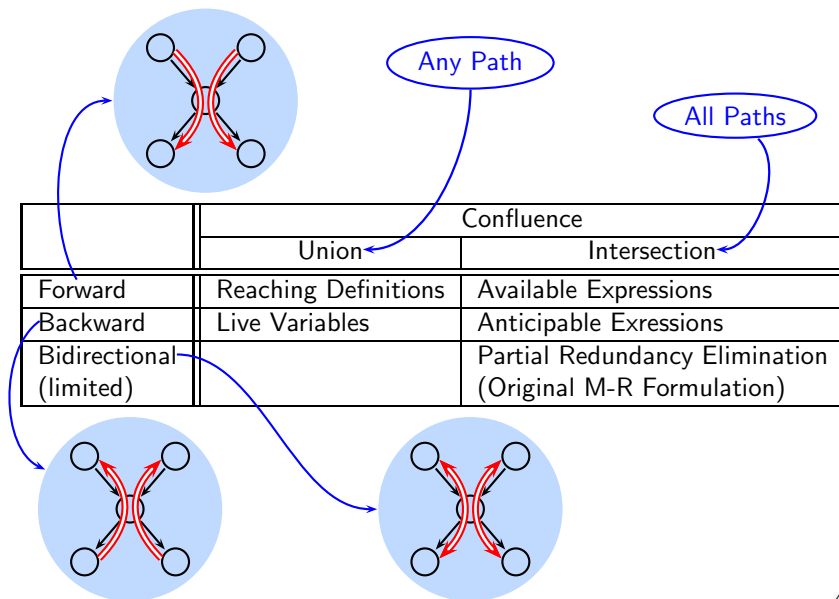
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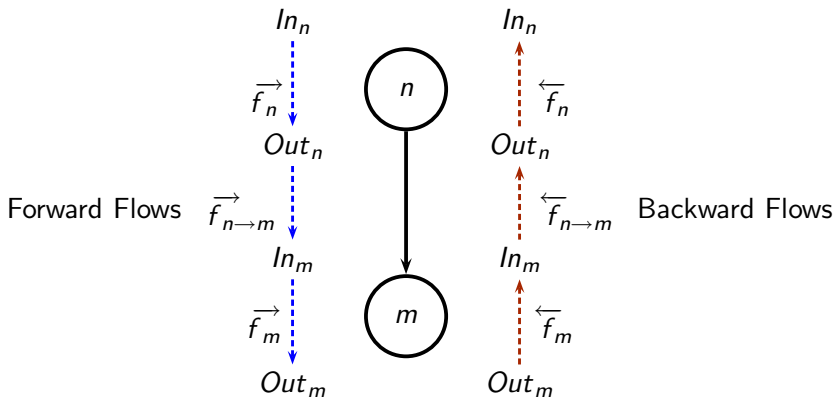
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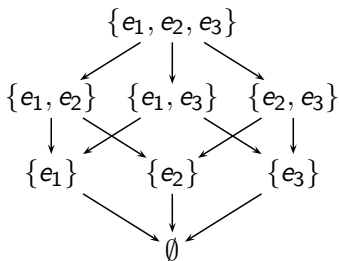


The Abstraction of Flow Functions



The Abstraction of Data Flow Values

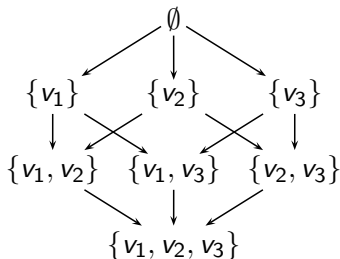
Available Expressions Analysis



$\sqsubseteq$ is $\subseteq$

$\sqcap$ is $\cap$

Live Variables Analysis



$\sqsubseteq$ is $\supseteq$

$\sqcap$ is $\cup$



The Abstraction of Data Flow Equations

$$\begin{aligned} In_n &= \begin{cases} BStart \sqcap \overleftarrow{f}_n(Out_n) & n = Start \\ \left(\bigsqcap_{m \in pred(n)} \overrightarrow{f}_{m \rightarrow n}(Out_m) \right) \sqcap \overleftarrow{f}_n(Out_n) & \text{otherwise} \end{cases} \\ Out_n &= \begin{cases} BEnd \sqcap \overrightarrow{f}_n(In_n) & n = End \\ \left(\bigsqcap_{m \in succ(n)} \overleftarrow{f}_{m \rightarrow n}(In_m) \right) \sqcap \overrightarrow{f}_n(In_n) & \text{otherwise} \end{cases} \end{aligned}$$



Iterative Methods of Performing Data Flow Analysis

Successive recomputation after conservative initialization ($\top$)

- *Round Robin*. Repeated traversals over nodes in a fixed order

Termination : After values stabilise

- + Simplest to understand and implement
- May perform unnecessary computations



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Termination : After values stabilise

- + Simplest to understand and implement
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Our examples use this method.

- *Work List*. Dynamic list of nodes which need recomputation

Termination : When the list becomes empty

- + Demand driven. Avoid unnecessary computations.
- Overheads of maintaining work list.



Common Form of Flow Functions

$$f_n(X) = (X - \text{Kill}_n(X)) \cup \text{Gen}_n(X)$$

- For General Data Flow Frameworks

$$\text{Gen}_n(X) = \text{ConstGen}_n \cup \text{DepGen}_n(X)$$

$$\text{Kill}_n(X) = \text{ConstKill}_n \cup \text{DepKill}_n(X)$$

- For bit vector frameworks

$$\text{Gen}_n(X) = \text{ConstGen}_n$$

$$\text{Kill}_n(X) = \text{ConstKill}_n$$



Defining Flow Functions for Bit Vector Frameworks

- Live variables analysis

	Entity	Manipulation	Exposition
$ConstGen_n$	Variable	Use	Upwards
$ConstKill_n$	Variable	Modification	Anywhere

- Available expressions analysis

	Entity	Manipulation	Exposition
Gen_n	Expression	Use	Downwards
$Kill_n$	Expression	Modification	Anywhere



Part 3

*Implementing Data Flow
Analysis using gdfa*

Implementing Available Expressions Analysis

1. Specifying available expressions analysis
2. Implementing the entry function of available expressions analysis pass
3. Registering the available expressions analysis pass
 - 3.1 Declaring the pass
 - 3.2 Registering the pass
 - 3.3 Positioning the pass



Step 1: Specifying Available Expressions Analysis

```
struct gimple_pfbv_dfa_spec gdfa_ave =  
{  
    entity_expr,          /* entity          */  
    ONES,                 /* top_value      */  
    ZEROS,                /* entry_info     */  
    ONES,                 /* exit_info      */  
    FORWARD,             /* traversal_order */  
    INTERSECTION,        /* confluence     */  
  
};
```



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    FORWARD,             /* traversal_order */  
    INTERSECTION,         /* confluence     */  
    entity_use,           /* gen_effect     */  
    down_exp,             /* gen_exposition */  
    entity_mod,           /* kill_effect    */  
    any_where,            /* kill_exposition */  
  
};
```



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    any_where,            /* kill_exposition */  
    global_only,          /* preserved_dfi  */  
  
};
```



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    entity_use,           /* gen_effect     */  
    down_exp,             /* gen_exposition */  
    entity_mod,           /* kill_effect    */  
    any_where,            /* kill_exposition */  
    global_only,          /* preserved_dfi  */  
    identity_forward_edge_flow, /* forward_edge_flow */  
    stop_flow_along_edge,  /* backward_edge_flow */  
    forward_gen_kill_node_flow, /* forward_node_flow */  
    stop_flow_along_node   /* backward_node_flow */  
};
```



Step 2: Implementing Available Expressions Analysis Pass

```
pfbv_dfi ** AV_pfbv_dfi = NULL;

static unsigned int
gimple_pfbv_ave_dfa(void)
{

    AV_pfbv_dfi = gdfa_driver(gdfa_ave);

    return 0;
}
```



Step 3.1: Declaring the Available Expressions Analysis Pass

```
struct gimple_opt_pass pass_gimple_pfbv_ave_dfa =
{
  { GIMPLE_PASS,
    "gdfa_ave",          /* name */
    NULL,                /* gate */
    gimple_pfbv_ave_dfa, /* execute */
    NULL,                /* sub */
    NULL,                /* next */
    0,                   /* static_pass_number */
    0,                   /* tv_id */
    0,                   /* properties_required */
    0,                   /* properties_provided */
    0,                   /* properties_destroyed */
    0,                   /* todo_flags_start */
    0,                   /* todo_flags_finish */
    0                     /* letter */
  }
};
```



Step 3.2: Registering the Available Expressions Analysis Pass

In file file tree-pass.h

```
extern struct gimple_opt_pass pass_gimple_pfbv_ave_dfa;
```



Step 3.3: Positioning the Pass

In function `init_optimization_passes` in file `passes.c`.

```
    NEXT_PASS (pass_build_cfg);  
/* Intraprocedural dfa passes begin */  
    NEXT_PASS (pass_init_gimple_pfbvdfa);  
    NEXT_PASS (pass_gimple_pfbv_ave_dfa);
```



Enabling gdfa and Examining Data Flow Values

- Enabling gdfa pass

```
gcc -S -fdump-tree-all -fgdfa
```

- Enabling gdfa dump

```
gcc -S -fdump-tree-all -fgdfa-details
```

- Dump file name extension for available expressions analysis

```
.gdfa_ave
```



Specifying Live Variables Analysis

- Entity should be `entity_var`
- T , BI_{Start} and BI_{End} should be ZEROS
- Direction should be BACKWARD
- Confluence should be UNION
- Exposition should be `up_exp`
- Forward edge flow should be `stop_flow_along_edge`
- Forward node flow should be `stop_flow_along_node`
- Backward edge flow should be `identity_backward_edge_flow`
- Backward node flow should be `backward_gen_kill_node_flow`



Part 4

gdfa: Design and Implementation

Specification Data Structure

```
struct gimple_pfbv_dfa_spec
{
    entity_name          entity;
    initial_value        top_value_spec;
    initial_value        entry_info;
    initial_value        exit_info;
    traversal_direction   traversal_order;
    meet_operation        confluence;
};
```



Specification Data Structure

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struct gimple_pfbv_dfa_spec
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    entity_name          entity;
    initial_value        top_value_spec;
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    initial_value        exit_info;
    traversal_direction   traversal_order;
    meet_operation        confluence;
    entity_manipulation   gen_effect;
    entity_occurrence     gen_exposition;
    entity_manipulation   kill_effect;
    entity_occurrence     kill_exposition;
};
```



Specification Data Structure

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struct gimple_pfbv_dfa_spec
{
    entity_name            entity;
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    initial_value          exit_info;
    traversal_direction     traversal_order;
    meet_operation         confluence;
    entity_manipulation     gen_effect;
    entity_occurrence       gen_exposition;
    entity_manipulation     kill_effect;
    entity_occurrence       kill_exposition;
    dfi_to_be_preserved     preserved_dfi;
    dfvalue (*forward_edge_flow)(basic_block src, basic_block dest);
    dfvalue (*backward_edge_flow)(basic_block src, basic_block dest);
    dfvalue (*forward_node_flow)(basic_block bb);
    dfvalue (*backward_node_flow)(basic_block bb);
};
```



Specification Primitives

Enumerated Type	Possible Values
entity_name	entity_expr, entity_var, entity_defn
initial_value	ONES, ZEROS
traversal_direction	FORWARD, BACKWARD, BIDIRECTIONAL
meet_operation	UNION, INTERSECTION
entity_manipulation	entity_use, entity_mod
entity_occurrence	up_exp, down_exp, any_where
dfi_to_be_preserved	all, global_only, no_value



Pre-Defined Edge Flow Functions

- Edge Flow Functions

Edge Flow Function	Returned value
<code>identity_forward_edge_flow(src, dest)</code>	<code>CURRENT_OUT(src)</code>
<code>identity_backward_edge_flow(src, dest)</code>	<code>CURRENT_IN(dest)</code>
<code>stop_flow_along_edge(src, dest)</code>	<code>top_value</code>



Pre-Defined Edge Flow Functions

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<code>stop_flow_along_edge(src, dest)</code>	<code>top_value</code>

- Node Flow Functions

Node Flow Function	Returned value
<code>identity_forward_node_flow(bb)</code>	<code>CURRENT_IN(bb)</code>
<code>identity_backward_node_flow(bb)</code>	<code>CURRENT_OUT(bb)</code>
<code>stop_flow_along_node(bb)</code>	<code>top_value</code>
<code>forward_gen_kill_node_flow(bb)</code>	$\text{CURRENT_GEN}(bb) \cup (\text{CURRENT_IN}(bb) - \text{CURRENT_KILL}(bb))$
<code>backward_gen_kill_node_flow(bb)</code>	$\text{CURRENT_GEN}(bb) \cup (\text{CURRENT_OUT}(bb) - \text{CURRENT_KILL}(bb))$



The Generic Driver for Global Data Flow Analysis

```
pfbv_dfi ** gdfa_driver(struct gimple_pfbv_dfa_spec dfa_spec)
{
    if (find_entity_size(dfa_spec) == 0) return NULL;
    initialize_special_values(dfa_spec);
    create_dfi_space();
    traversal_order = dfa_spec.traversal_order;
    confluence = dfa_spec.confluence;

    local_dfa(dfa_spec);

    forward_edge_flow = dfa_spec.forward_edge_flow;
    backward_edge_flow = dfa_spec.backward_edge_flow;
    forward_node_flow = dfa_spec.forward_node_flow;
    backward_node_flow = dfa_spec.backward_node_flow;
    perform_pfbvdfa();

    preserve_dfi(dfa_spec.preserved_dfi);
    return current_pfbv_dfi;
}
```



The Generic Driver for Local Data Flow Analysis

- **The Main Difficulty:** Interface with the intermediate representation details



The Generic Driver for Local Data Flow Analysis

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- **State of Art:** The user is expected to supply the flow function implementation



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- **Our Key Ideas:**
 - ▶ Local data flow analysis is a special case of global data flow analysis
Other than the start and end blocks ($\equiv$ statements), every block has just one predecessor and one successor



The Generic Driver for Local Data Flow Analysis

- **The Main Difficulty:** Interface with the intermediate representation details
- **State of Art:** The user is expected to supply the flow function implementation
- **Our Key Ideas:**
 - ▶ Local data flow analysis is a special case of global data flow analysis Other than the start and end blocks ($\equiv$ statements), every block has just one predecessor and one successor
 - ▶ $ConstGen_n$ and $ConstKill_n$ are just different names given to particular sets of entities accumulated by traversing these basic blocks



The Generic Driver for Local Data Flow Analysis

- Traverse statements in a basic block in appropriate order

Exposition	Direction
up_exp	backward
down_exp	forward
any_where	don't care

- Solve the recurrence

$$\begin{aligned} \text{accumulated_entities} &= (\text{accumulated_entities} \\ &\quad - \text{remove_entities}) \\ &\quad \cup \text{add_entities} \end{aligned}$$


Example for Available Expressions Analysis

Entity is `entity_expr`.

Let $\text{expr}(x)$ denote the set of all expressions of x

Exposition	Manipulation	$a = b * c$		$b = b * c$	
		add	remove	add	remove
upwards	use				
downwards	use				
upwards	modification				
downwards	modification				



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downwards	use	$b * c$	$\text{expr}(a)$	$\emptyset$	$\text{expr}(b)$
upwards	modification	$\text{expr}(a)$	$b * c$	$\text{expr}(b) - \{b * c\}$	
downwards	modification				



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Example for Available Expressions Analysis

Entity is `entity_expr`.

Let $\text{expr}(x)$ denote the set of all expressions of x

Exposition	Manipulation	$a = b * c$		$b = b * c$	
		add	remove	add	remove
upwards	use	$b * c$	$\text{expr}(a)$	$b * c$	$\text{expr}(b)$
downwards	use	$b * c$	$\text{expr}(a)$	$\emptyset$	$\text{expr}(b)$
upwards	modification	$\text{expr}(a)$	$b * c$	$\text{expr}(b) - \{b * c\}$	$b * c$
downwards	modification	$\text{expr}(a)$	$b * c$	$\text{expr}(b)$	$\emptyset$

Note: In the case of modifications, if we first add then remove the entities modification, the set difference is not required



Future Work

Main thrust

- Supporting general data flow frameworks
- Supporting interprocedural analysis

