Workshop on Essential Abstractions in GCC

Introduction to Parallelization and Vectorization

GCC Resource Center

(www.cse.iitb.ac.in/grc)

Department of Computer Science and Engineering,

Indian Institute of Technology, Bombay

July 2010

Outline

- Transformation for parallel and vector execution
- Data dependence

The Scope of this Tutorial

- What this tutorial does not address
  - Algorithms used for parallelization and vectoriation
  - Machine level issues related to parallelization and vectoriation
- What this tutorial addresses

  Basics of Discovering Parallelism using GCC

Part 1

Transformations for Parallel and Vector Execution

A Taxonomy of Parallel Computation

	Single Program	Multiple Programs
Single Data	SPSD	MPSD
Multiple Data	SPMD	MPMD

A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	MISD	MPSD
Multiple Data	SIMD	MIMD	MPMD

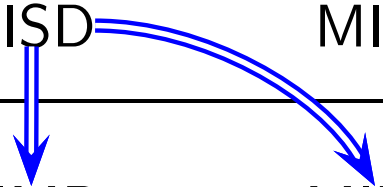
A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	?	?
Multiple Data	SIMD	MIMD	MPMD

Redundant computation for validation of intermediate steps

A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	MISD	MPSD
Multiple Data	SIMD	MIMD	MPMD



Transformations performed by a compiler

Vectorization: SISD ⇒ SIMD

- Parallelism in executing operation on shorter operands (8-bit, 16-bit, 32-bit operands)
- Existing 32 or 64-bit arithmetic units used to perform multiple operations in parallel
  A 64 bit word ≡ a vector of 2×(32 bits), 4×(16 bits), or 8×(8 bits)

Example 1

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : Yes

Original Code

```
int A[N], B[N], i;
for (i=1; i<N; i++)
  A[i] = A[i] + B[i-1];
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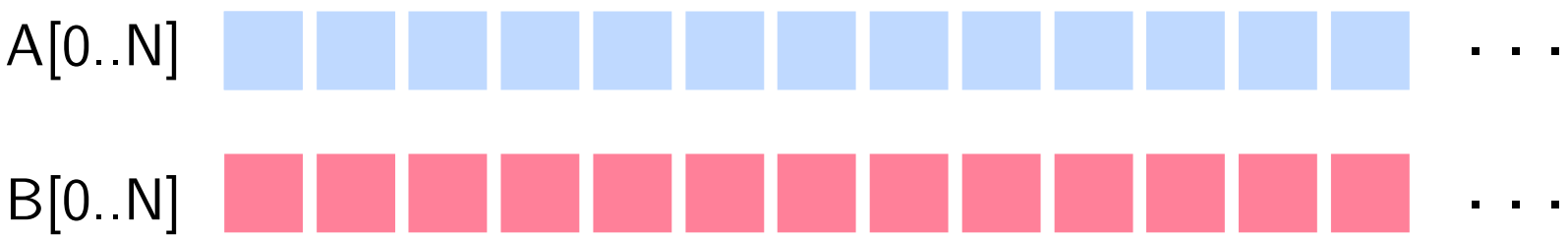

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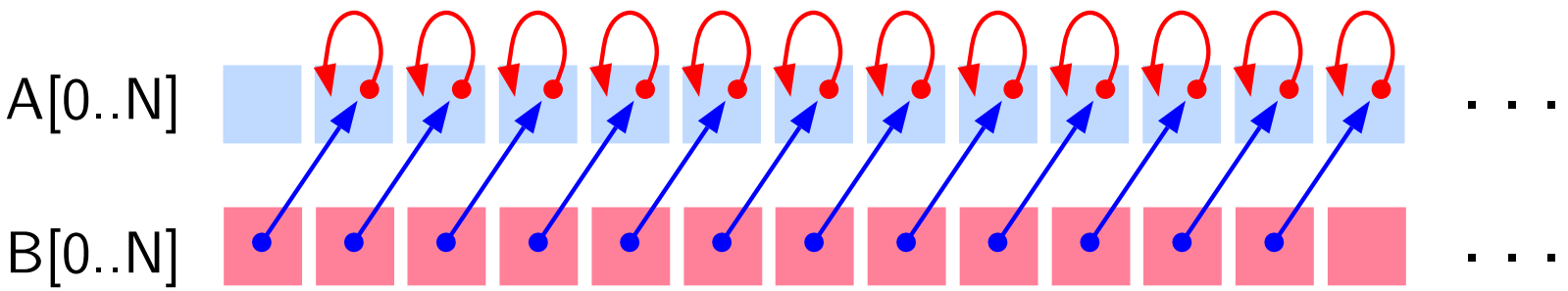

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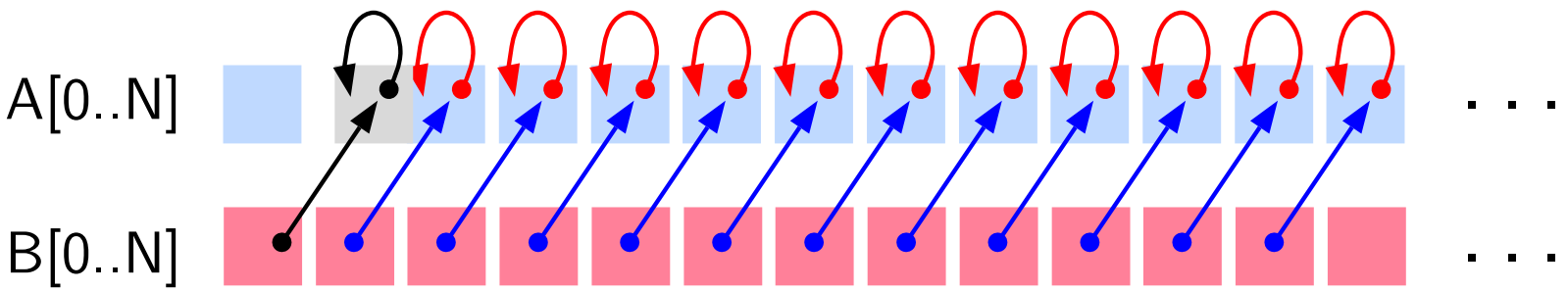

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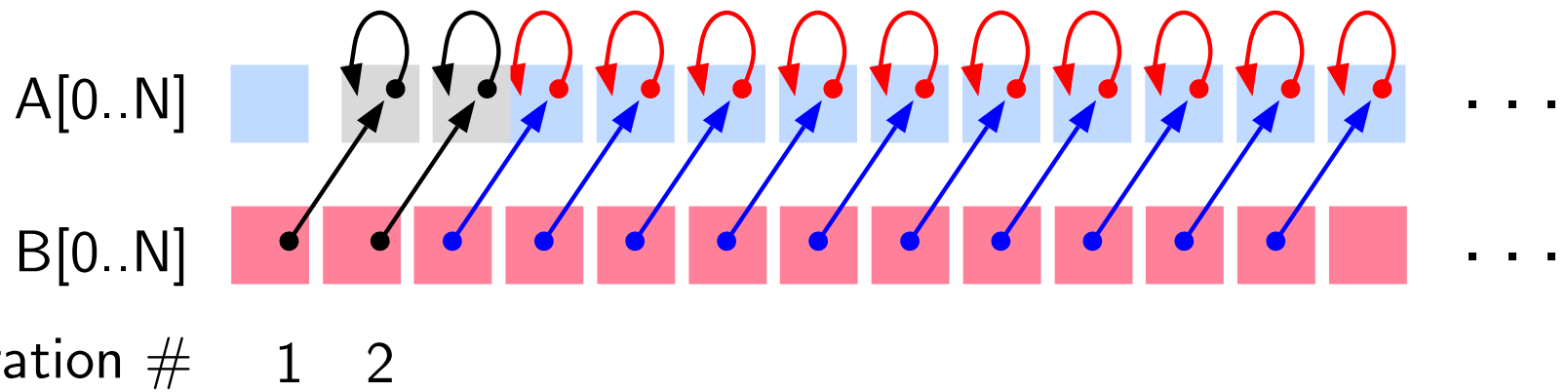

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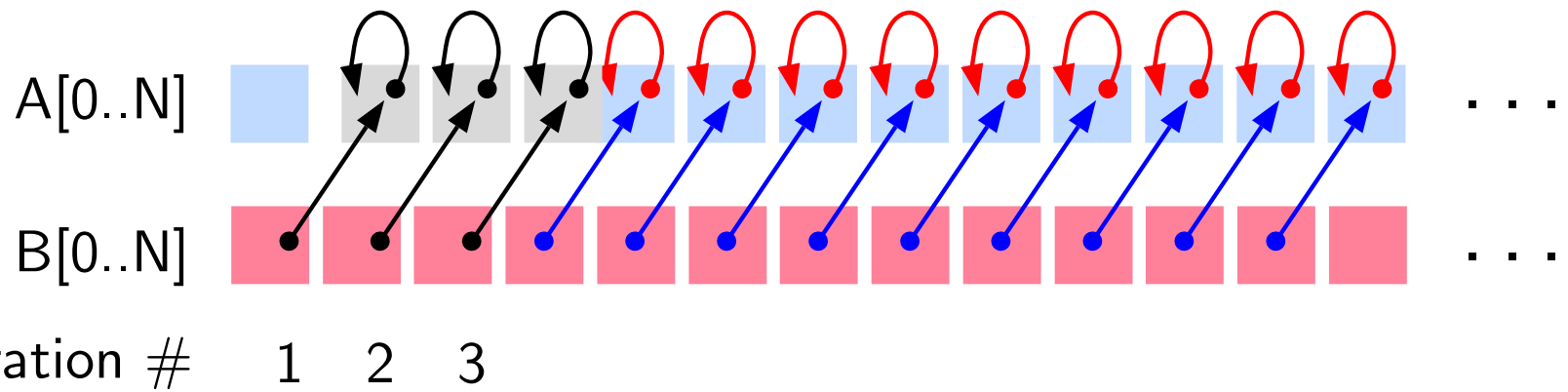

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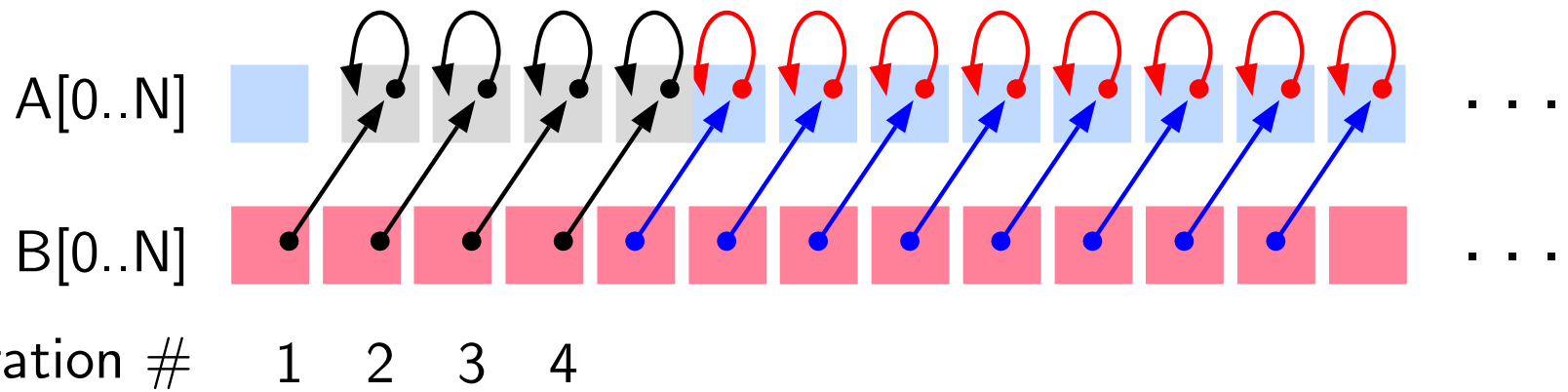

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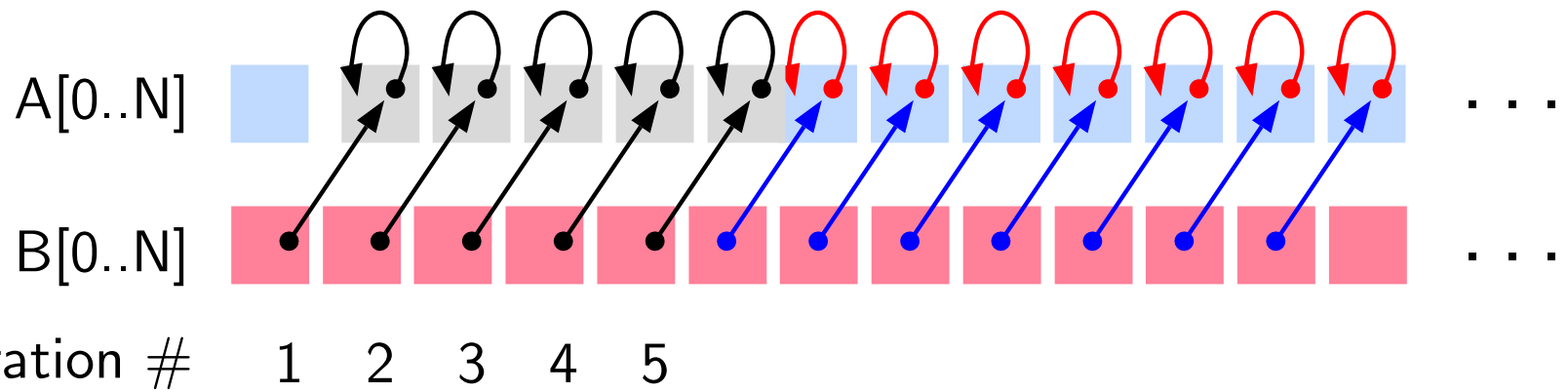

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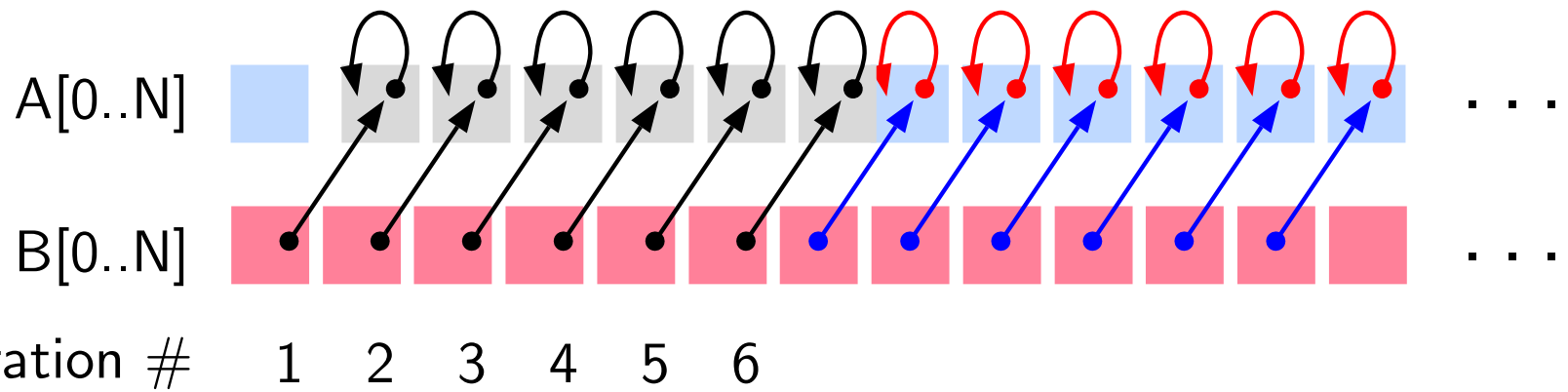

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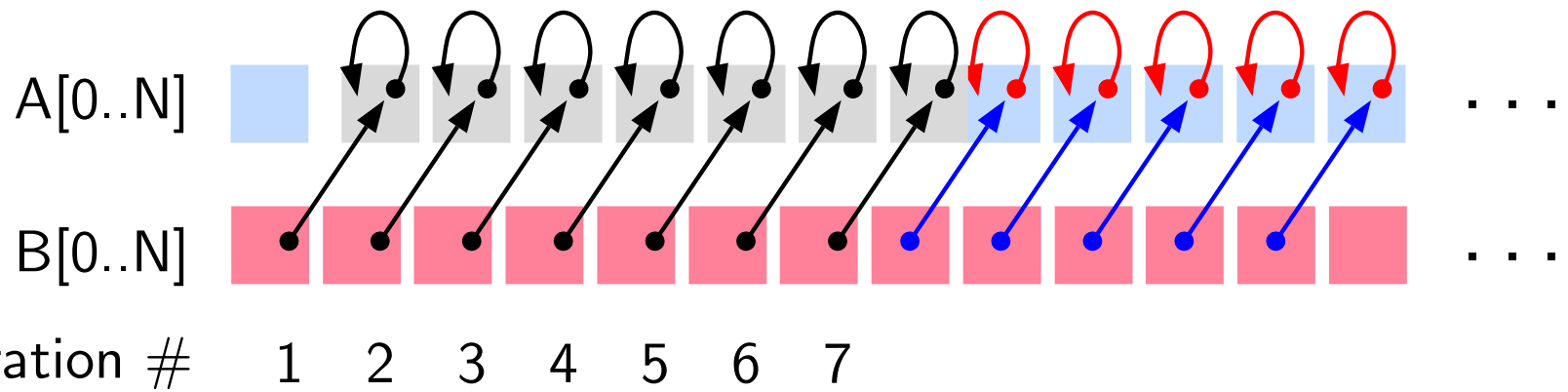

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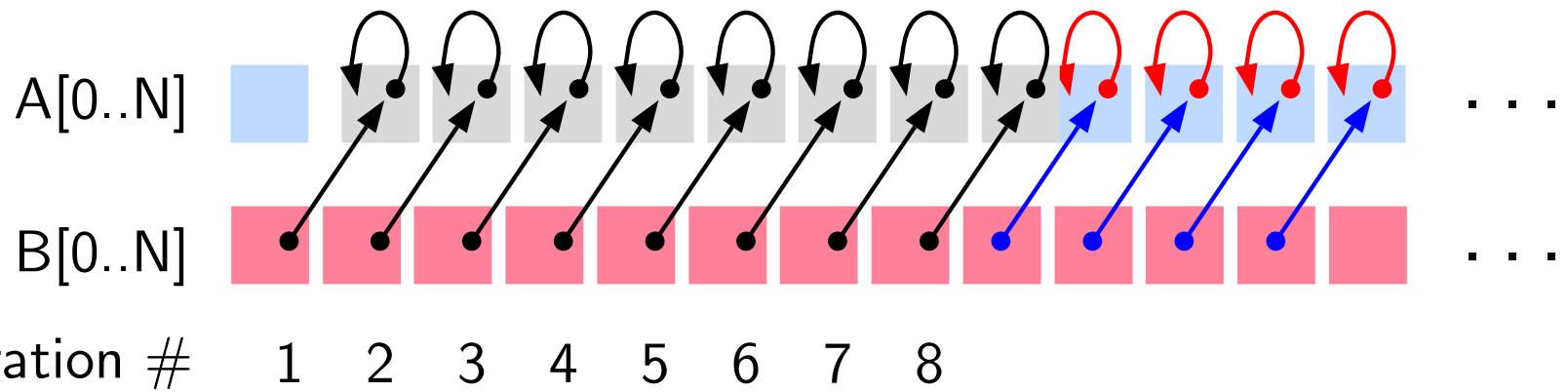

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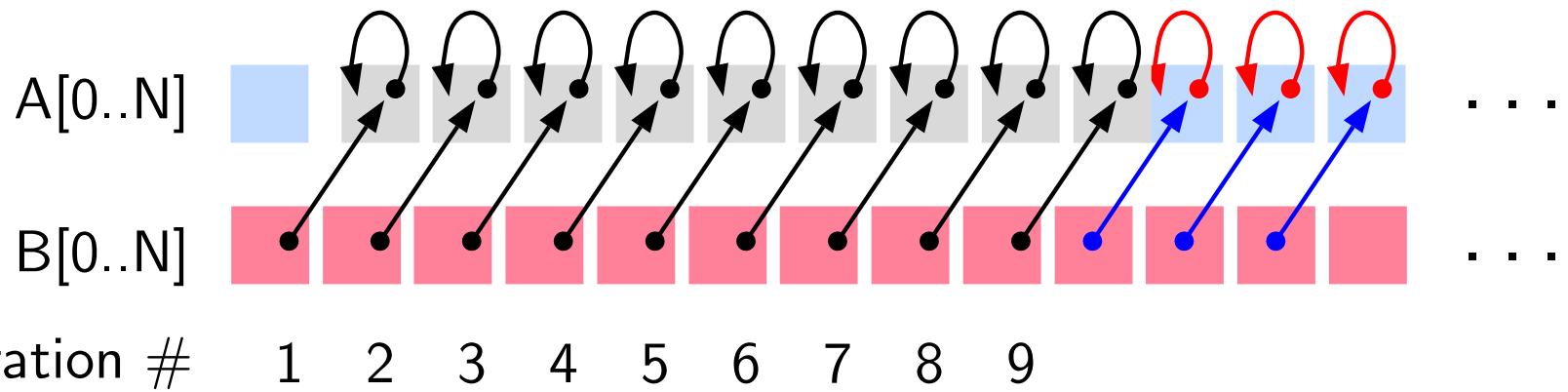

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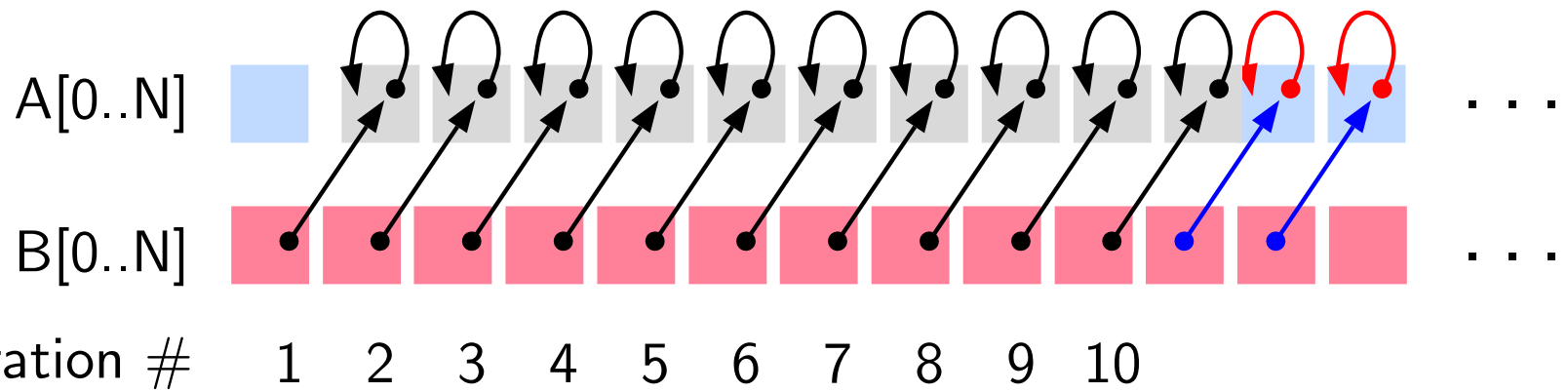

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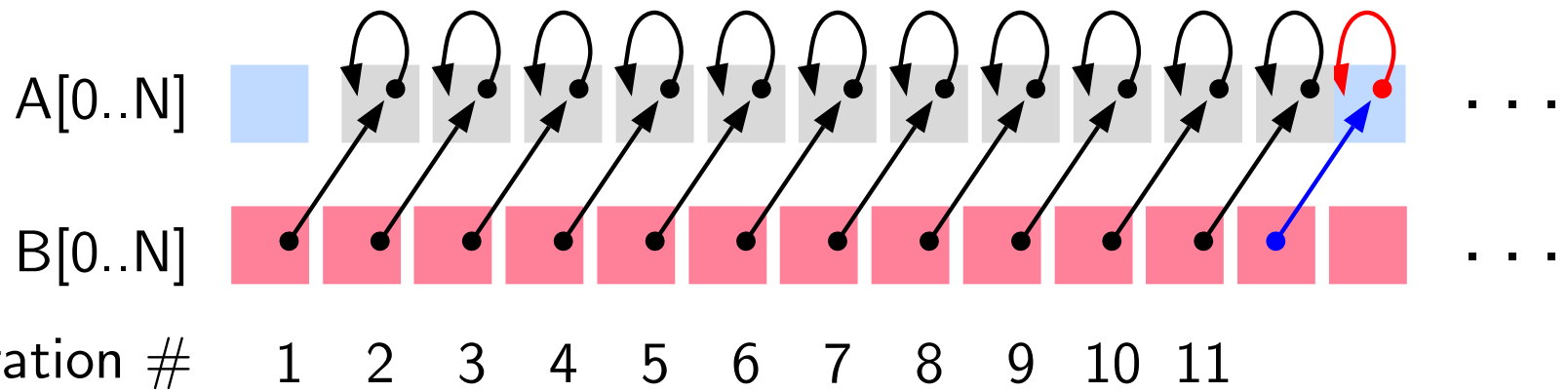

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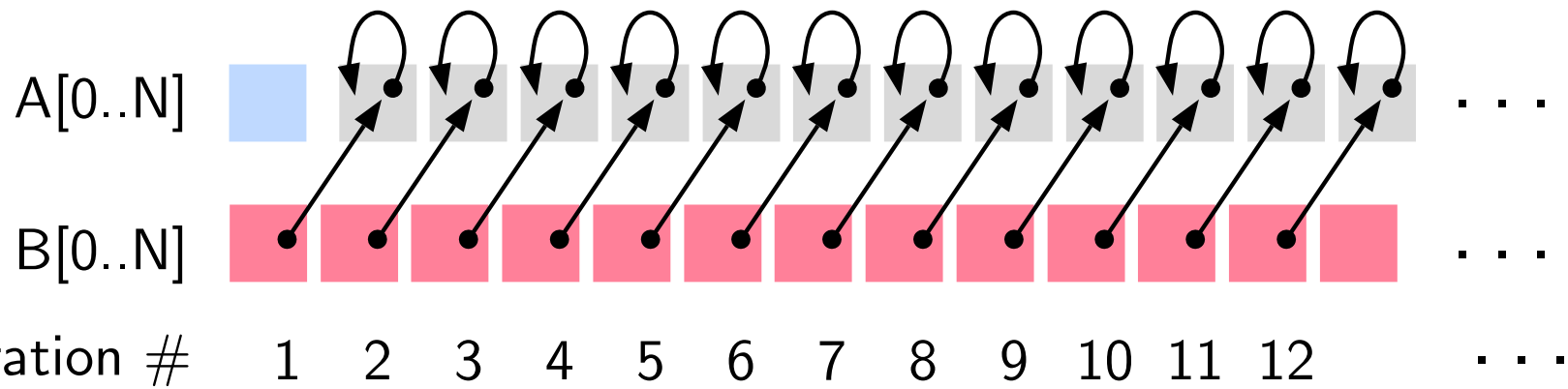

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```

Vectorized Code

```
int A[N], B[N], i;
for (i=1; i<N; i=i+4)
  A[i:i+3] = A[i:i+3] + B[i-1:i+2];
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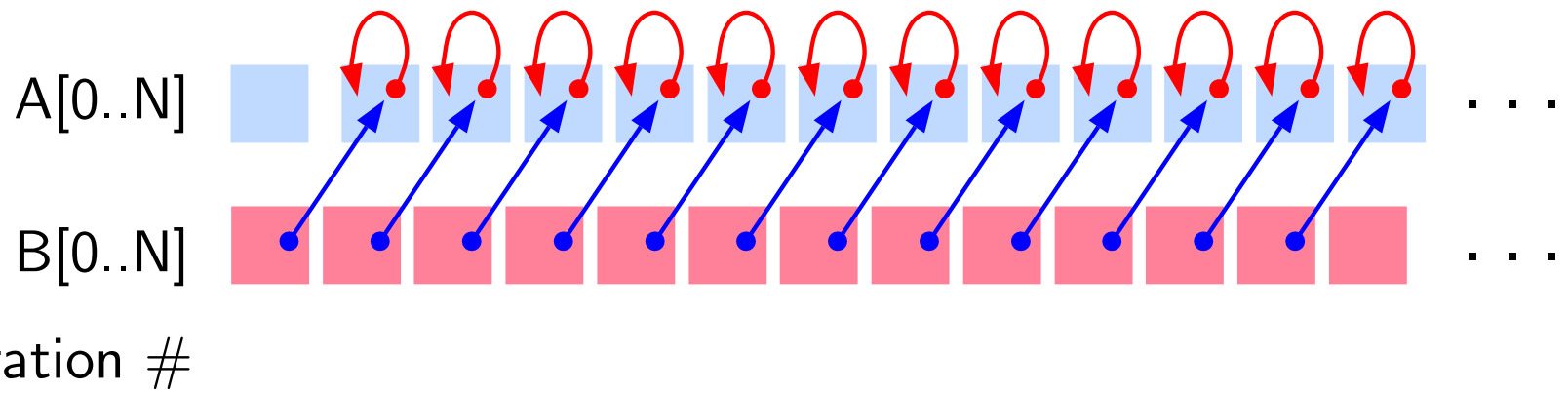

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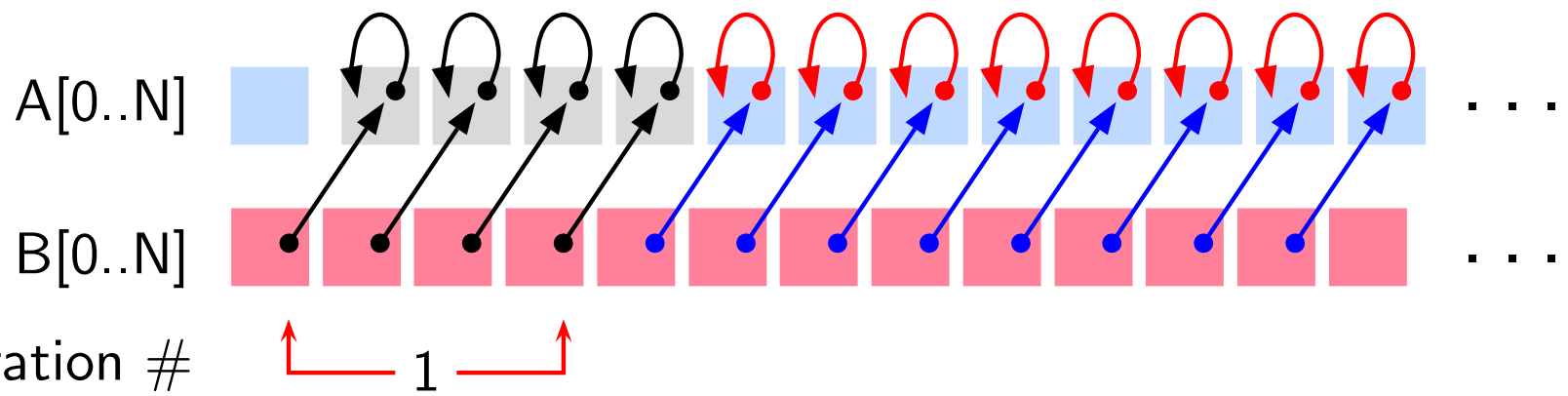

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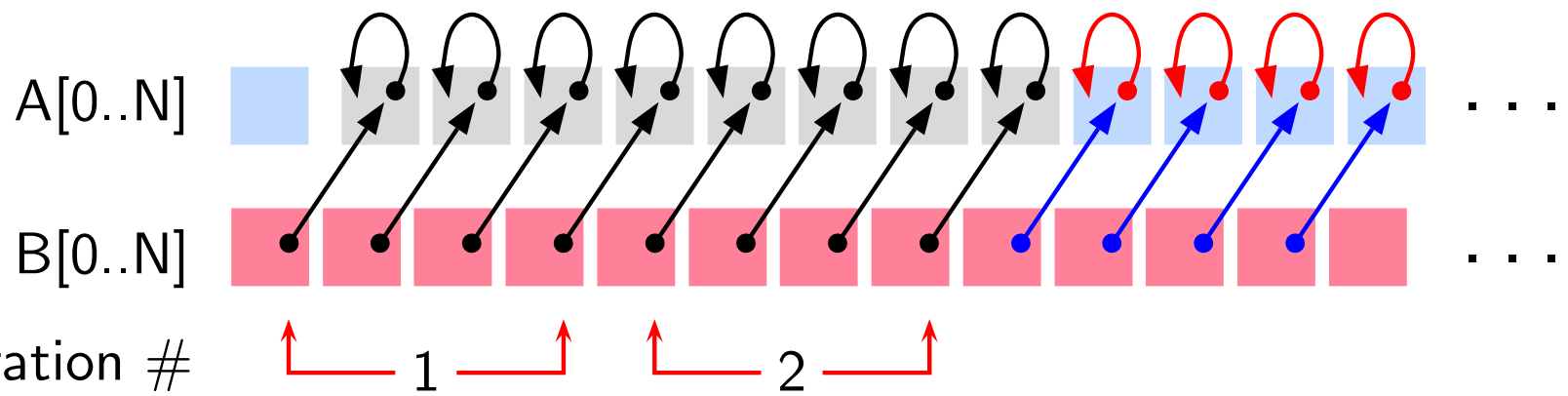

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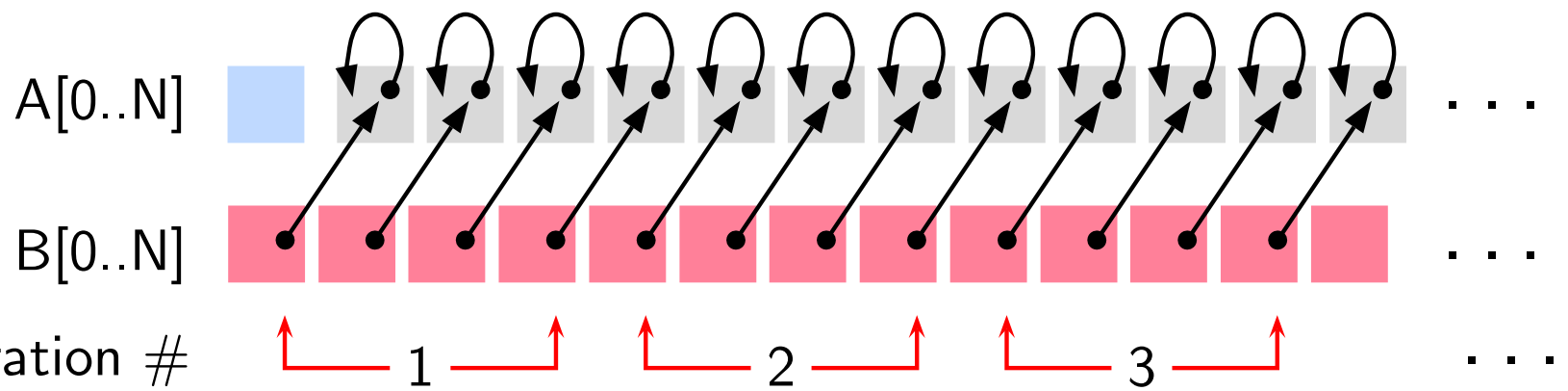

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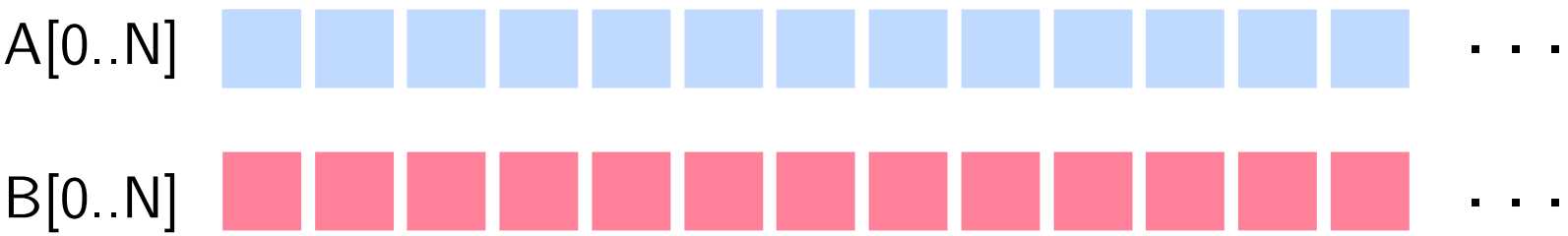

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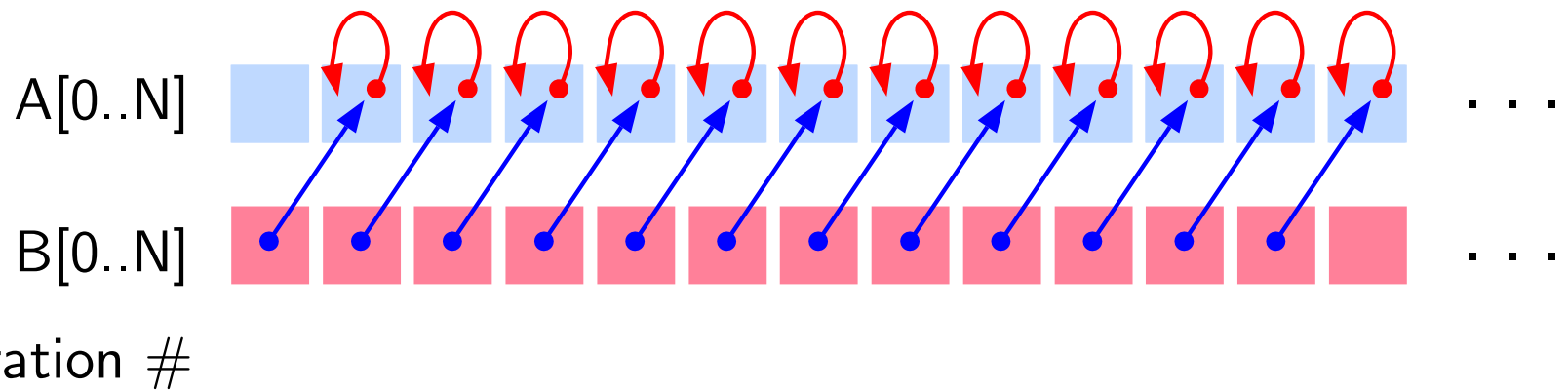

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Parallelized Code

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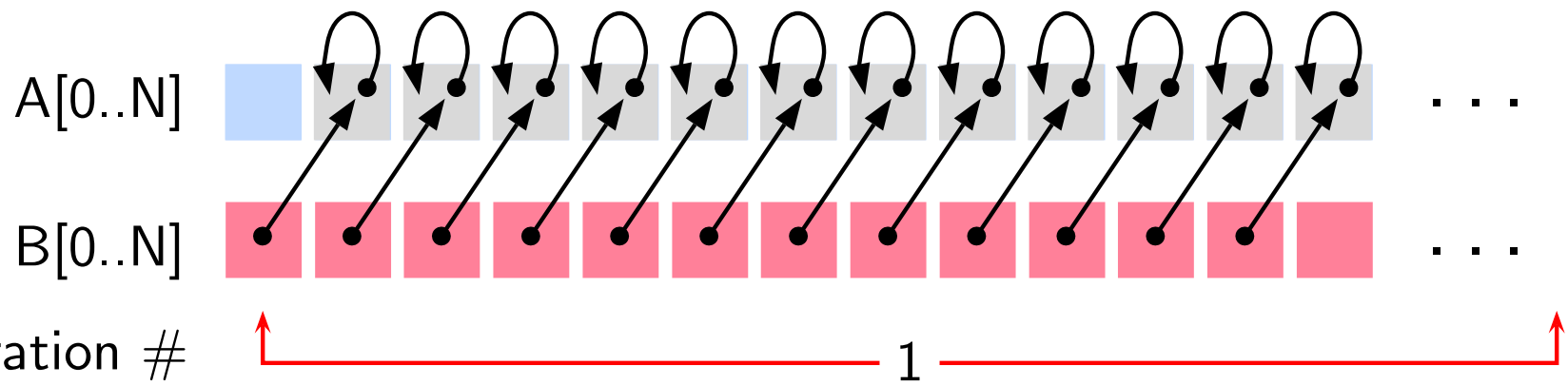

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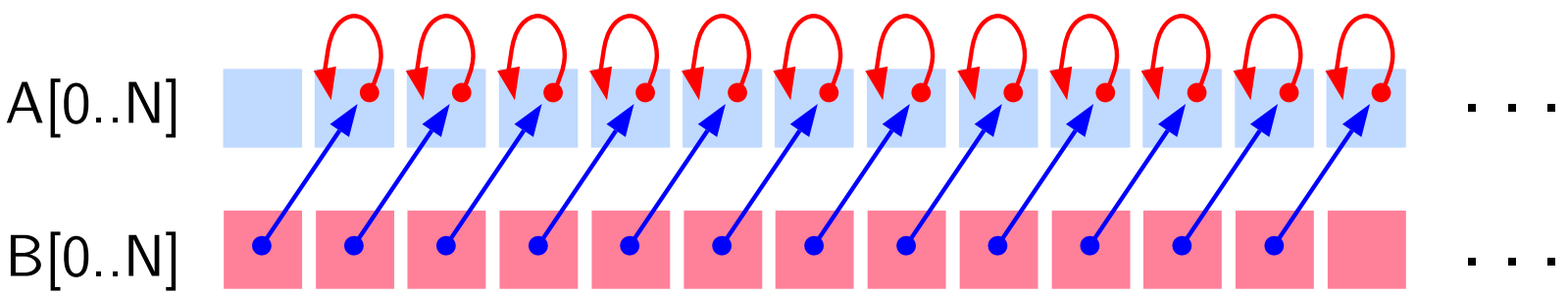

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Vectorization (SISD ⇒ SIMD) : Yes
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When the same location is accessed across different iterations, the order of reads and writes must be preserved

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for (i
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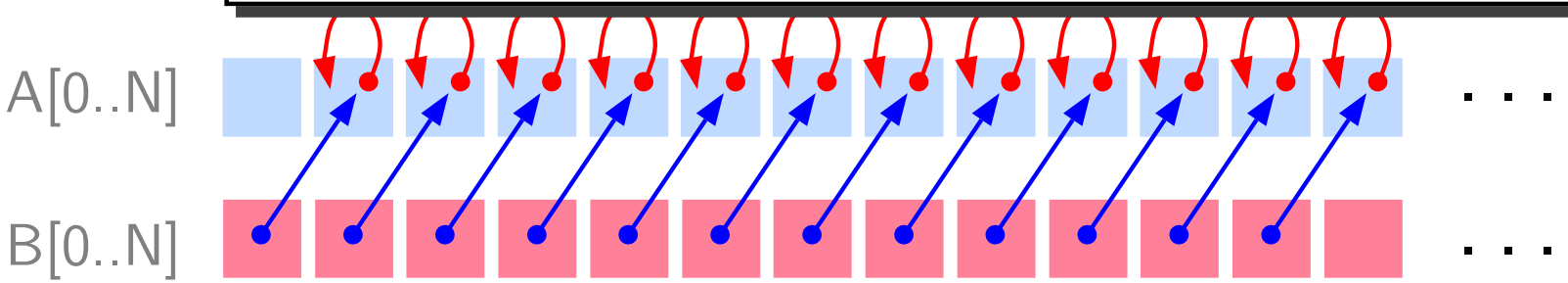
Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	
Write	Read	
Write	Write	
Read	Read	



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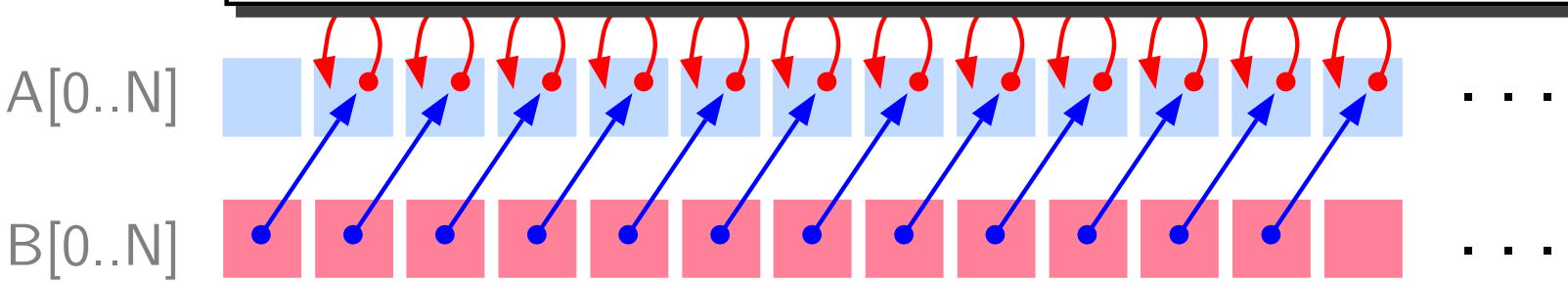

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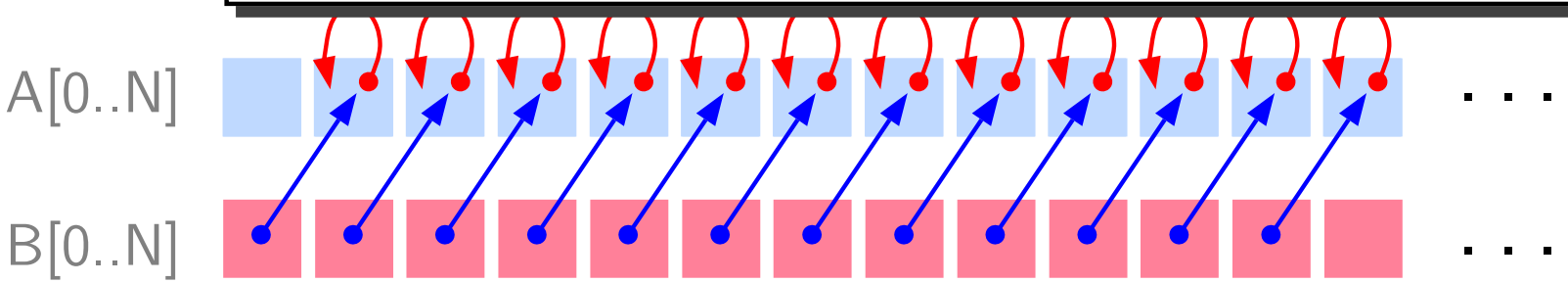

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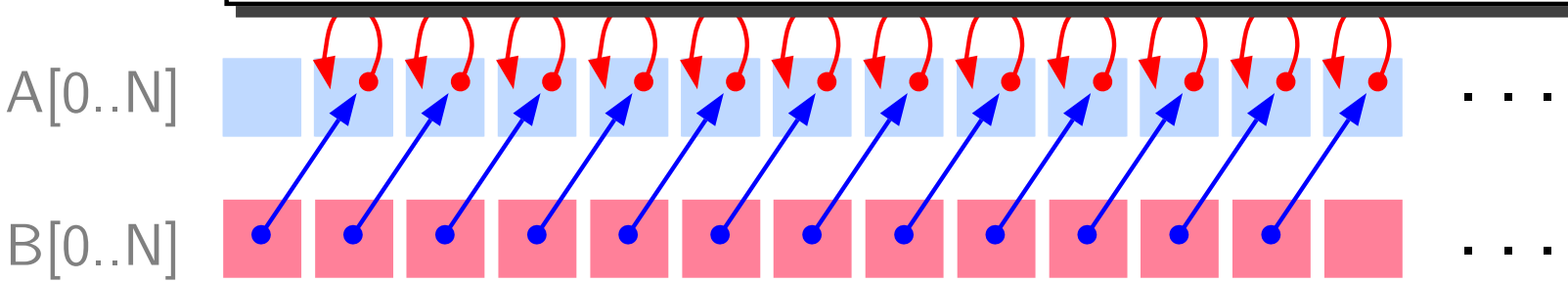

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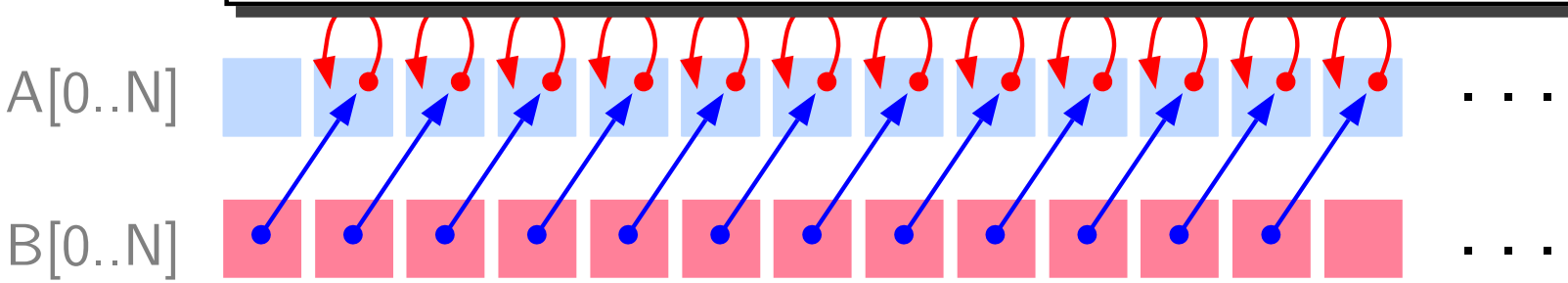
Read	Read	Does not matter



Example 2

Vectorization (SISD $\Rightarrow$ SIMD) : Yes
Parallelization (SISD $\Rightarrow$ MIMD) : No

Original Code

```
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for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
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Example 2

Vectorization (SISD ⇒ SIMD) : Yes
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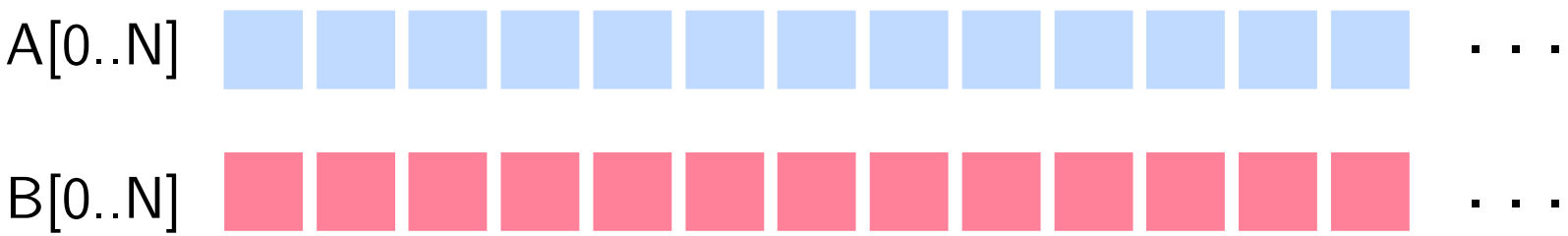

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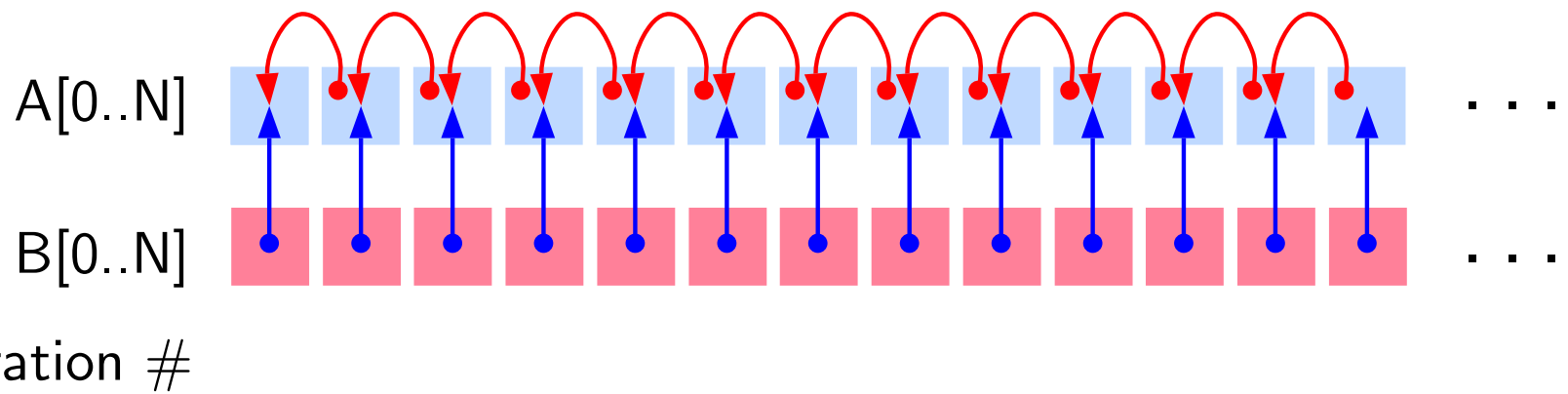
```

Observe reads and writes into a given location



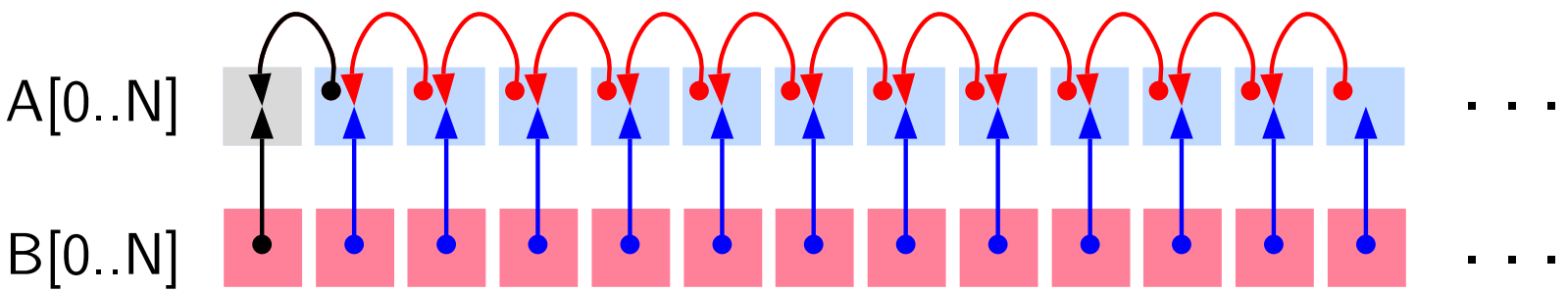
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



Iteration # 1

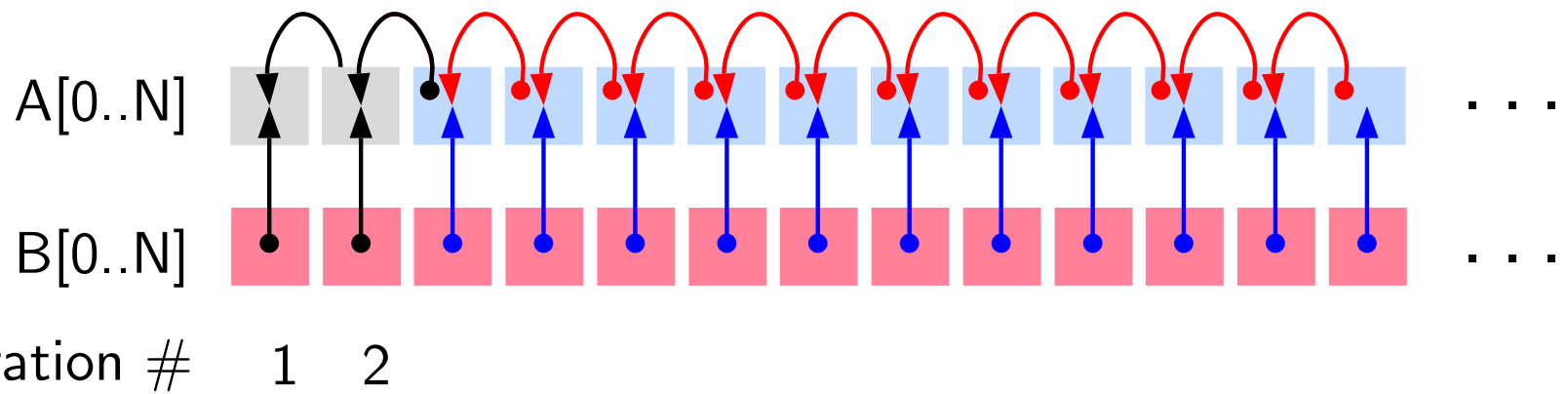
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



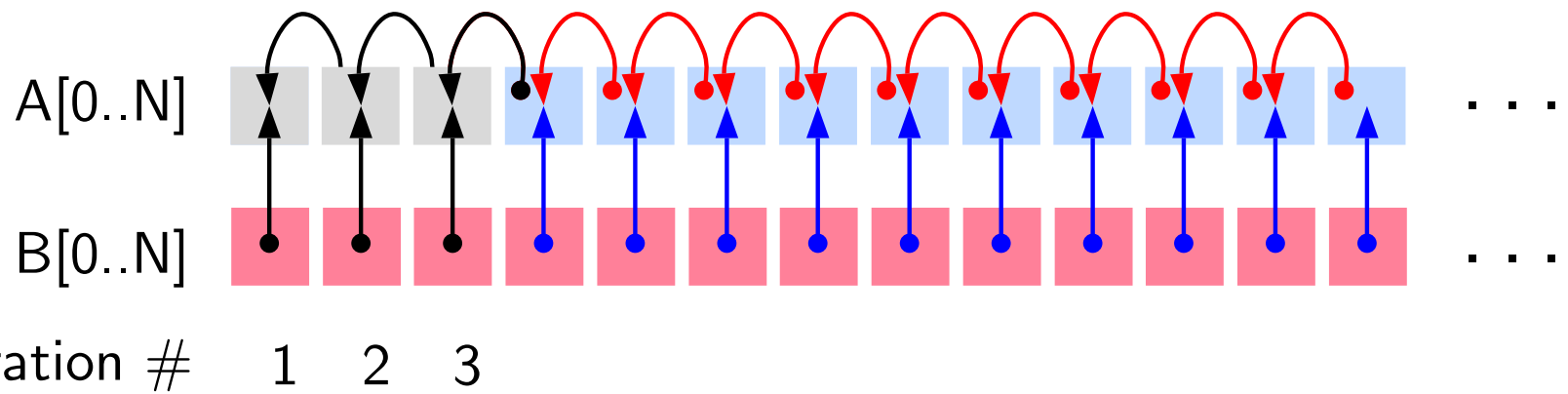
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



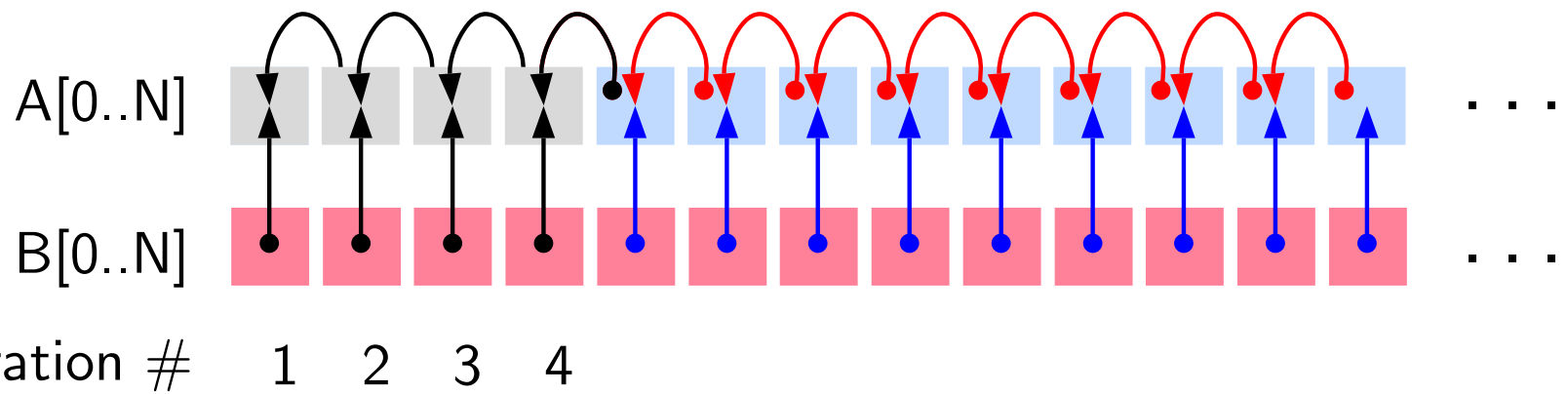
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



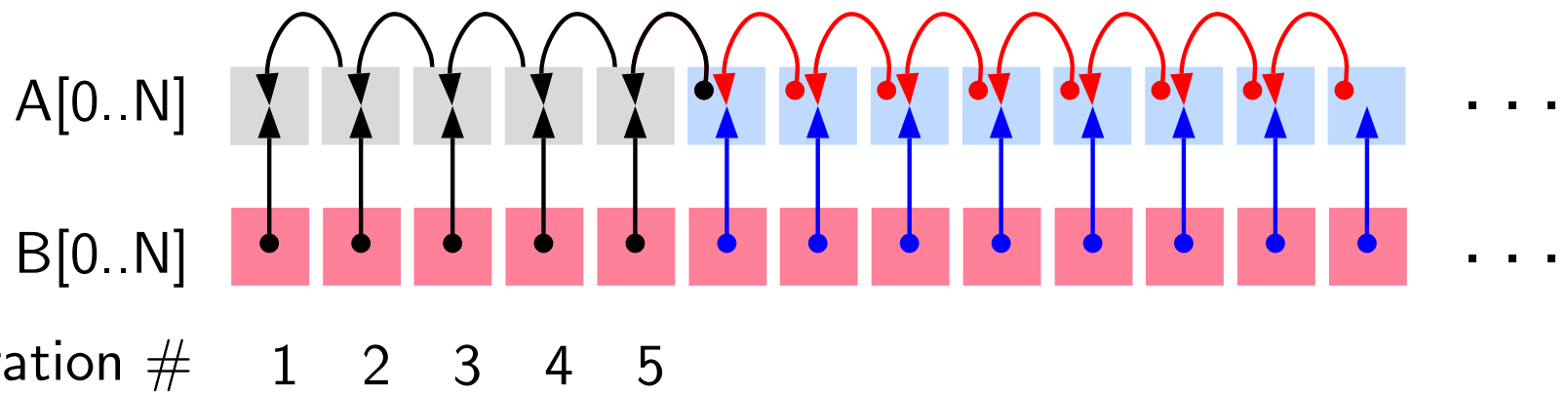
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



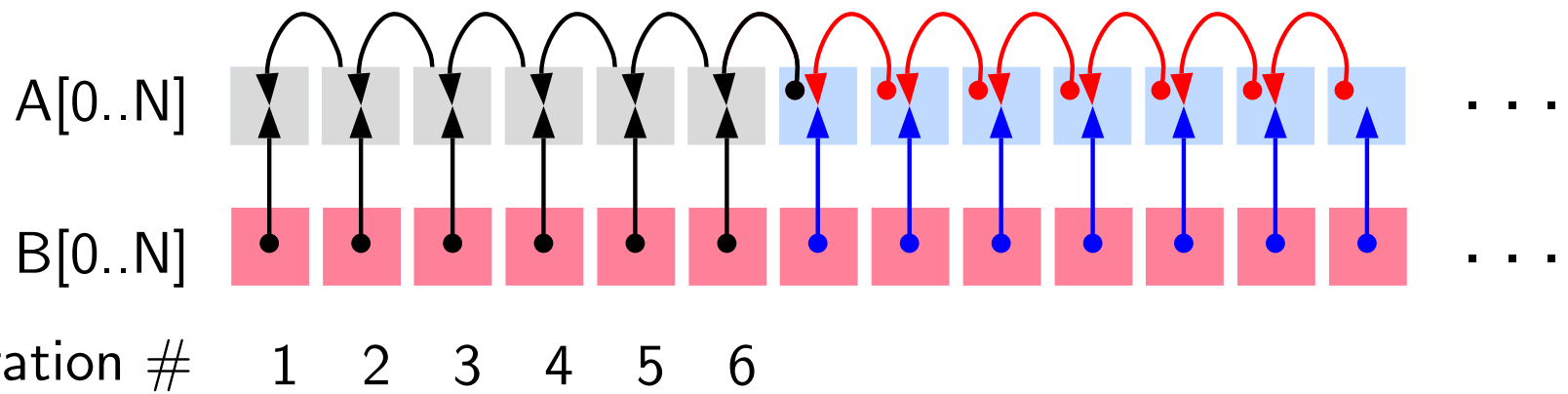
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



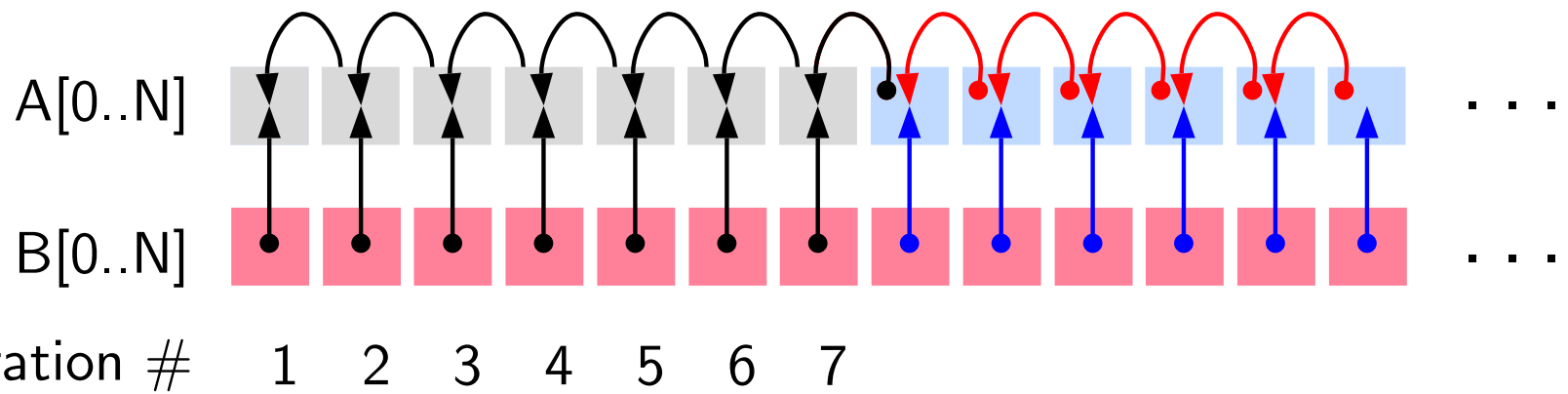
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



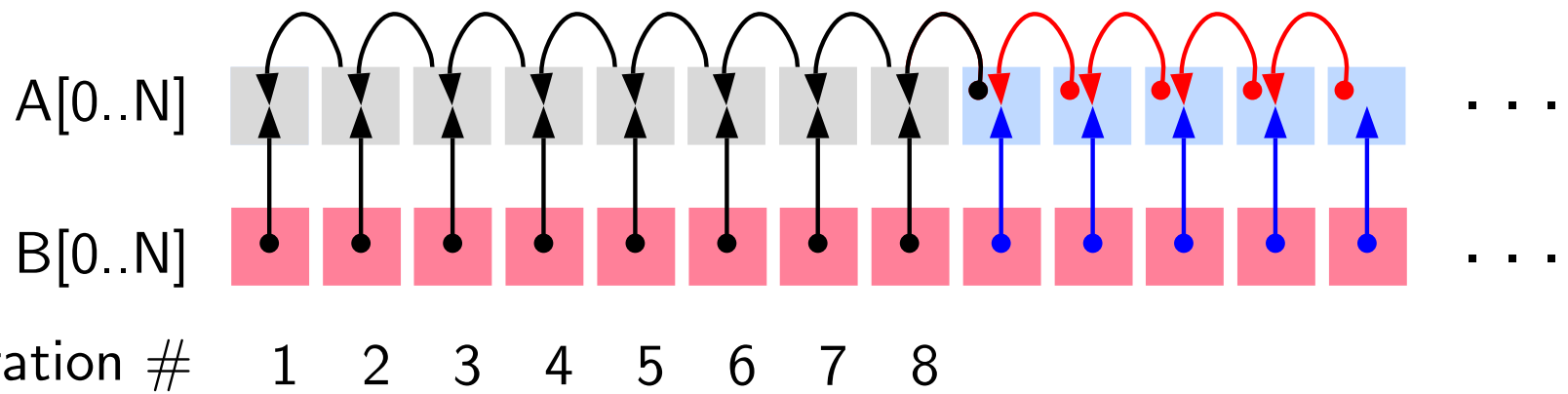
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



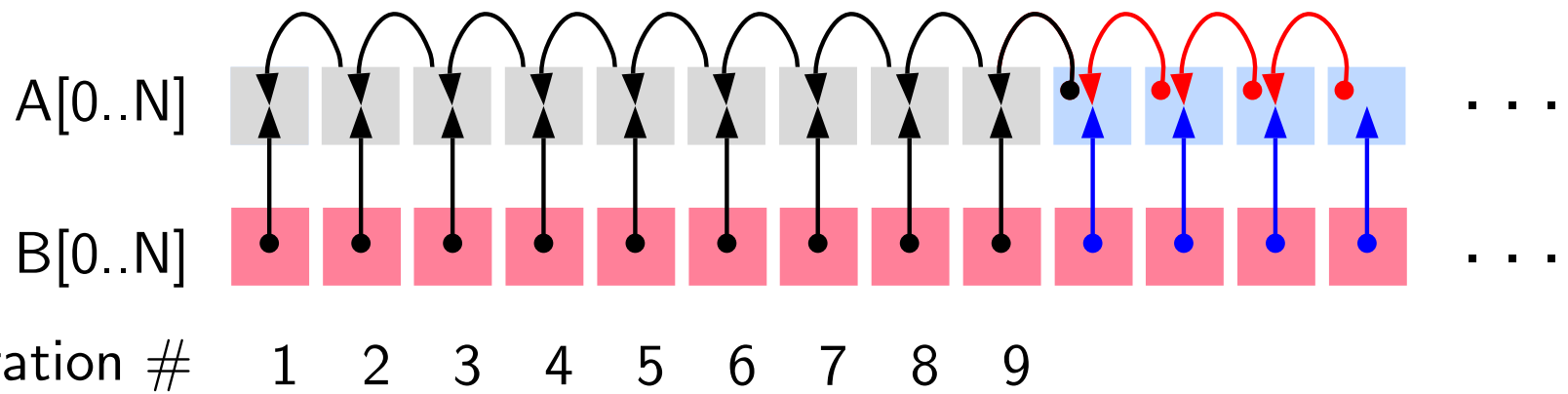
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



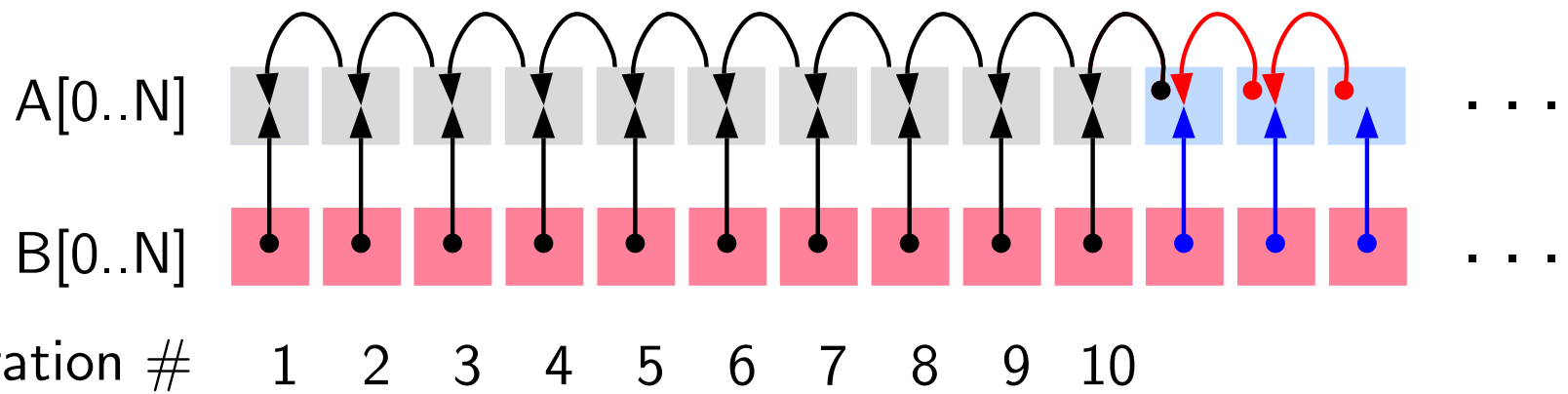
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



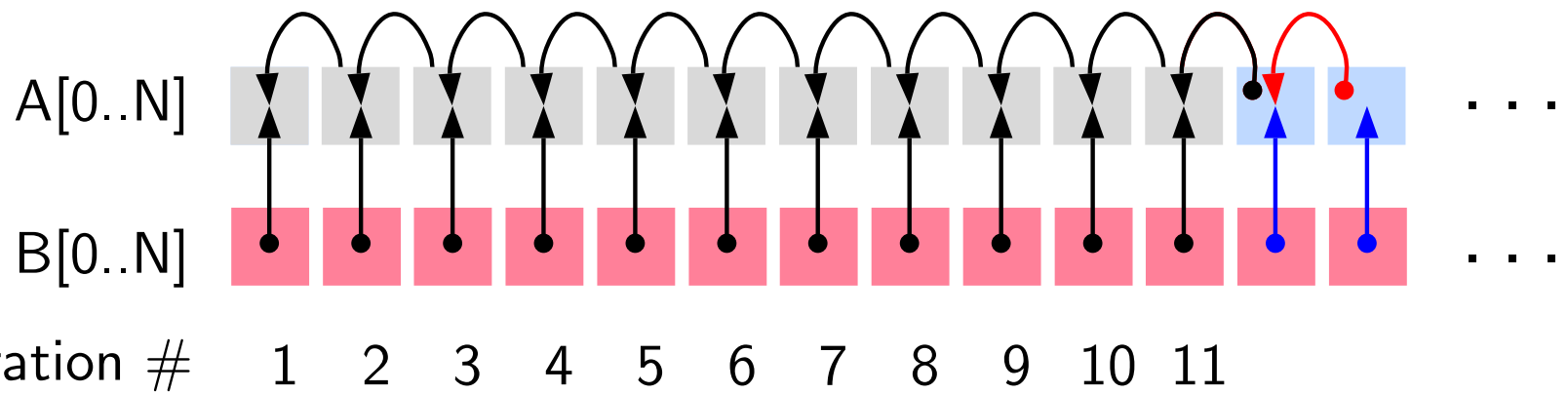
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



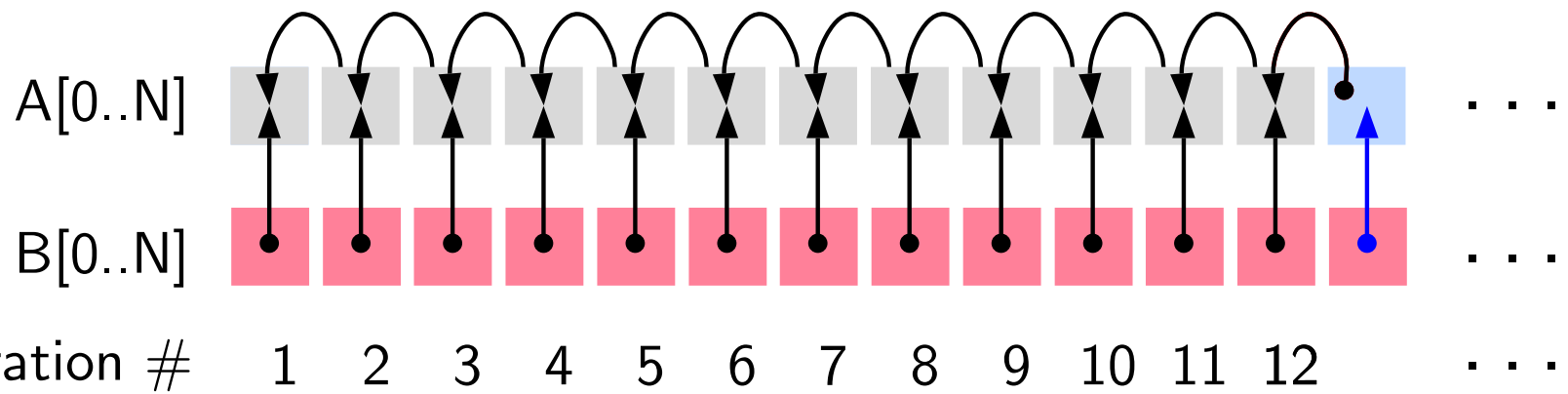
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



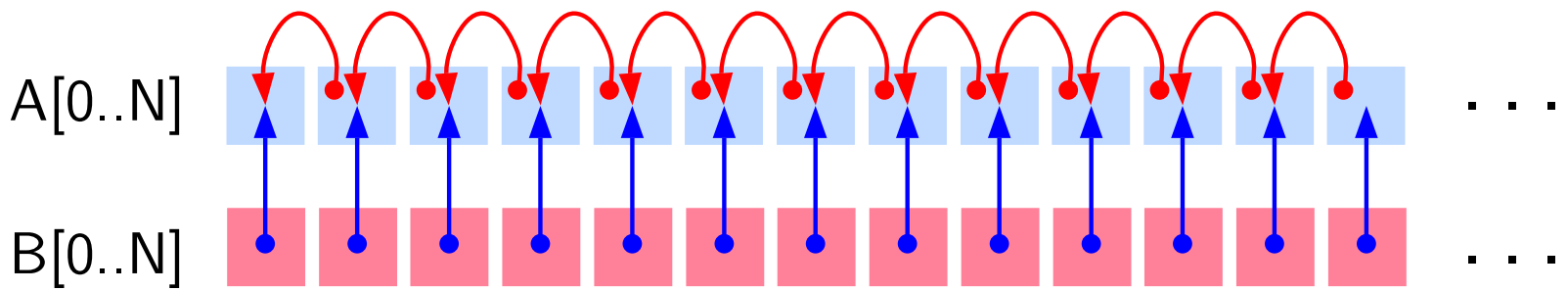
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



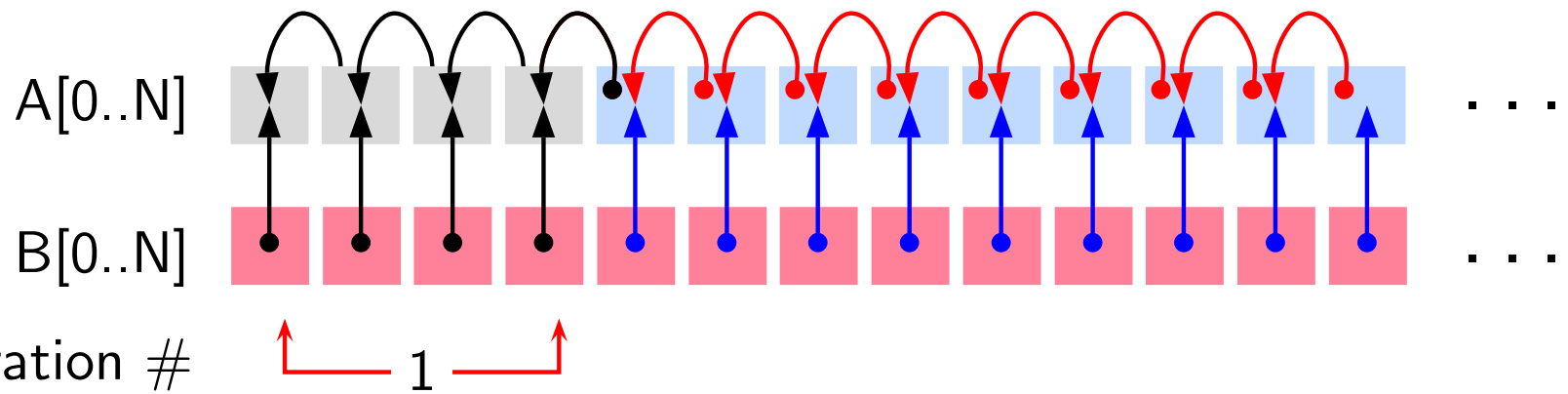
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



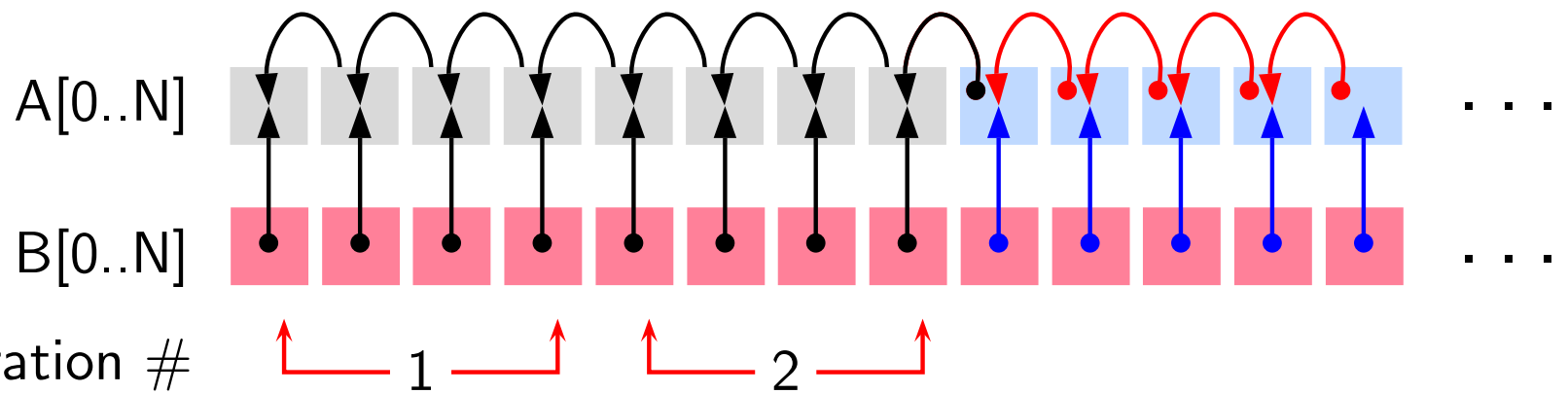
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



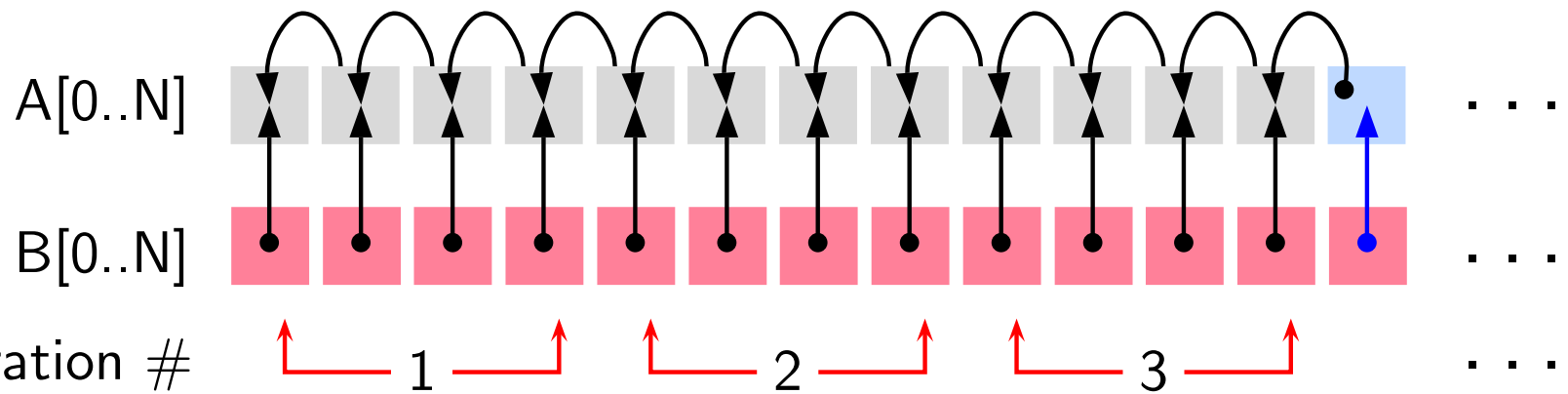
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



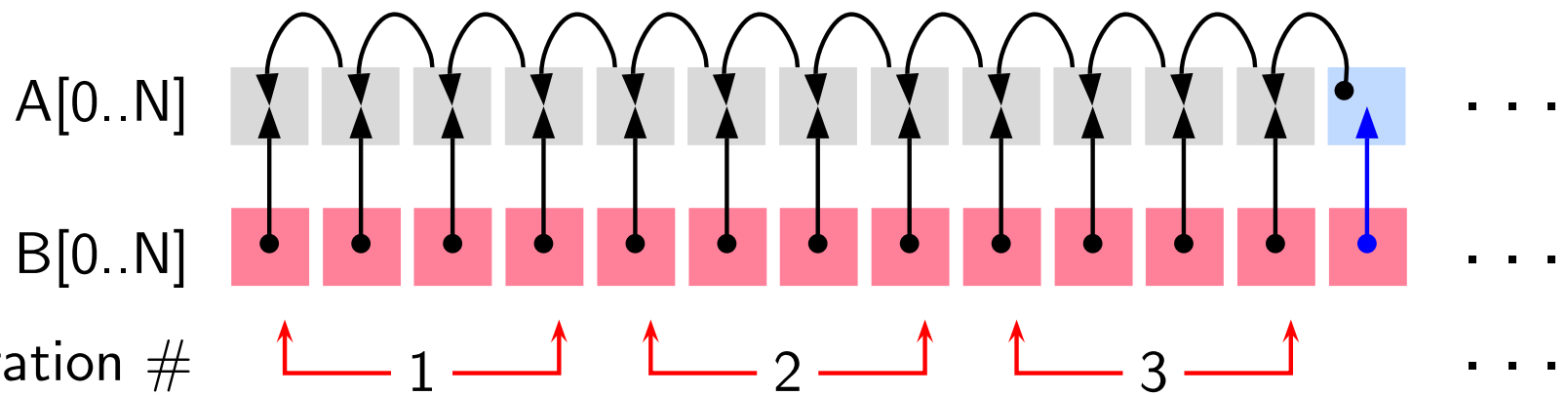
Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction
- Read-writes across multiple instructions executing in parallel may not be synchronized



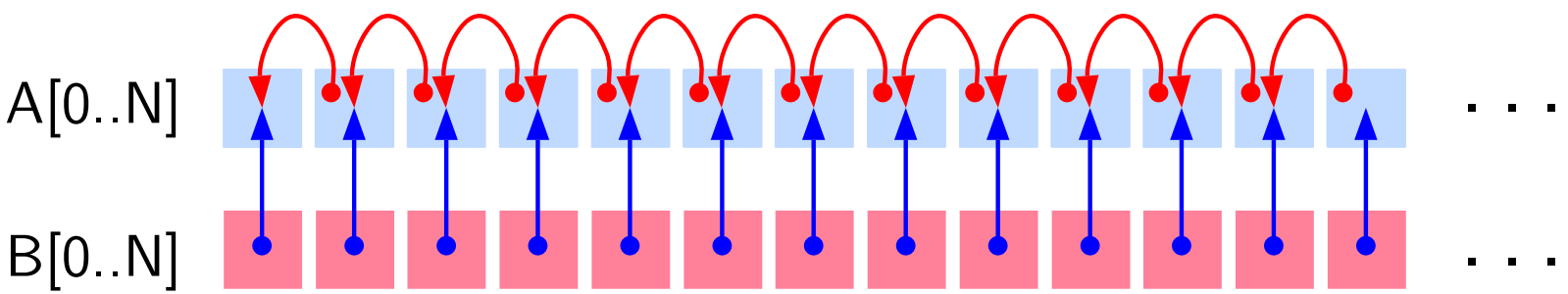
Example 2: The Moral of the Story

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



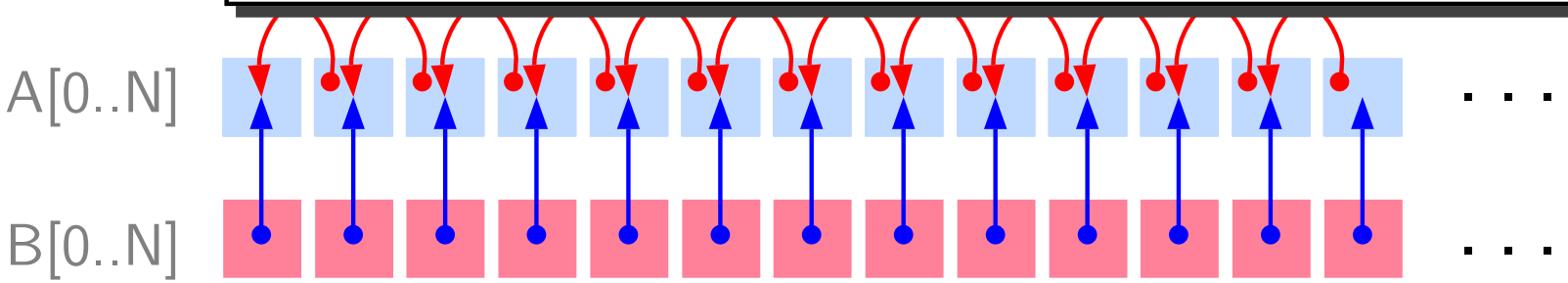
Example 2: The Moral of the Story

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

When the same location is accessed across different iterations, the order of reads and writes must be preserved

```
int A[
for (i
  A[i]
```

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	
Write	Read	
Write	Write	
Read	Read	



Example 2: The Moral of the Story

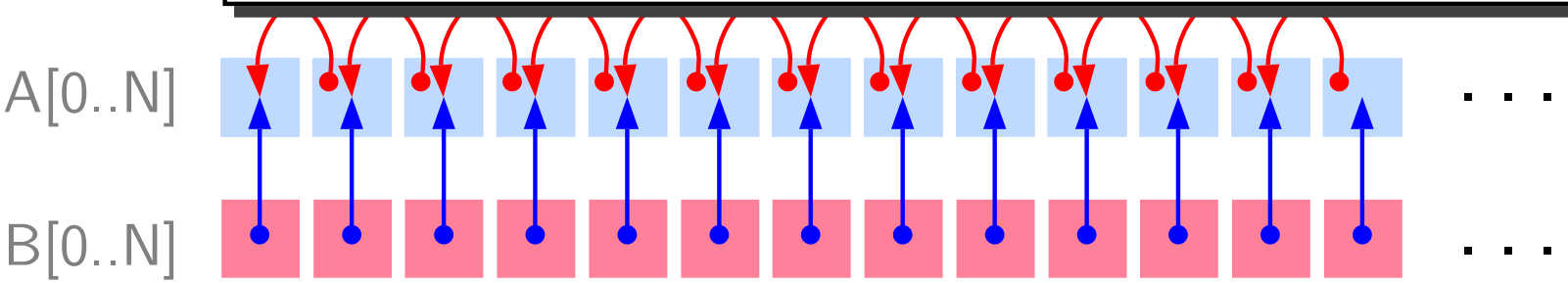
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

When the same location is accessed across different iterations, the order of reads and writes must be preserved

```
int A[
for (i
  A[i]
```

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	Yes
Write	Read	
Write	Write	
Read	Read	



Example 2: The Moral of the Story

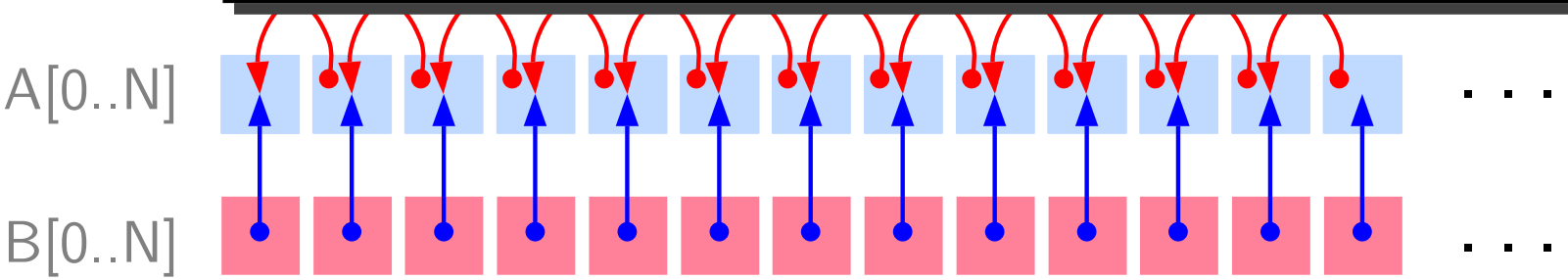
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

When the same location is accessed across different iterations, the order of reads and writes must be preserved

```
int A[
for (i
  A[i]
```

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	
Read	Read	



Example 2: The Moral of the Story

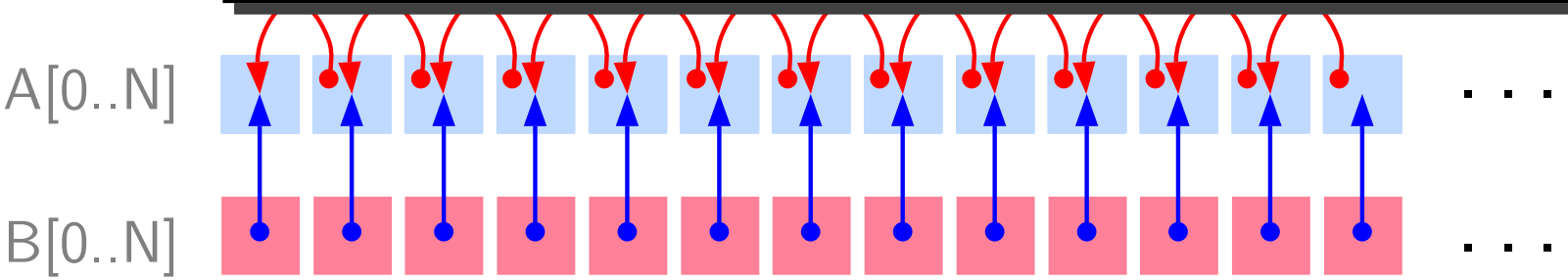
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

When the same location is accessed across different iterations, the order of reads and writes must be preserved

```
int A[
for (i
  A[i]
```

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	No
Read	Read	



Example 2: The Moral of the Story

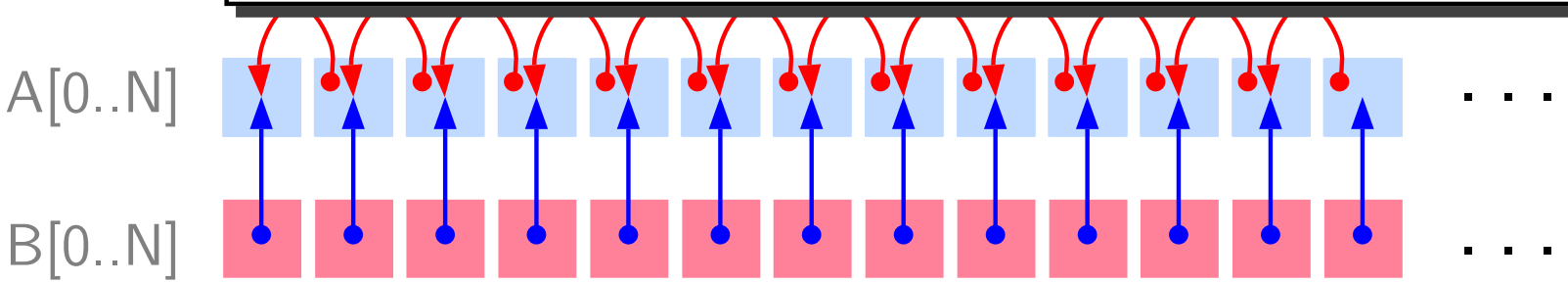
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

When the same location is accessed across different iterations, the order of reads and writes must be preserved

```
int A[
for (i
  A[i]
```

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	No
Read	Read	Does not matter



Example 3

Vectorization (SISD $\Rightarrow$ SIMD) : No
Parallelization (SISD $\Rightarrow$ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

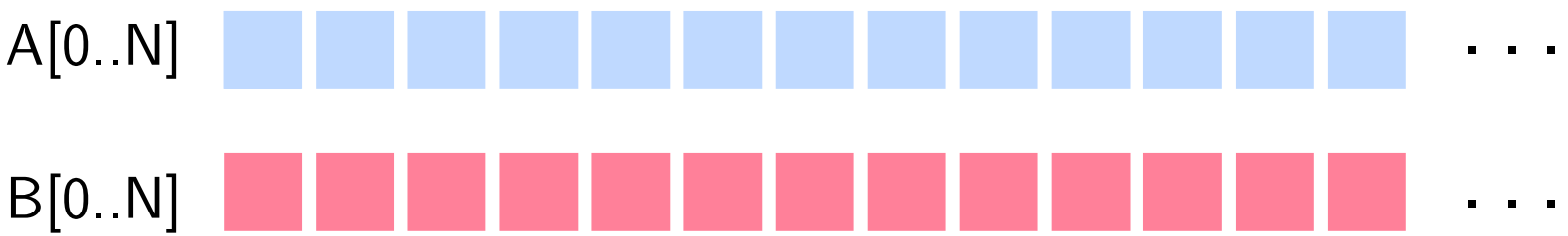
Observe reads and writes into a given location

Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

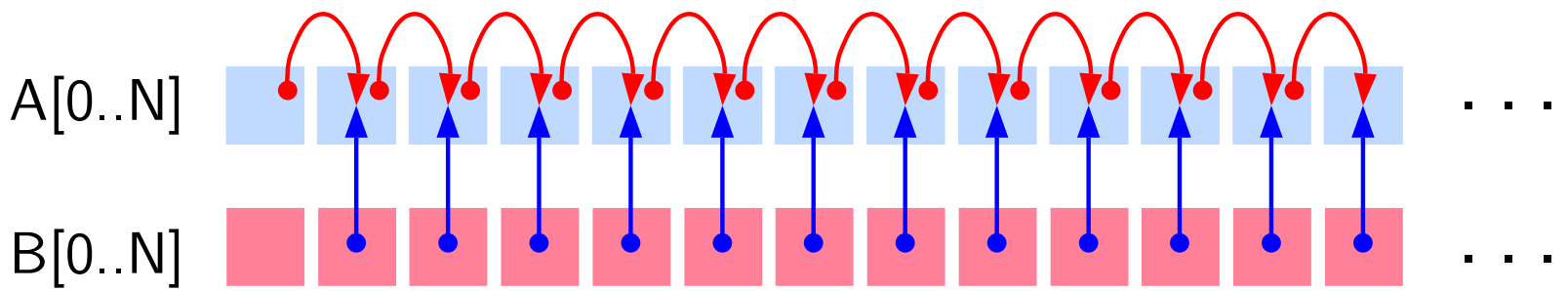


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

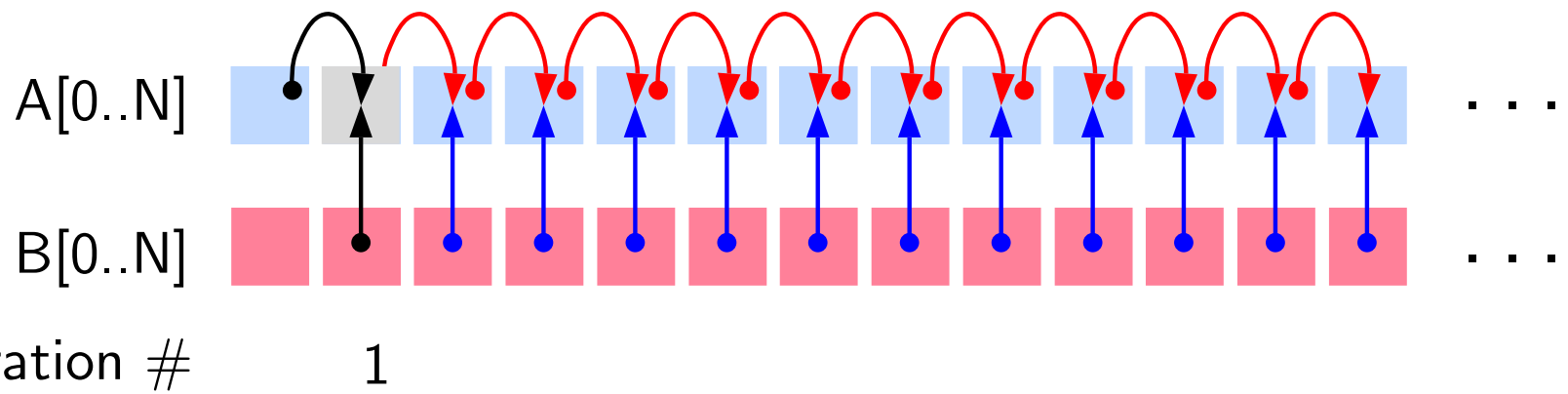


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

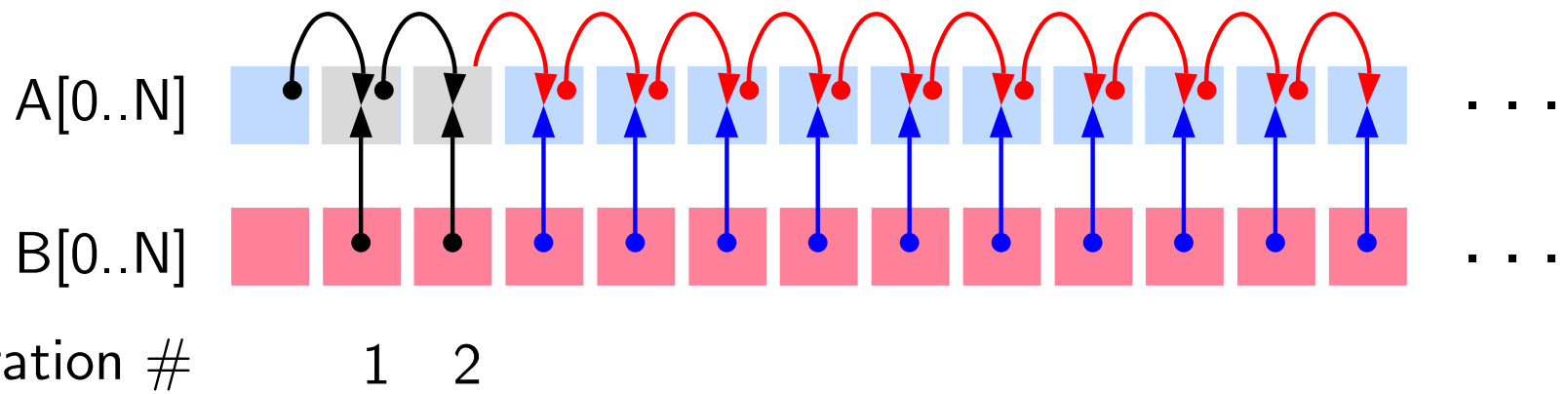


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

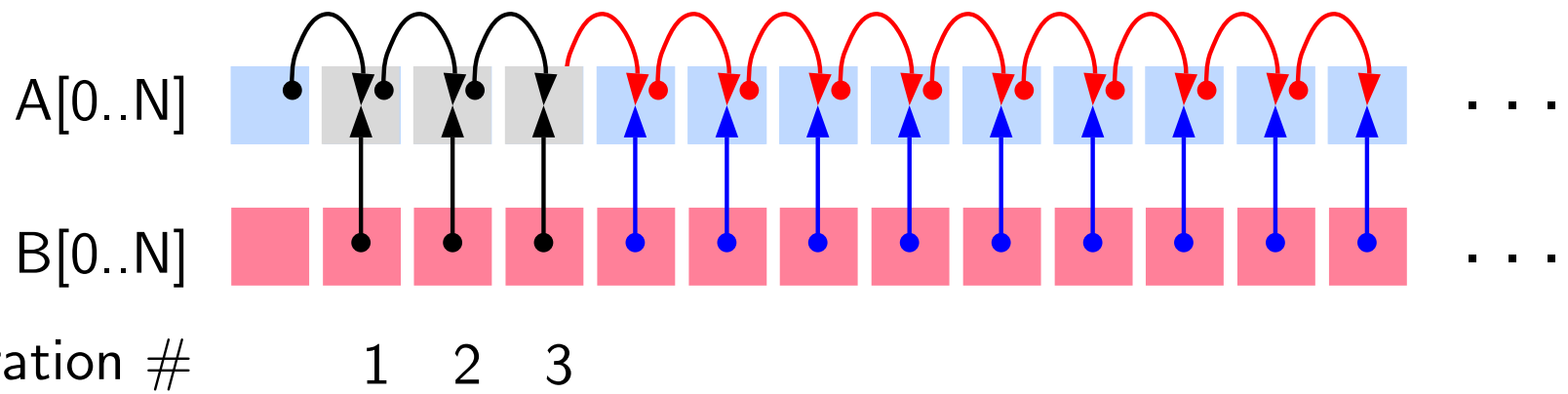


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

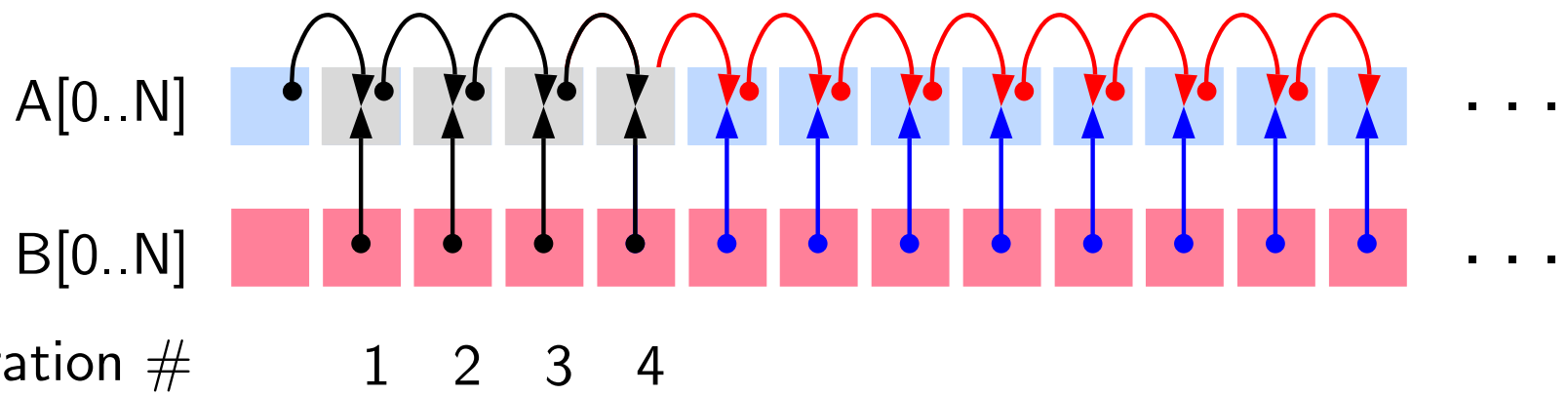


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location



Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
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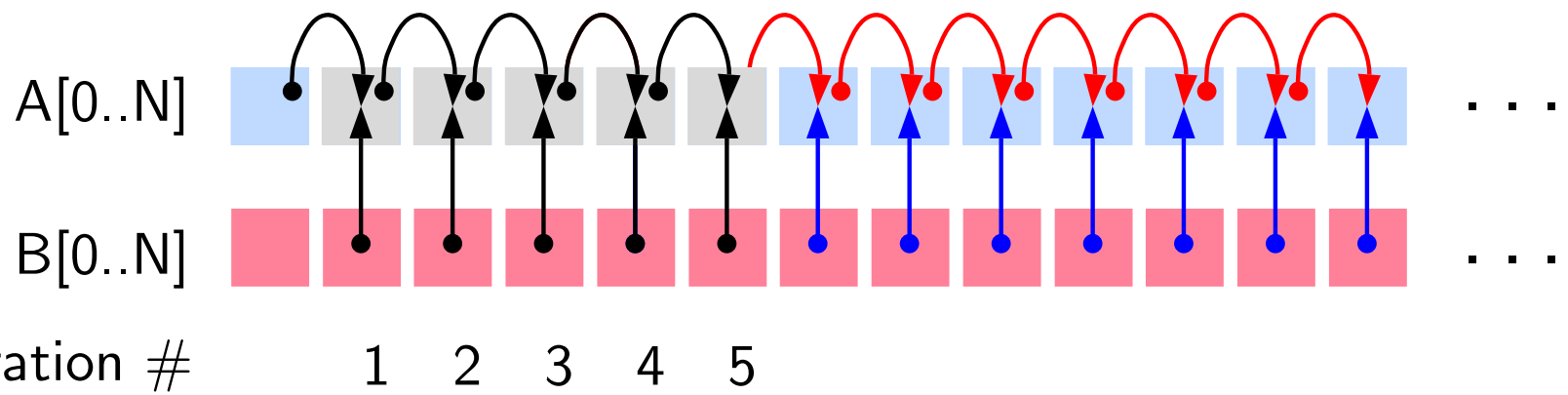
Observe reads and writes into a given location

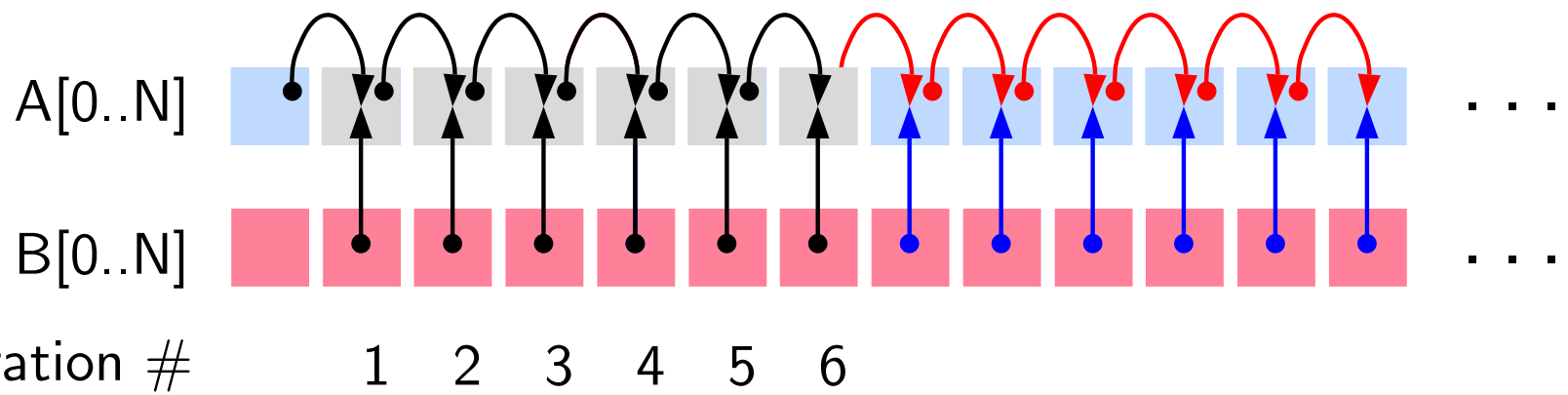


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

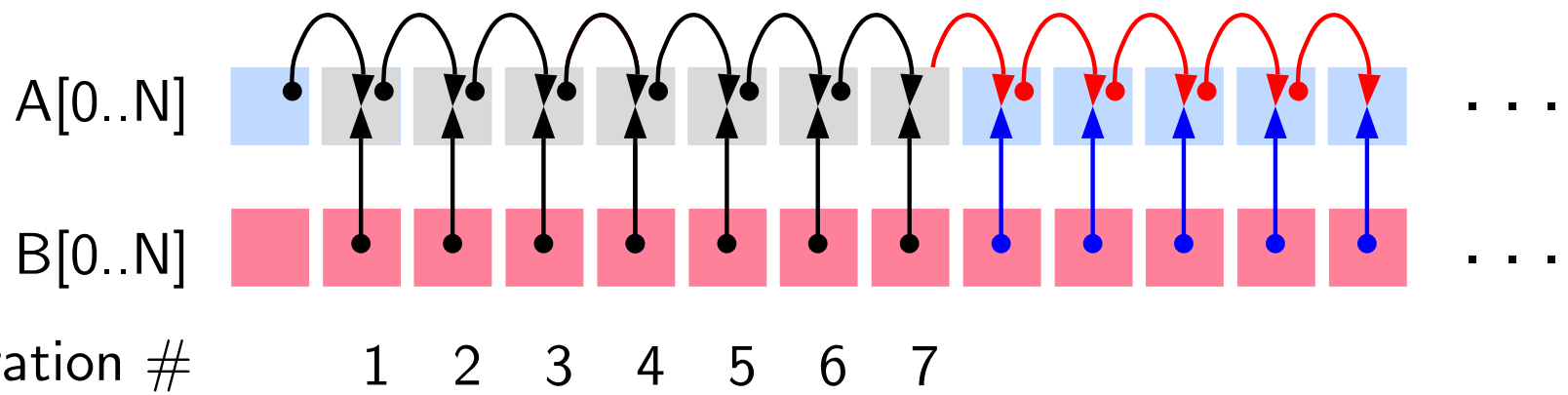


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

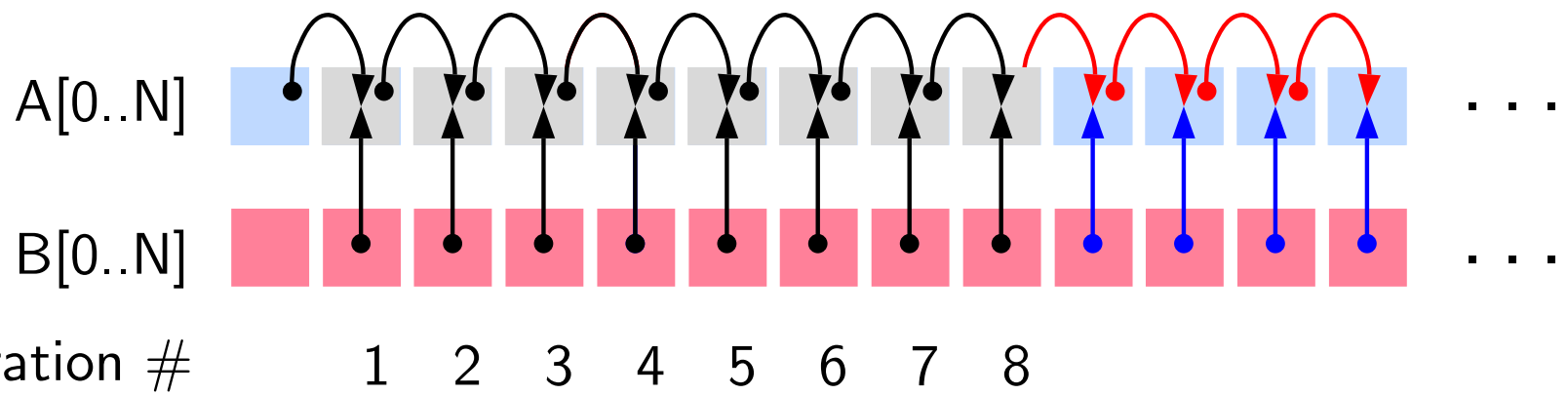


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location



Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

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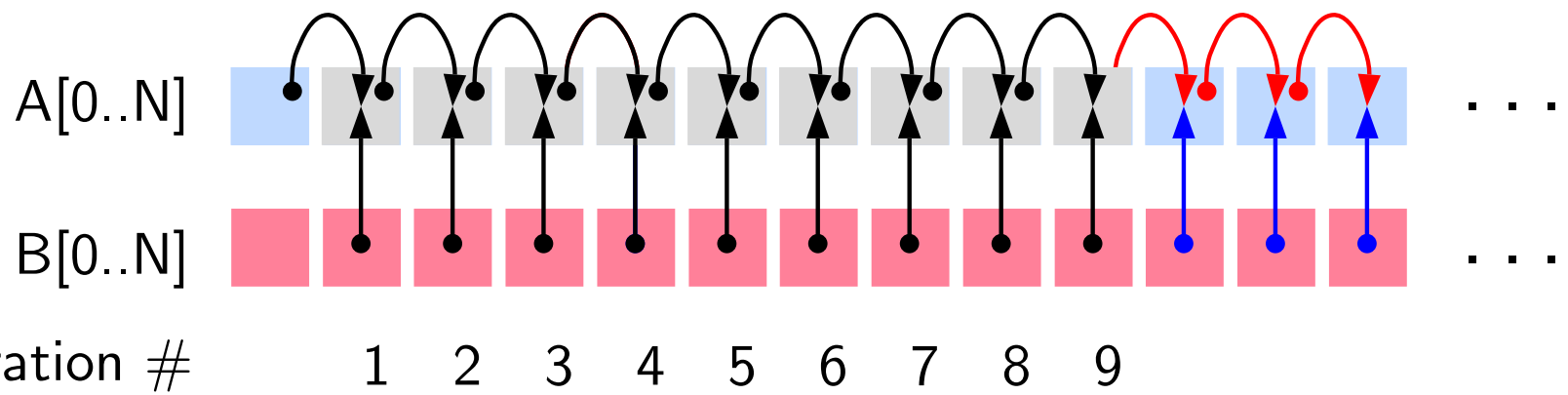

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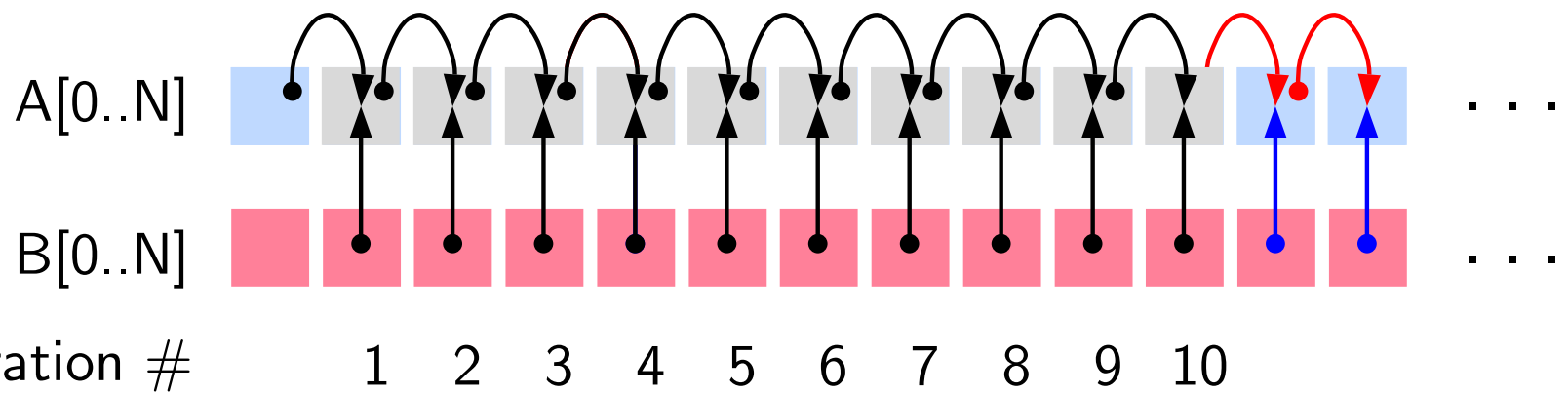

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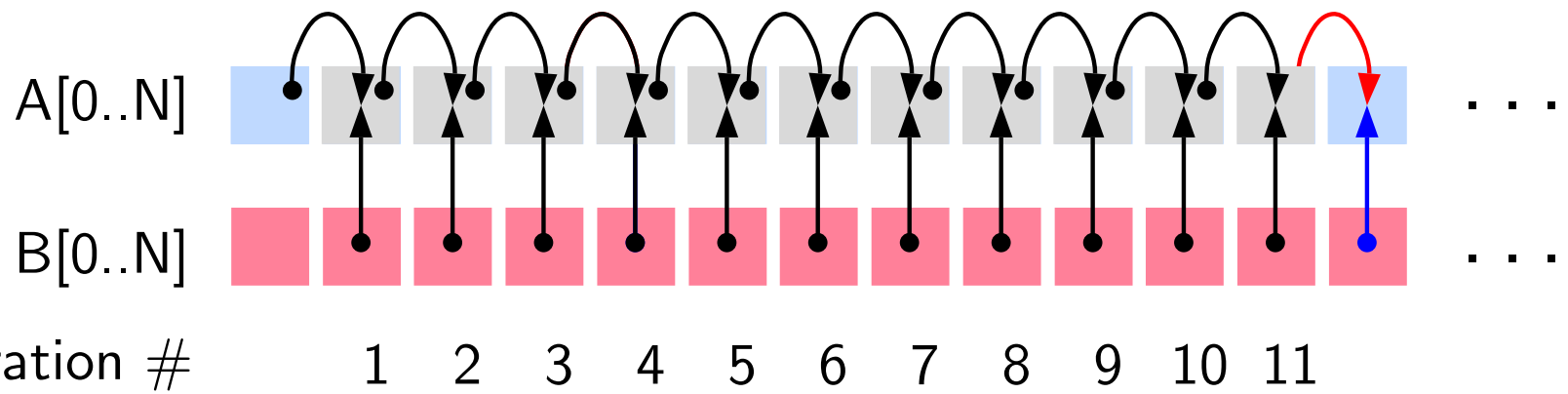

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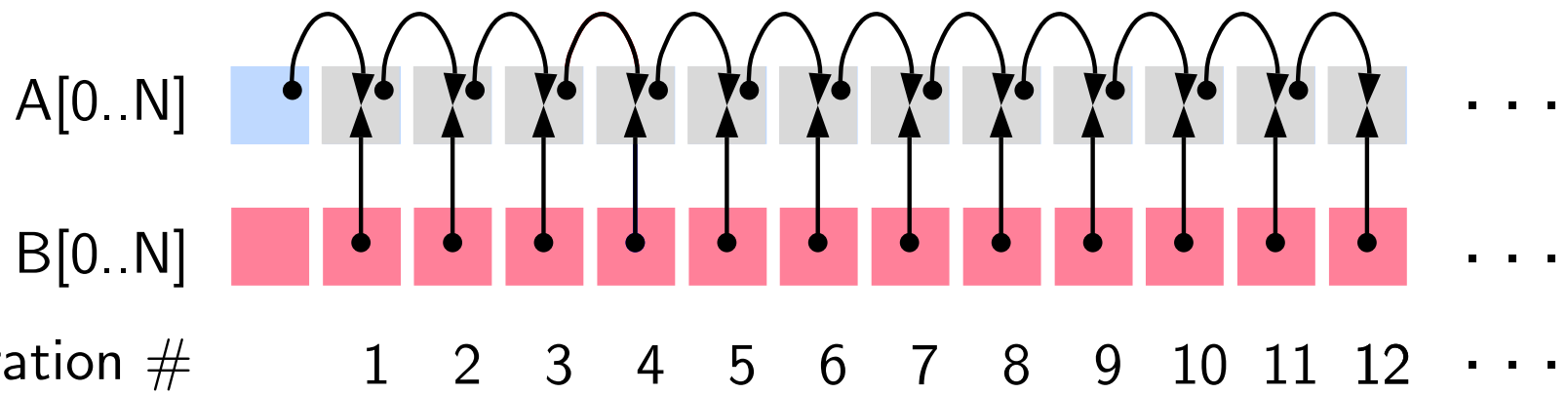


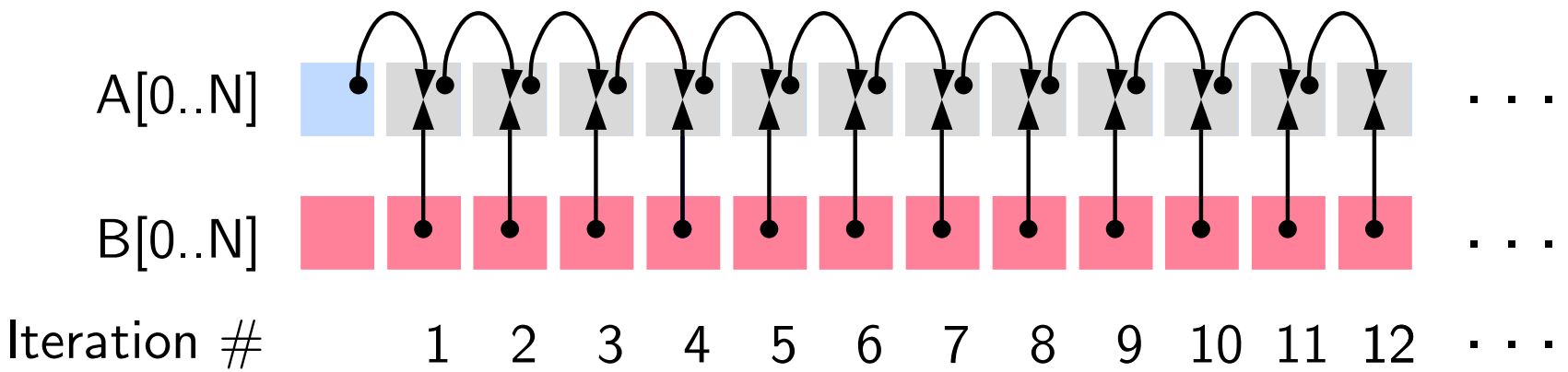
Example 3

Vectorization (SISD ⇒ SIMD) : No

Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B
for (i=0; i
  A[i+1] =
```

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	No
Write	Read	
Write	Write	
Read	Read	



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Vectorization (SISD ⇒ SIMD) : **No**

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```

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Iteration i	Iteration $i+k$	Observation
Read	Write	No
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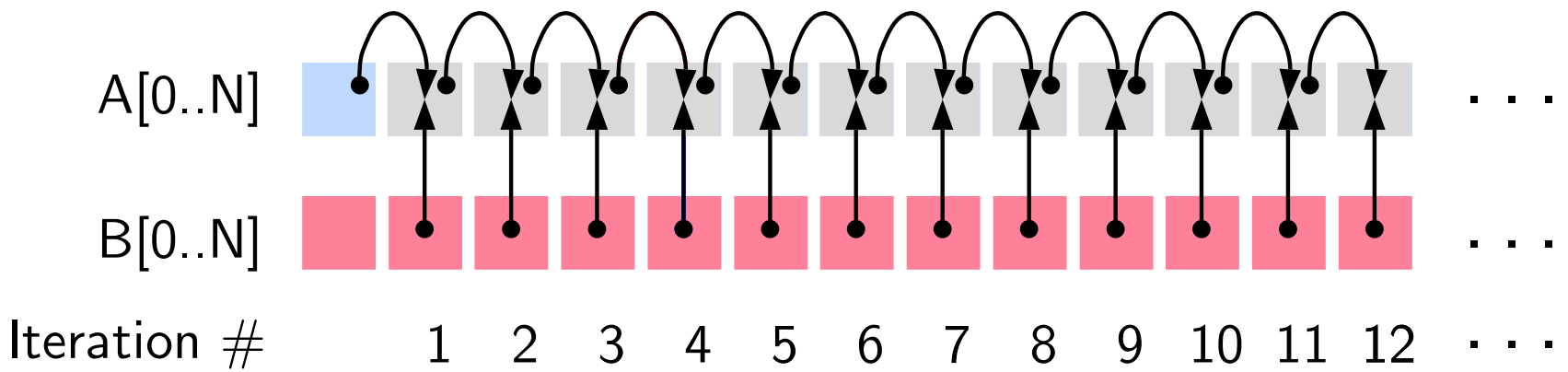
Write	Write	
Read	Read	



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```
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```

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Iteration i	Iteration $i+k$	Observation
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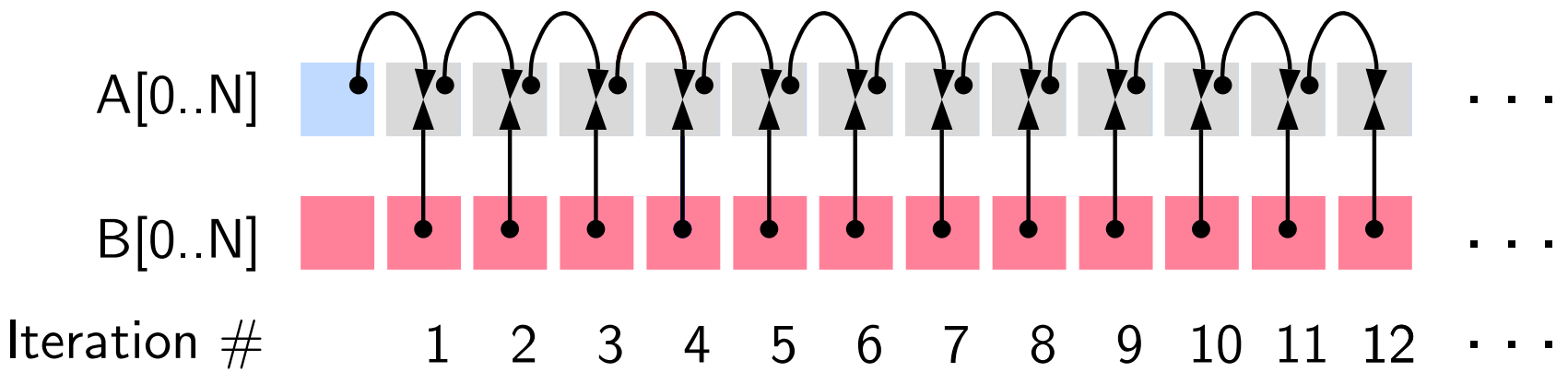

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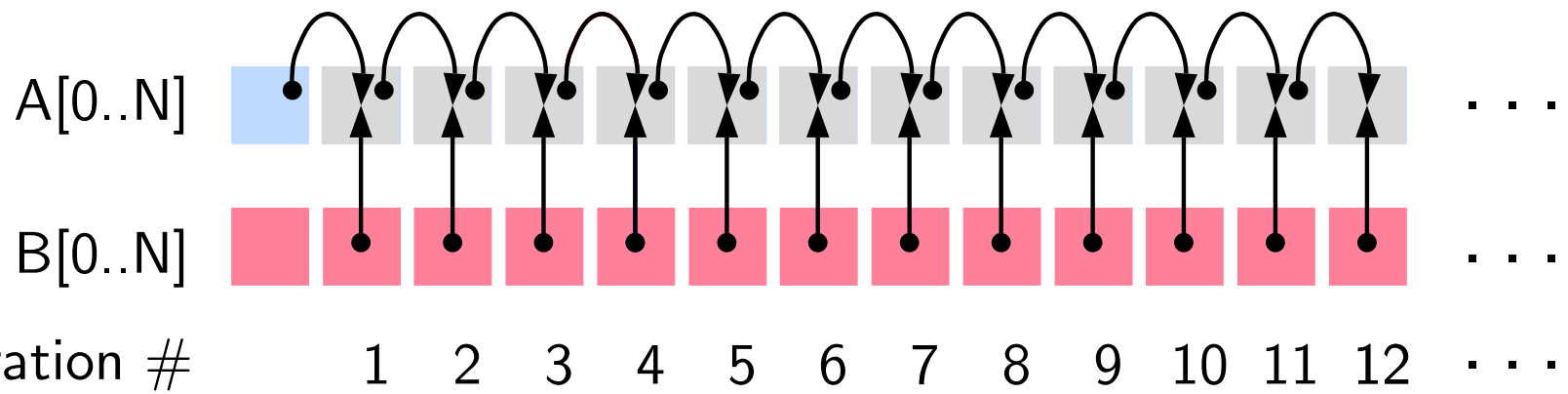
```
int A[N], B
for (i=0; i
  A[i+1] =
```

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	No
Write	Read	Yes
Write	Write	No
Read	Read	Does not matter



Example 4

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: Yes

Example 4

Vectorization (SISD $\Rightarrow$ SIMD) : No
Parallelization (SISD $\Rightarrow$ MIMD) : Yes

- This case is not possible

Example 4

Vectorization (SISD $\Rightarrow$ SIMD) : No
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- This case is not possible
- Vectorization is a limited granularity parallelization

Example 4

Vectorization (SISD $\Rightarrow$ SIMD) : No
Parallelization (SISD $\Rightarrow$ MIMD) : Yes

- This case is not possible
- Vectorization is a limited granularity parallelization
- If parallelization is possible then vectorization is trivially possible

Data Dependence

Let statements S_i and S_j access memory location m at time instants t and $t + k$

Access in S_i	Access in S_j	Dependence	Notation
Read m	Write m	Anti (or Pseudo)	$S_i \ \bar{\delta} \ S_j$
Write m	Read m	Flow (or True)	$S_i \ \delta \ S_j$
Write m	Write m	Output (or Pseudo)	$S_i \ \delta^O \ S_j$
Read m	Read m	Does not matter	

- Pseudo dependences may be eliminated by some transformations
- True dependence prohibits parallel execution of S_i and S_j

Loop Carried and Loop Independent Dependences

Consider dependence between statements S_i and S_j in a loop

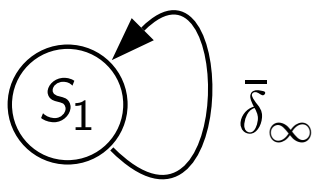
- Loop independent dependence. t and $t + k$ occur in the same iteration of a loop
  - S_i and S_j must be executed sequentially
  - Different iterations of the loop can be parallelized
- Loop carried dependence. t and $t + k$ occur in the different iterations of a loop
  - Within an iteration, S_i and S_j can be executed in parallel
  - Different iterations of the loop must be executed sequentially
- S_i and S_j may have both loop carried and loop independent dependences

Dependence in Example 1

- Program

```
int A[N], B[N], i;
for (i=1; i<N; i++)
    A[i] = A[i] + B[i-1];    /* S1 */
```

- Dependence graph



- No loop carried dependence
  Both vectorization and parallelization are possible

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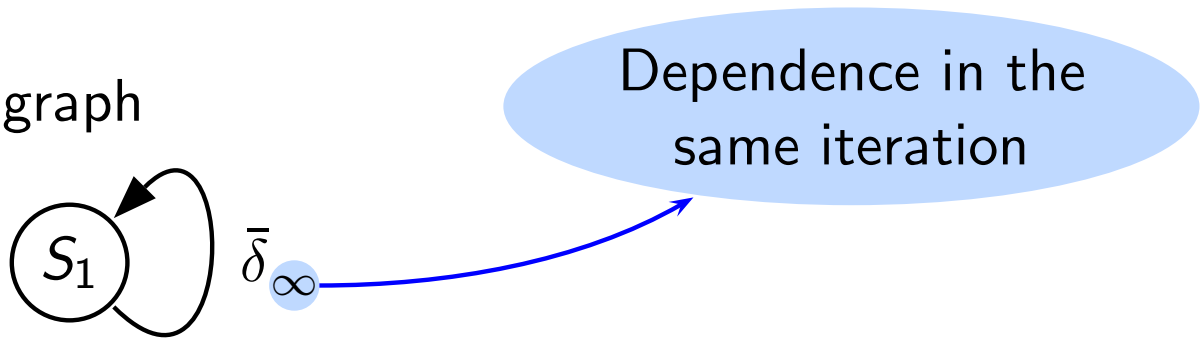


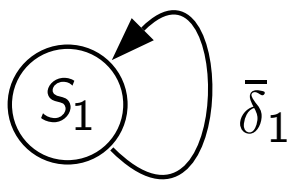
- No loop carried dependence
  Both vectorization and parallelization are possible

Dependence in Example 2

- Program

```
int A[N], B[N], i;
  for (i=0; i<N; i++)
     A[i] = A[i+1] + B[i];  /* S1 */
```

- Dependence graph



- Loop carried anti-dependence
  Parallelization is not possible
  Vectorization is possible since all reads are done before all writes

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```
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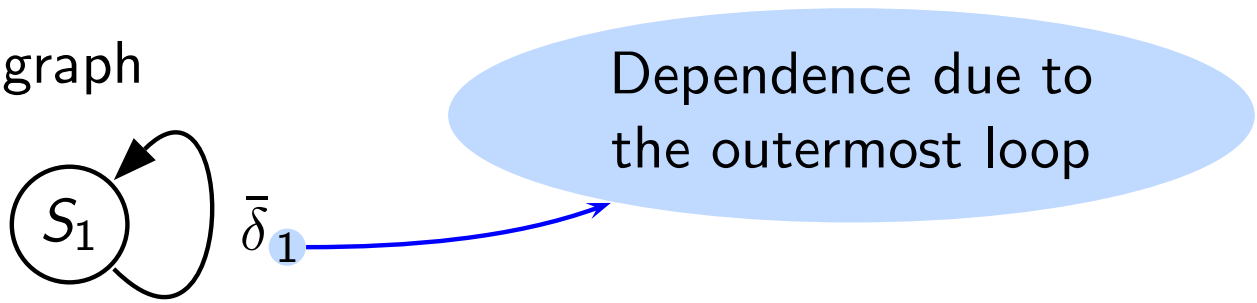

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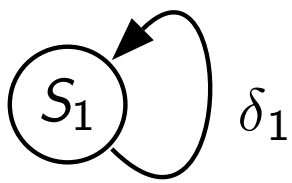
Dependence in Example 3

- Program

```
int A[N], B[N], i;
  for (i=0; i<N; i++)
     A[i+1] = A[i] + B[i+1];  /* S1 */
```

- Dependence graph



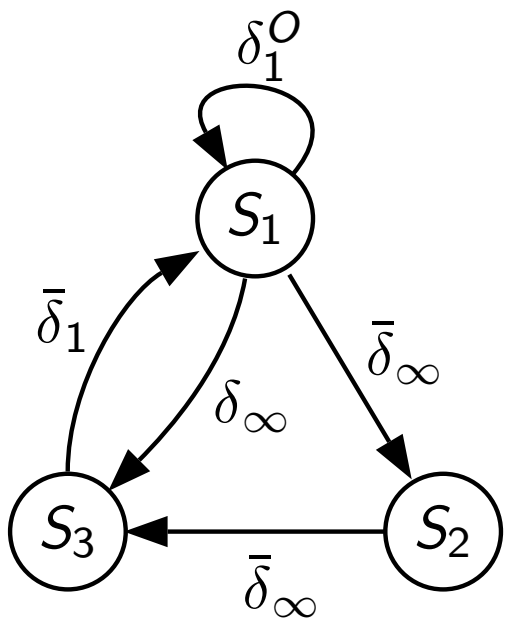
- Loop carried flow-dependence
  Neither parallelization not vectorization is possible

Example 4: Dependence

Program to swap arrays

```
for (i=0; i<N; i++)
{    T = A[i];          /* S1 */
     A[i] = B[i];       /* S2 */
     B[i] = T;          /* S3 */
}
```

Dependence Graph



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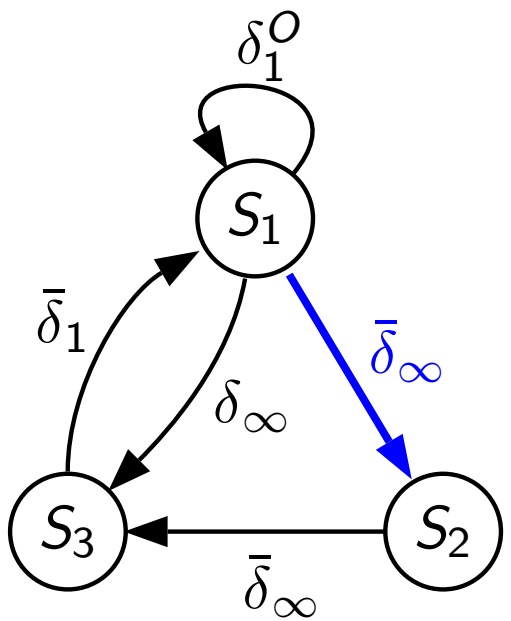

Loop independent anti dependence due to `A[i]`

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Program to swap arrays

```
for (i=0; i<N; i++)
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```
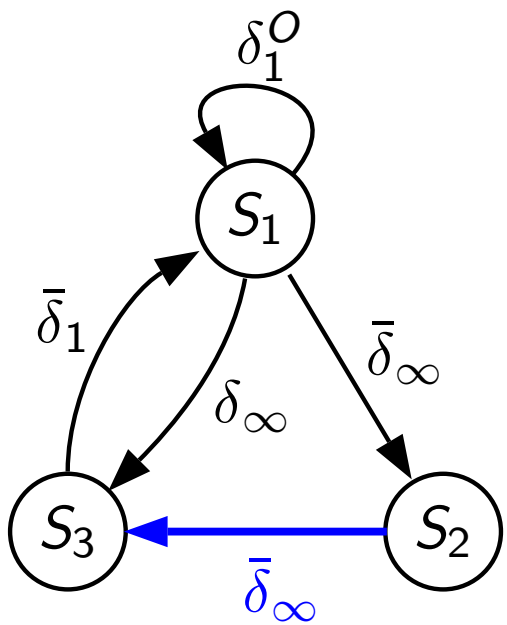

Dependence Graph



Loop independent anti dependence due to `B[i]`

Example 4: Dependence

Program to swap arrays

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for (i=0; i<N; i++)
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     A[i] = B[i];       /* S2 */
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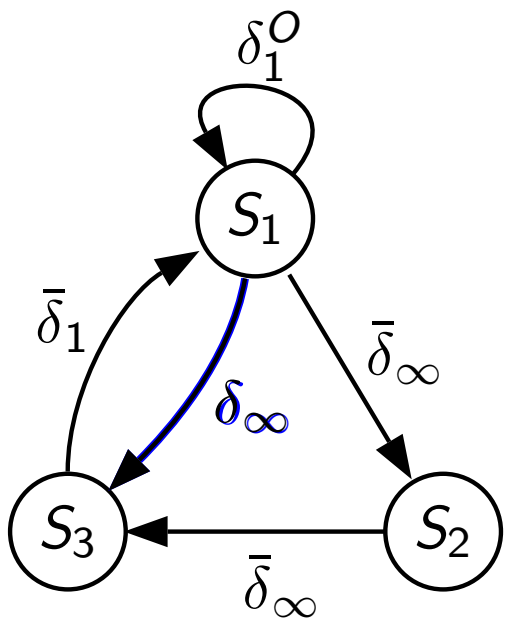
```

Dependence Graph



Loop independent flow dependence due to T

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Program to swap arrays

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for (i=0; i<N; i++)
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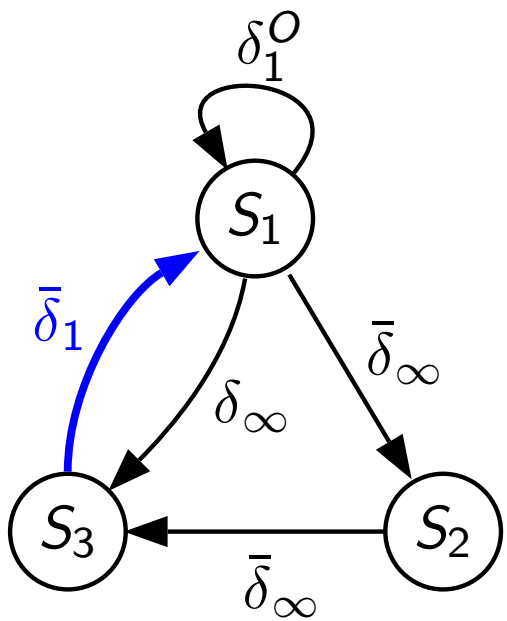

Loop carried anti dependence due to T

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```
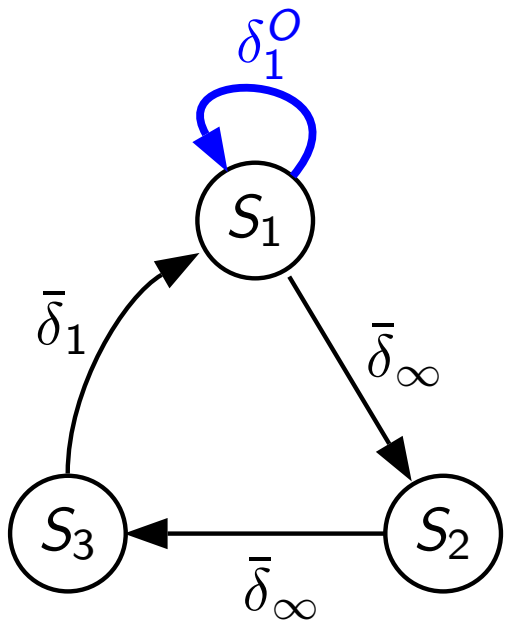

Dependence Graph



Loop carried output dependence due to T

Example 4: Dependence

Program to swap arrays

```
for (i=0; i<N; i++)
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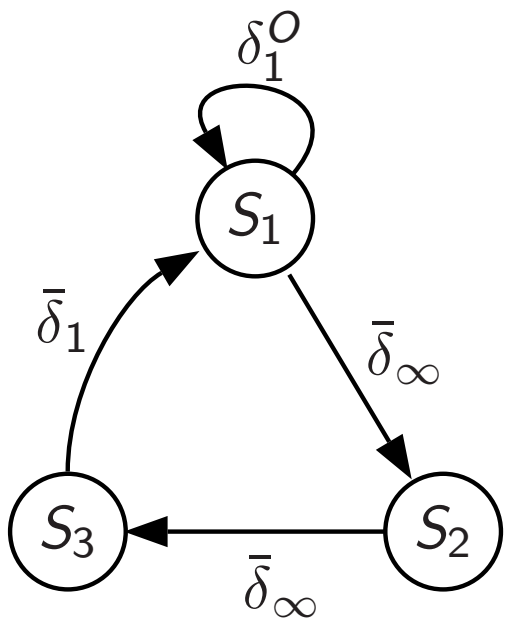
Dependence Graph



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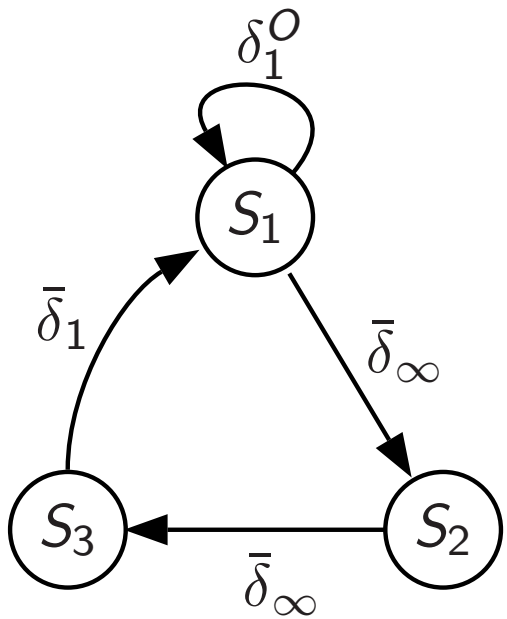
Dependence Graph



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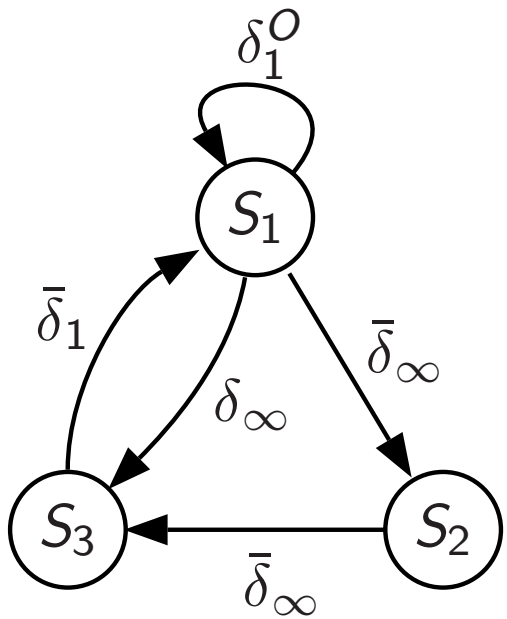

Tutorial Problem for Discovering Dependence

Draw the dependence graph for the following program
(Earlier program modified to swap 2-dimensional arrays)

```
for (i=0; i<N; i++)
{
    for (j=0; j<N; j++)
    {   T = A[i][j];          /* S1 */
        A[i][j] = B[i][j]; /* S2 */
        B[i][j] = T;          /* S3 */
    }
}
```

Data Dependence in Loops

- Analysis in loop is tricky, as
  - Loops may be nested
  - Different loop iterations may access same memory location
  - Arrays occur frequently
  - Far too many array locations to be treated as independent scalars

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- Consider array location `A[4][9]` in the following program

```
for(i = 0; i <= 5; i ++)
  for(j = 0; j <= 4; j ++)
  {
    A[i+1][3*j] = ... ;           /* S1 */
          ...   = A[i+3][2*j+1]; /* S2 */
  }
```

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for(i = 0; i <= 5; i ++)
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  {
    A[i+1][3*j] = ... ;           /* S1 */
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  }
```

S2 accesses in iteration (1,4), S1 accesses in iteration (3,3)

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

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```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
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Conclusion: Dependence exists

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0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

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0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
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```
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1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i][j] = a[i][j] + 2;
  }
```

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
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  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Conclusion: No dependence

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Data Dependence Theorem

There exists a dependence from statement S_1 to statement S_2 in common nest of loops if and only if there exist two iteration vectors **i** and **j** for the nest, such that

1. **i** $<$ **j** or **i** $=$ **j** and there exists a path from S_1 to S_2 in the body of the loop,
2. statement S_1 accesses memory location M on iteration **i** and statement S_2 accesses location M on iteration **j**, and
3. one of these accesses is a write access.