Workshop on Essential Abstractions in GCC

Introduction to Parallelization and Vectorization

GCC Resource Center

(www.cse.iitb.ac.in/grc)

Department of Computer Science and Engineering,

Indian Institute of Technology, Bombay

3 July 2011

Outline

- Transformation for parallel and vector execution
- Data dependence

The Scope of this Tutorial

- What this tutorial does not address
  - Algorithms used for parallelization and vectorization
  - Code or data structures of the parallelization and vectorization pass of GCC
  - Machine level issues related to parallelization and vectorization
- What this tutorial addresses

Basics of Discovering Parallelism using GCC

Part 1

Transformations for Parallel and Vector Execution

A Taxonomy of Parallel Computation

	Single Program	Multiple Programs
Single Data	SPSD	MPSD
Multiple Data	SPMD	MPMD

A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	MISD	MPSD
Multiple Data	SIMD	MIMD	MPMD

A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	?	?
Multiple Data	SIMD	MIMD	MPMD

Redundant computation for validation of intermediate steps

A Taxonomy of Parallel Computation

	Single Program		Multiple Programs
	Single Instruction	Multiple Instructions	
Single Data	SISD	MISD	MPSD
Multiple Data	SIMD	MIMD	MPMD

Transformations performed by a compiler

Vectorization: SISD $\Rightarrow$ SIMD

- Parallelism in executing operation on shorter operands
  (8-bit, 16-bit, 32-bit operands)
- Existing 32 or 64-bit arithmetic units used to perform multiple operations in parallel
  A 64 bit word $\equiv$ a vector of $2\times$(32 bits), $4\times$(16 bits), or $8\times$(8 bits)

Example 1

Vectorization	(SISD $\Rightarrow$ SIMD)	:	Yes
Parallelization	(SISD $\Rightarrow$ MIMD)	:	Yes

Original Code

```
int A[N], B[N], i;
for (i=1; i<N; i++)
  A[i] = A[i] + B[i-1];
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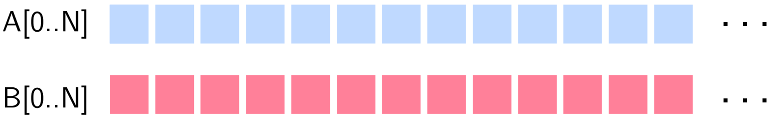

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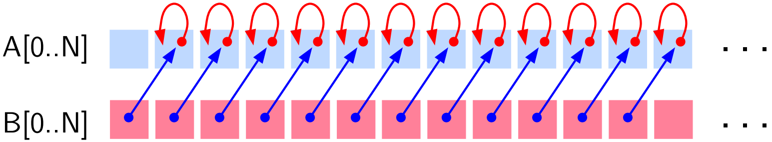

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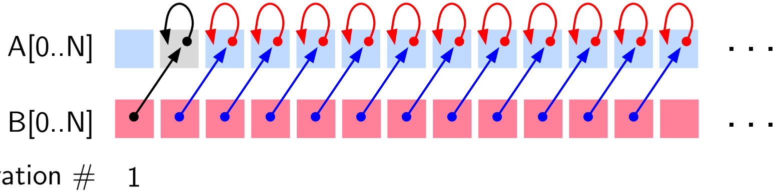

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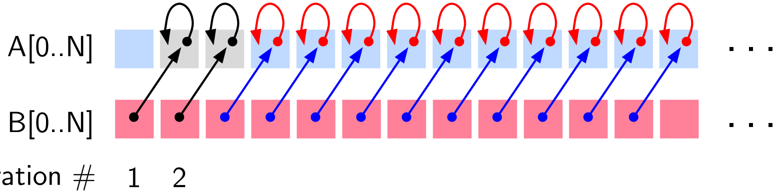

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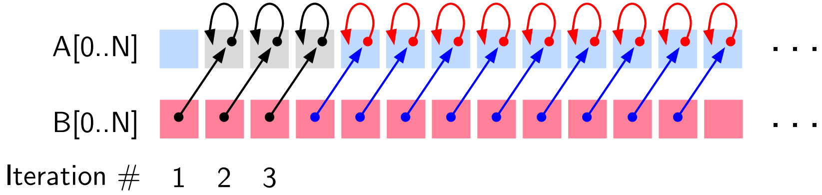

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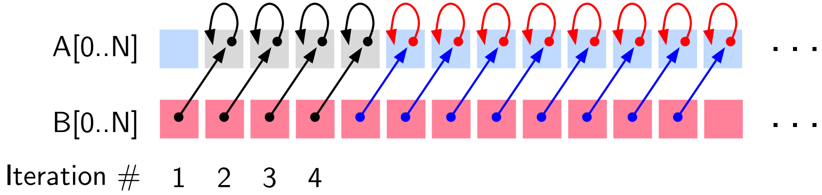

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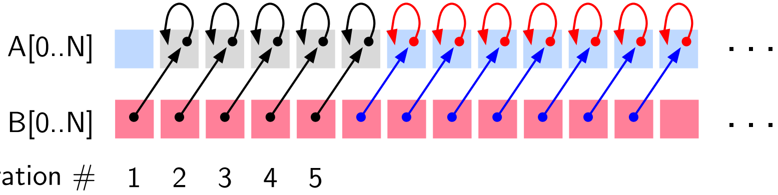

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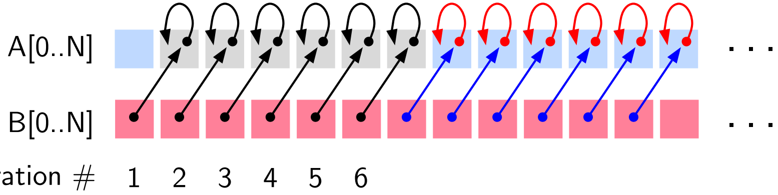

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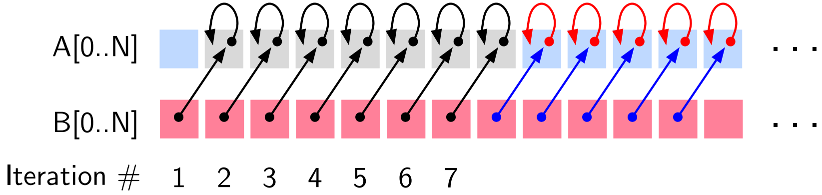

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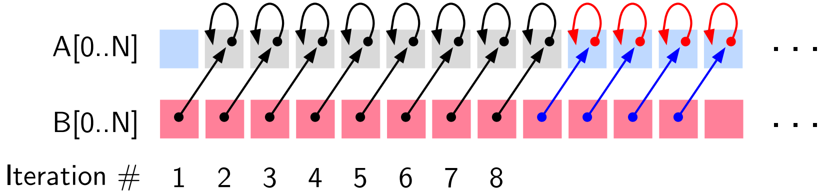

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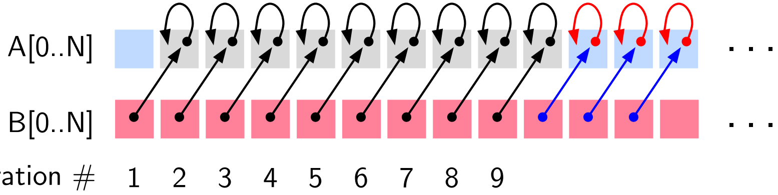

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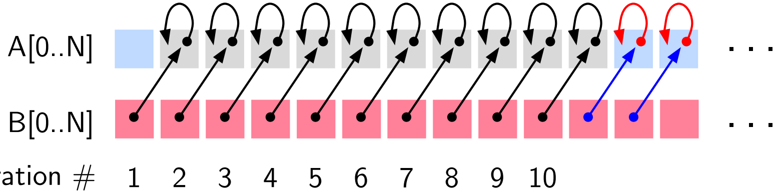

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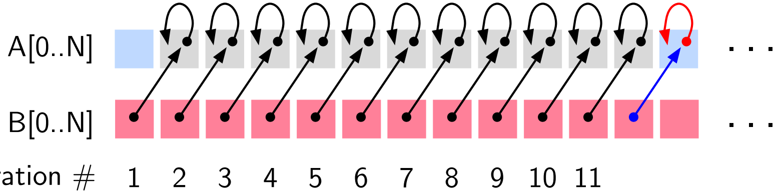

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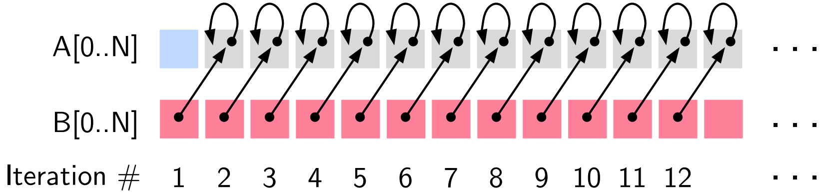

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Vectorization Factor

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Vectorized Code

```
int A[N], B[N], i;
for (i=1; i<N; i=i+4)
  A[i:i+3] = A[i:i+3] +
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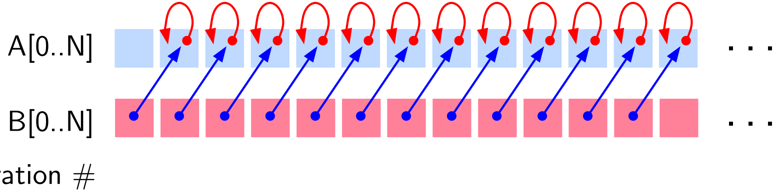

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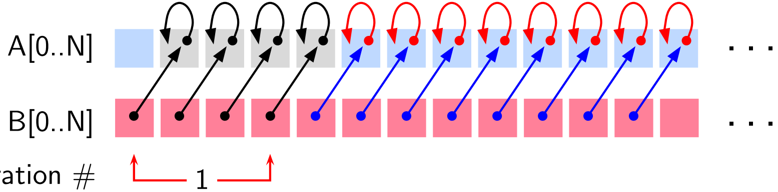

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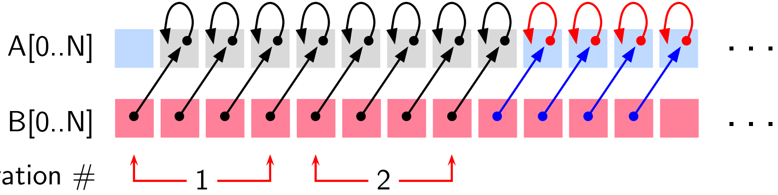

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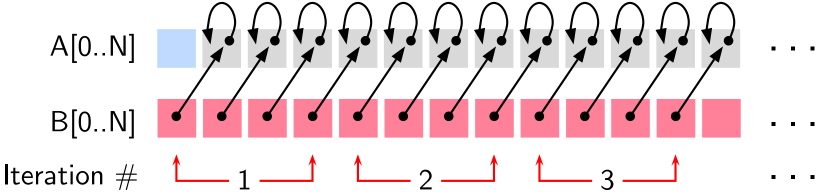

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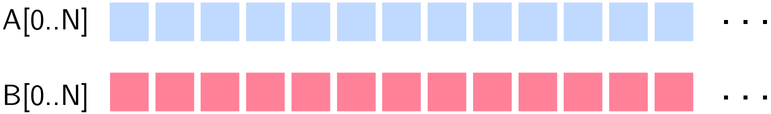

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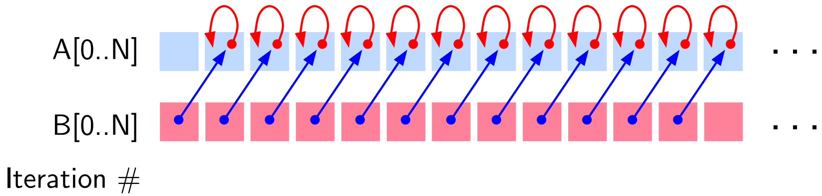

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Parallelized Code

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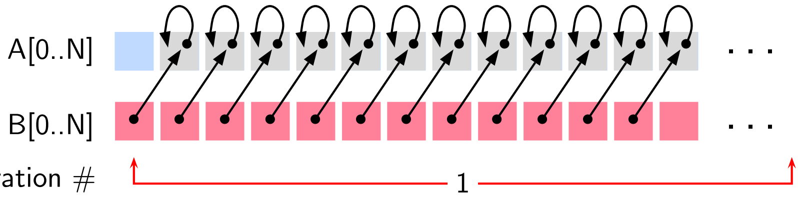

Example 1: The Moral of the Story

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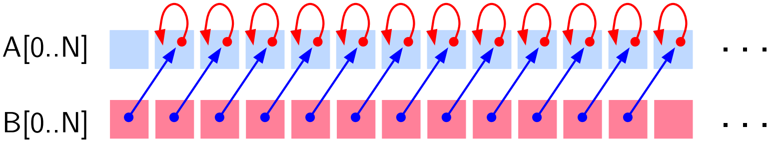

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Vectorization (SISD ⇒ SIMD) : Yes
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When the same location is accessed across different iterations, the order of reads and writes must be preserved

Nature of accesses in our example		
Iteration i	Iteration $i + k$	Observation
Read	Write	
Write	Read	
Write	Write	
Read	Read	

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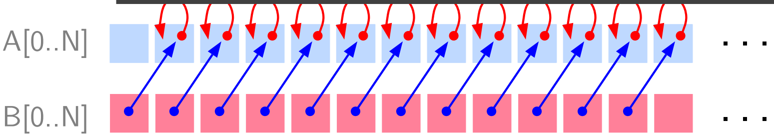

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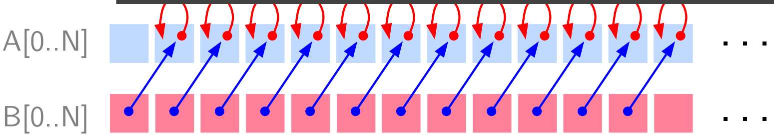

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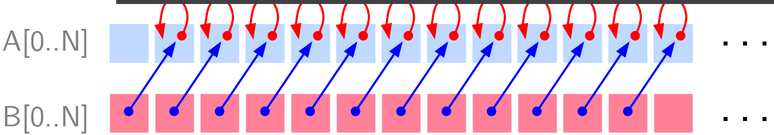

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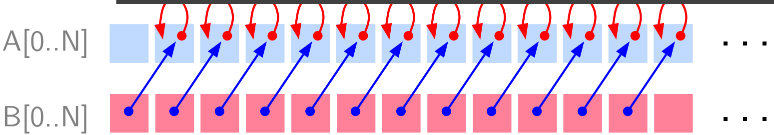

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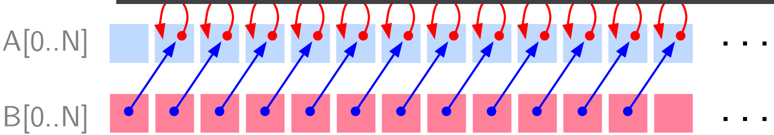

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Vectorization	(SISD $\Rightarrow$ SIMD)	: Yes
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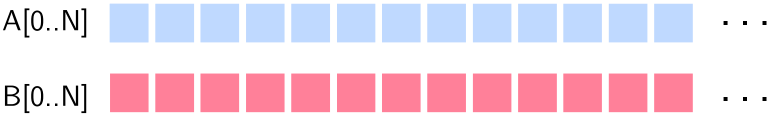

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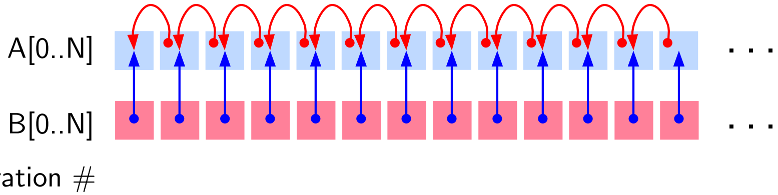

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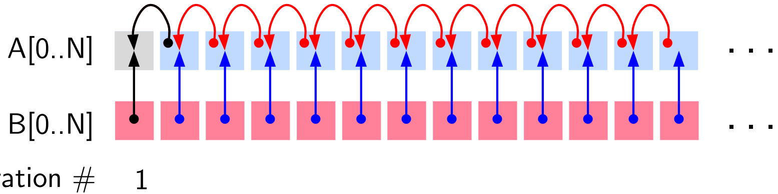

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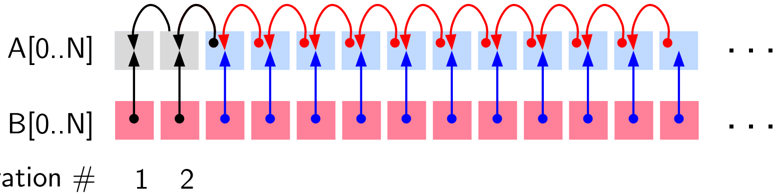

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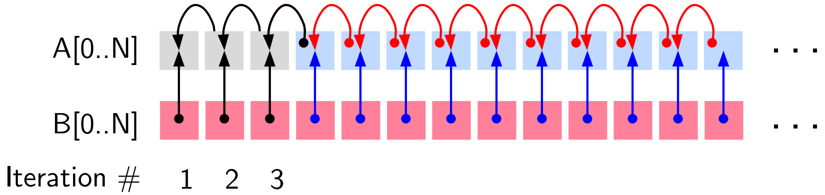

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  A[i] = A[i+1] + B[i];
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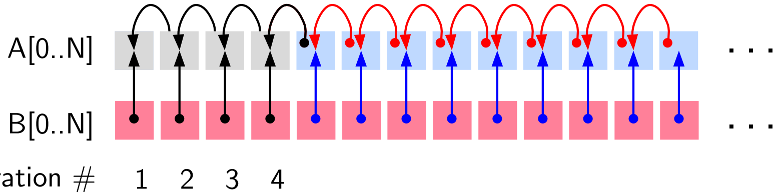
Observe reads and writes into a given location



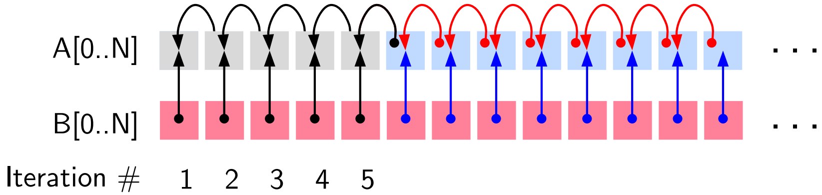
Example 2

Vectorization	(SISD ⇒ SIMD)	:	Yes
Parallelization	(SISD ⇒ MIMD)	:	No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



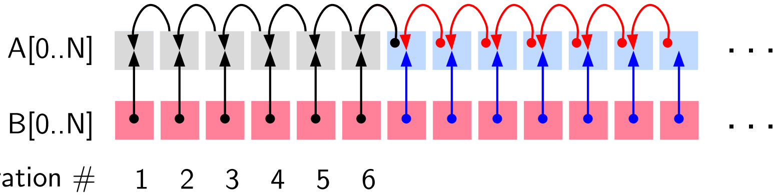
Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



Example 2

Vectorization	(SISD $\Rightarrow$ SIMD)	:	Yes
Parallelization	(SISD $\Rightarrow$ MIMD)	:	No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
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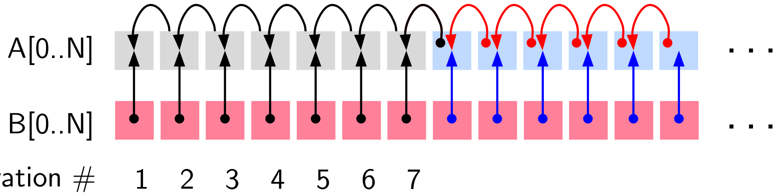

Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
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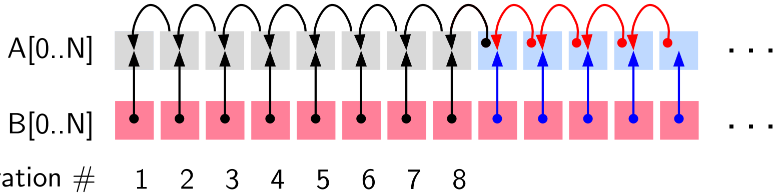

Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

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for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
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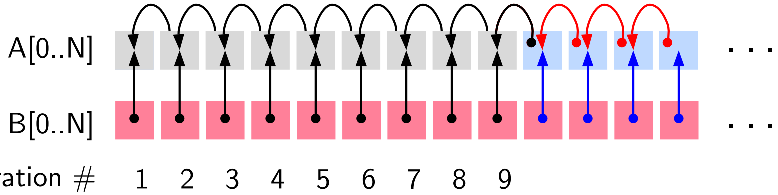
Observe reads and writes into a given location



Example 2

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
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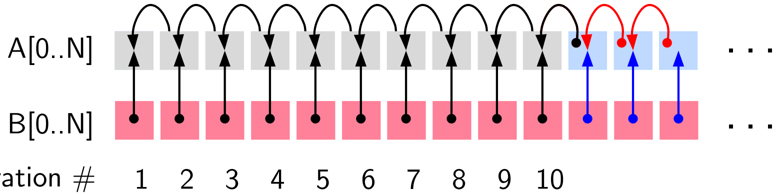
Observe reads and writes into a given location



Example 2

Vectorization	(SISD $\Rightarrow$ SIMD)	: Yes
Parallelization	(SISD $\Rightarrow$ MIMD)	: No

Original Code

```
int A[N], B[N], i;
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  A[i] = A[i+1] + B[i];
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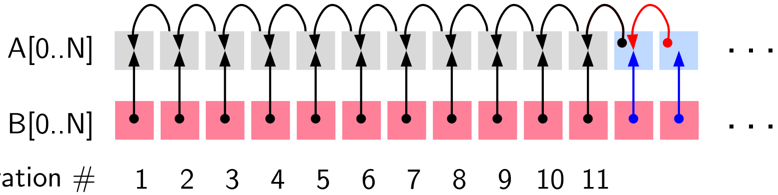
Observe reads and writes into a given location



Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
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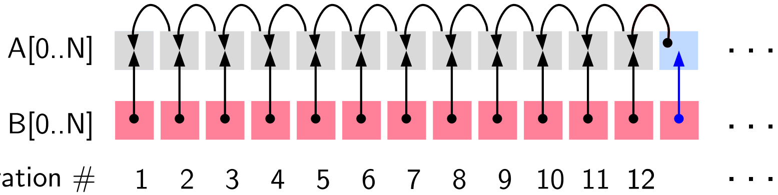


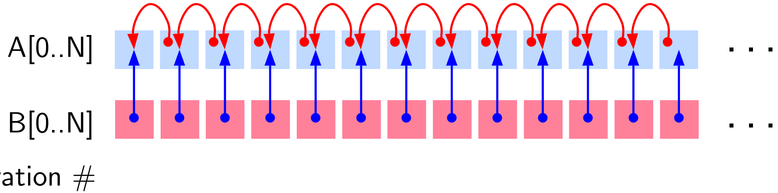
Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



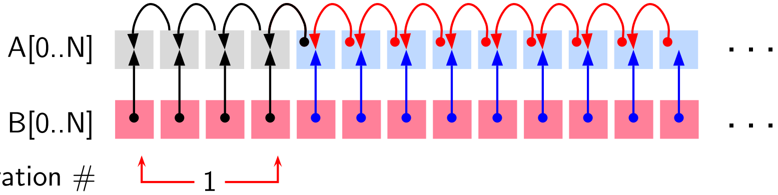
Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction



Example 2

Vectorization	(SISD $\Rightarrow$ SIMD)	: Yes
Parallelization	(SISD $\Rightarrow$ MIMD)	: No

Original Code

```
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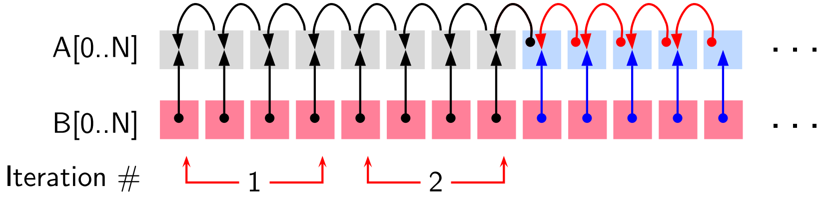

Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
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  A[i] = A[i+1] + B[i];
```
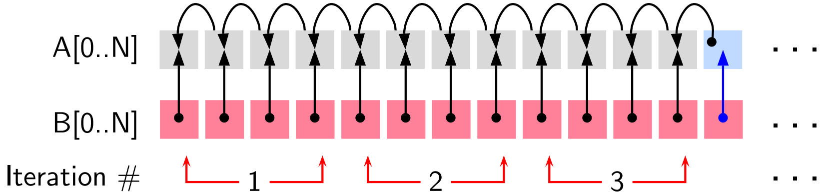

- Vector instruction is synchronized: All reads before writes in a given instruction



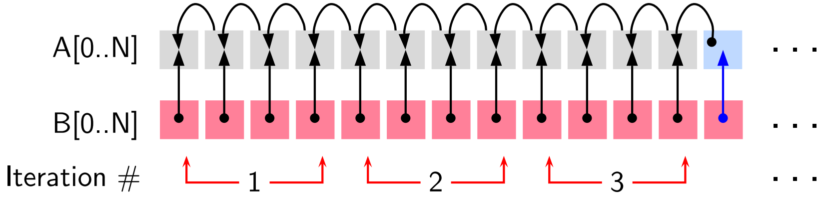
Example 2

Vectorization	(SISD ⇒ SIMD)	: Yes
Parallelization	(SISD ⇒ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

- Vector instruction is synchronized: All reads before writes in a given instruction
- Read-writes across multiple instructions executing in parallel may not be synchronized



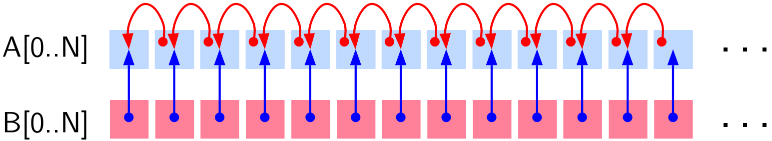
Example 2: The Moral of the Story

Vectorization	(SISD $\Rightarrow$ SIMD)	: Yes
Parallelization	(SISD $\Rightarrow$ MIMD)	: No

Original Code

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i] = A[i+1] + B[i];
```

Observe reads and writes into a given location



Example 2: The Moral of the Story

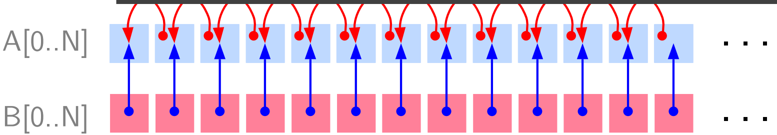
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : No

```
int A[
for (i
  A[i]
```

When the same location is accessed across different iterations, the order of reads and writes must be preserved

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	
Write	Read	
Write	Write	
Read	Read	



Example 2: The Moral of the Story

Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

```
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for (i
  A[i]
```

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Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	Yes
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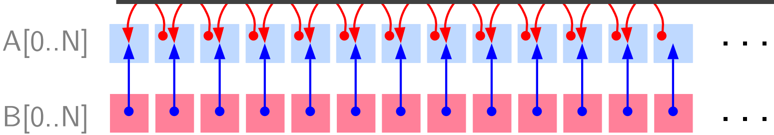
Read	Read	



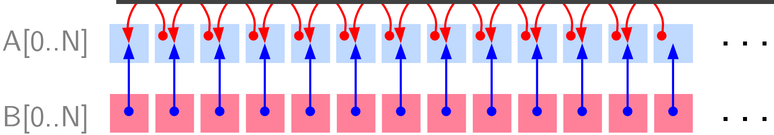
Example 2: The Moral of the Story

Vectorization (SISD ⇒ SIMD) : Yes
Parallelization (SISD ⇒ MIMD) : **No**

```
int A[
for (i
  A[i]
```

When the same location is accessed across different iterations, the order of reads and writes must be preserved

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	
Read	Read	



Example 2: The Moral of the Story

Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

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for (i
  A[i]
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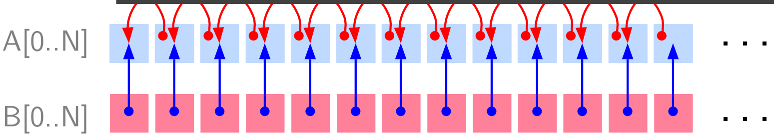
Iteration i	Iteration $i+k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	No
Read	Read	



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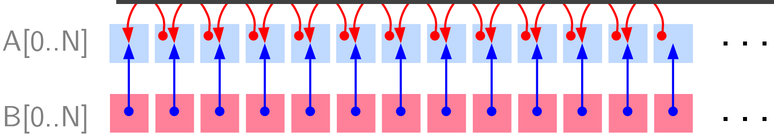
Vectorization (SISD ⇒ SIMD) : Yes

Parallelization (SISD ⇒ MIMD) : **No**

```
int A[
for (i
  A[i]
```

When the same location is accessed across different iterations, the order of reads and writes must be preserved

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	Yes
Write	Read	No
Write	Write	No
Read	Read	Does not matter



Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: No

```
int A[N], B[N], i;
for (i=0; i<N; i++)
  A[i+1] = A[i] + B[i+1];
```

Observe reads and writes into a given location

Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: No

```
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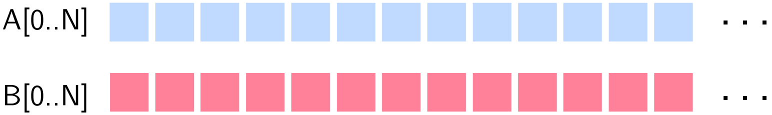
Observe reads and writes into a given location



Example 3

Vectorization	(SISD ⇒ SIMD)	: No
Parallelization	(SISD ⇒ MIMD)	: No

```
int A[N], B[N], i;
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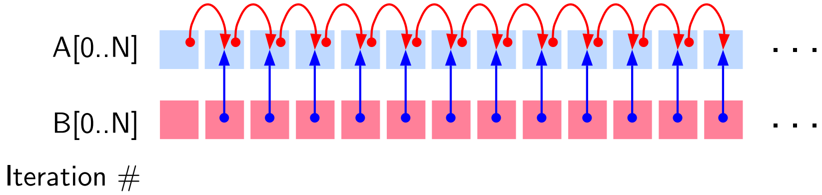
Observe reads and writes into a given location



Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	:	No
Parallelization	(SISD $\Rightarrow$ MIMD)	:	No

```
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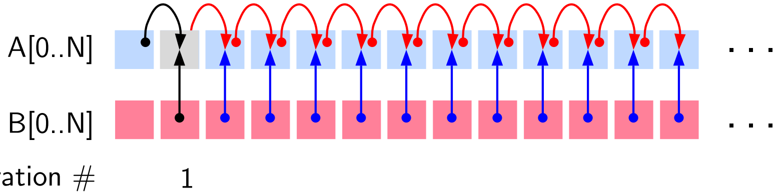
Observe reads and writes into a given location



Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	:	No
Parallelization	(SISD $\Rightarrow$ MIMD)	:	No

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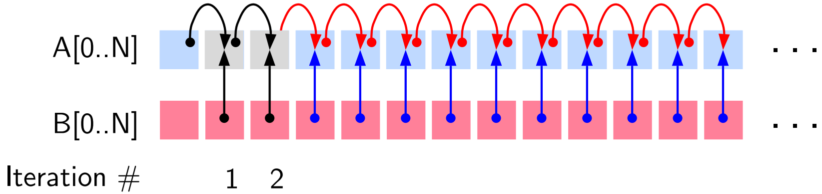

Example 3

Vectorization	(SISD ⇒ SIMD)	: No
Parallelization	(SISD ⇒ MIMD)	: No

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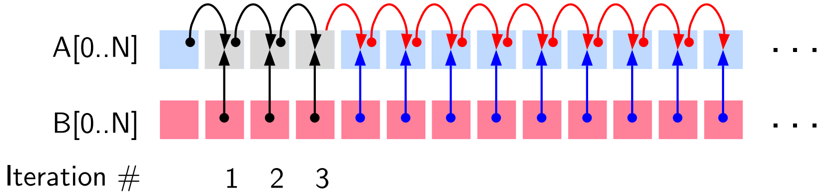

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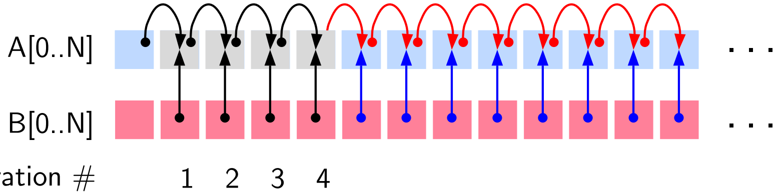

Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	:	No
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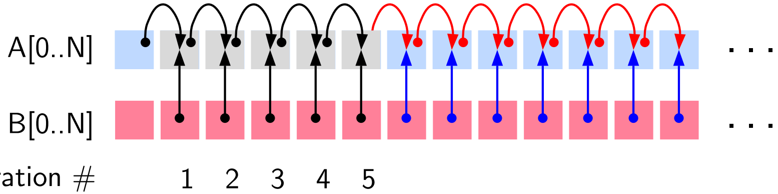

Example 3

Vectorization	(SISD ⇒ SIMD)	:	No
Parallelization	(SISD ⇒ MIMD)	:	No

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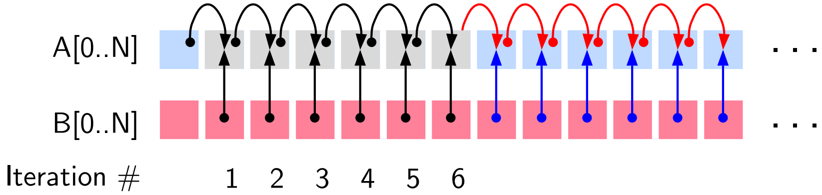

Example 3

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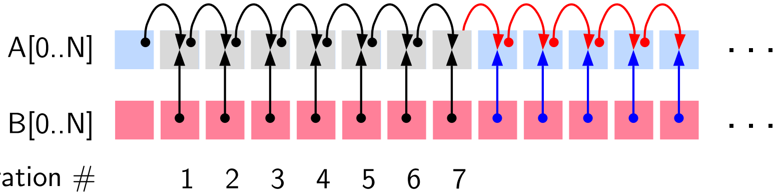

Example 3

Vectorization	(SISD ⇒ SIMD)	: No
Parallelization	(SISD ⇒ MIMD)	: No

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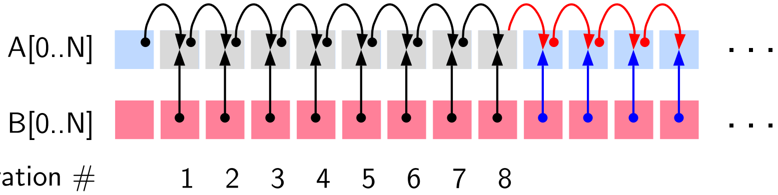

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Vectorization	(SISD $\Rightarrow$ SIMD)	: No
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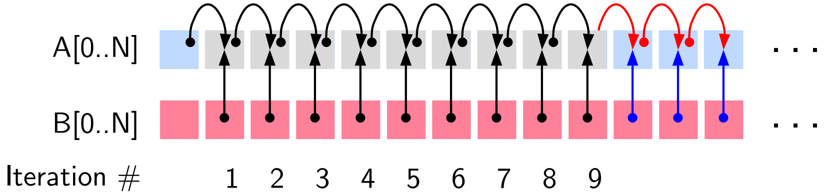

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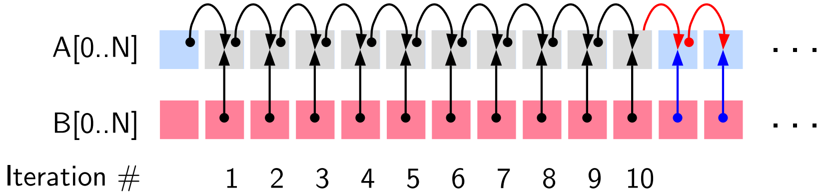

Example 3

Vectorization	(SISD ⇒ SIMD)	: No
Parallelization	(SISD ⇒ MIMD)	: No

```
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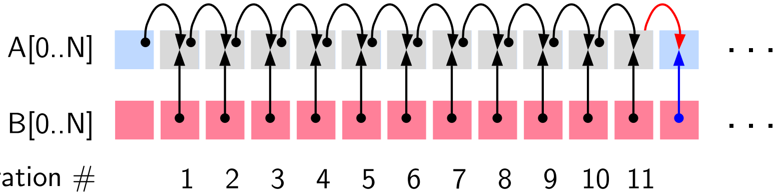
Observe reads and writes into a given location



Example 3

Vectorization	(SISD ⇒ SIMD)	: No
Parallelization	(SISD ⇒ MIMD)	: No

```
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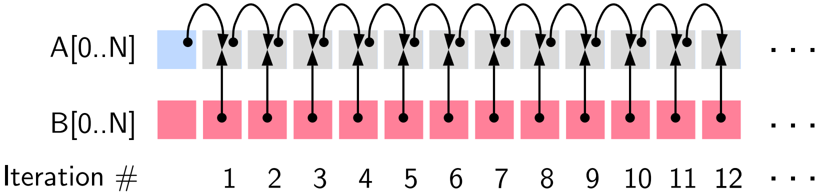
Observe reads and writes into a given location

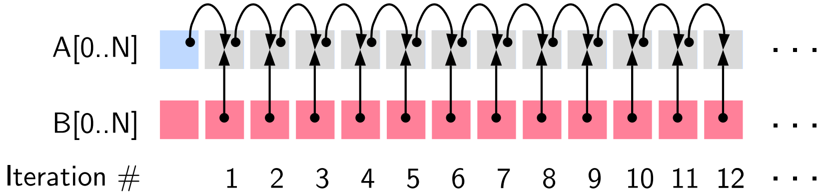


Example 3

Vectorization (SISD ⇒ SIMD) : No
Parallelization (SISD ⇒ MIMD) : No

```
int A[N], B
for (i=0; i
  A[i+1] =
```

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	No
Write	Read	
Write	Write	
Read	Read	



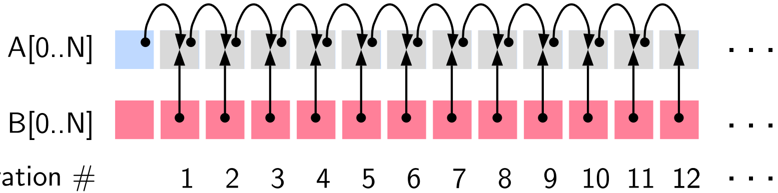
Example 3

Vectorization (SISD ⇒ SIMD) : **No**

Parallelization (SISD ⇒ MIMD) : **No**

```
int A[N],
for (i=0;
  A[i+1] =
```

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	No
Write	Read	Yes
Write	Write	
Read	Read	



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Vectorization (SISD ⇒ SIMD) : **No**

Parallelization (SISD ⇒ MIMD) : **No**

```
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  A[i+1] =
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Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
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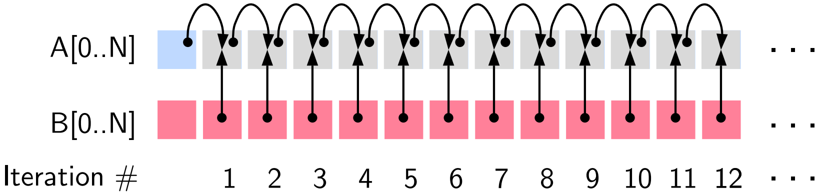
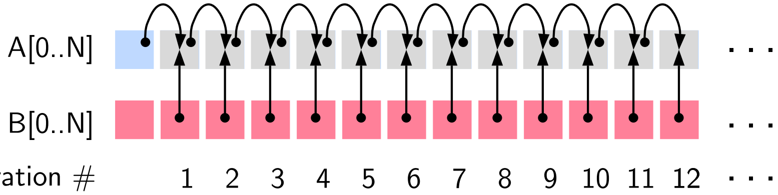


Example 3

Vectorization	(SISD $\Rightarrow$ SIMD)	:	**No**
Parallelization	(SISD $\Rightarrow$ MIMD)	:	**No**

```
int A[N], B
for (i=0; i
  A[i+1] =
```

Nature of accesses in our example		
Iteration i	Iteration $i+k$	Observation
Read	Write	No
Write	Read	Yes
Write	Write	No
Read	Read	Does not matter



Example 4

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: Yes

Example 4

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: Yes

- This case is not possible

Example 4

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: Yes

- This case is not possible
- Vectorization is a limited granularity parallelization

Example 4

Vectorization	(SISD $\Rightarrow$ SIMD)	: No
Parallelization	(SISD $\Rightarrow$ MIMD)	: Yes

- This case is not possible
- Vectorization is a limited granularity parallelization
- If parallelization is possible then vectorization is trivially possible

Data Dependence

Let statements S_i and S_j access memory location m at time instants t and $t + k$

Access in S_i	Access in S_j	Dependence	Notation
Read m	Write m	Anti (or Pseudo)	$S_i\ \bar{\delta}\ S_j$
Write m	Read m	Flow (or True)	$S_i\ \delta\ S_j$
Write m	Write m	Output (or Pseudo)	$S_i\ \delta^O\ S_j$
Read m	Read m	Does not matter	

- Pseudo dependences may be eliminated by some transformations
- True dependence prohibits parallel execution of S_i and S_j

Consider dependence between statements S_i and S_j in a loop

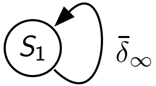
- Loop independent dependence. t and $t + k$ occur in the same iteration of a loop
  - S_i and S_j must be executed sequentially
  - Different iterations of the loop can be parallelized
- Loop carried dependence. t and $t + k$ occur in the different iterations of a loop
  - Within an iteration, S_i and S_j can be executed in parallel
  - Different iterations of the loop must be executed sequentially
- S_i and S_j may have both loop carried and loop independent dependences

Dependence in Example 1

- Program

```
int A[N], B[N], i;
for (i=1; i<N; i++)
    A[i] = A[i] + B[i-1];   /* S1 */
```

- Dependence graph



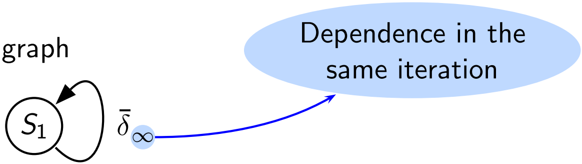
- No loop carried dependence
  Both vectorization and parallelization are possible

Dependence in Example 1

- Program

```
int A[N], B[N], i;
for (i=1; i<N; i++)
    A[i] = A[i] + B[i-1];    /* S1 */
```

- Dependence graph



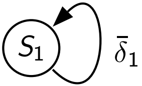
- No loop carried dependence
  Both vectorization and parallelization are possible

Dependence in Example 2

- Program

```
int A[N], B[N], i;
  for (i=0; i<N; i++)
     A[i] = A[i+1] + B[i];  /* S1 */
```

- Dependence graph



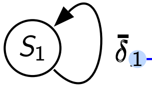
- Loop carried anti-dependence
  Parallelization is not possible
  Vectorization is possible since all reads are done before all writes

Dependence in Example 2

- Program

```
int A[N], B[N], i;
  for (i=0; i<N; i++)
      A[i] = A[i+1] + B[i];  /* S1 */
```

- Dependence graph



Dependence due to the outermost loop

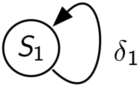
- Loop carried anti-dependence
  Parallelization is not possible
  Vectorization is possible since all reads are done before all writes

Dependence in Example 3

- Program

```
int A[N], B[N], i;
  for (i=0; i<N; i++)
     A[i+1] = A[i] + B[i+1];  /* S1 */
```

- Dependence graph



- Loop carried flow-dependence
  Neither parallelization not vectorization is possible

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Conclusion: Dependence exists

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Iteration Vector	Index Vector LHS	Index Vector RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Conclusion: Dependence exists

Iteration Vectors and Index Vectors: Example 1

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i+1][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Conclusion: Dependence exists

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	1, 0	0, 0
0, 1	1, 1	0, 1
0, 2	1, 2	0, 2
0, 3	1, 3	0, 3
1, 0	2, 0	1, 0
1, 1	2, 1	1, 1
1, 2	2, 2	1, 2
1, 3	2, 3	1, 3
2, 0	3, 0	2, 0
2, 1	3, 1	2, 1
2, 2	3, 2	2, 2
2, 3	3, 3	2, 3
3, 0	4, 0	3, 0
3, 1	4, 1	3, 1
3, 2	4, 2	3, 2
3, 3	4, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i][j] = a[i][j] + 2;
  }
```

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Iteration Vectors and Index Vectors: Example 2

```
for (i=0, i<4; i++)
  for (j=0; j<4; j++)
  {
     a[i][j] = a[i][j] + 2;
  }
```

Loop carried dependence exists if

- there are two distinct iteration vectors such that
- the index vectors of LHS and RHS are identical

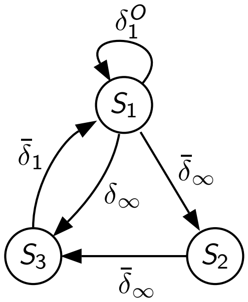
Conclusion: No dependence

Iteration Vector	Index Vector	
	LHS	RHS
0, 0	0, 0	0, 0
0, 1	0, 1	0, 1
0, 2	0, 2	0, 2
0, 3	0, 3	0, 3
1, 0	1, 0	1, 0
1, 1	1, 1	1, 1
1, 2	1, 2	1, 2
1, 3	1, 3	1, 3
2, 0	2, 0	2, 0
2, 1	2, 1	2, 1
2, 2	2, 2	2, 2
2, 3	2, 3	2, 3
3, 0	3, 0	3, 0
3, 1	3, 1	3, 1
3, 2	3, 2	3, 2
3, 3	3, 3	3, 3

Example 4: Dependence

Program to swap arrays	Dependence Graph

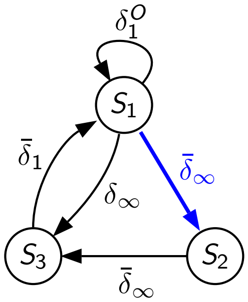
```
for (i=0; i<N; i++)
{
    T = A[i];         /* S1 */
    A[i] = B[i];      /* S2 */
    B[i] = T;         /* S3 */
}
```



Example 4: Dependence

Program to swap arrays	Dependence Graph

```
for (i=0; i<N; i++)
{
    T = A[i];         /* S1 */
    A[i] = B[i];      /* S2 */
    B[i] = T;         /* S3 */
}
```

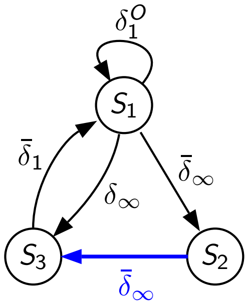


Loop independent anti dependence due to A[i]

Example 4: Dependence

Program to swap arrays	Dependence Graph
see code below	see graph below

```
for (i=0; i<N; i++)
{
    T = A[i];        /* S1 */
    A[i] = B[i];     /* S2 */
    B[i] = T;        /* S3 */
}
```

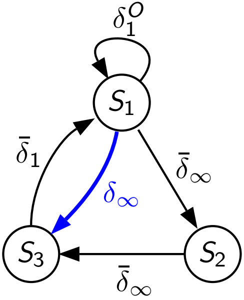


Loop independent anti dependence due to B[i]

Example 4: Dependence

Program to swap arrays	Dependence Graph
see code below	see graph below

```
for (i=0; i<N; i++)
{
    T = A[i];        /* S1 */
    A[i] = B[i];     /* S2 */
    B[i] = T;        /* S3 */
}
```



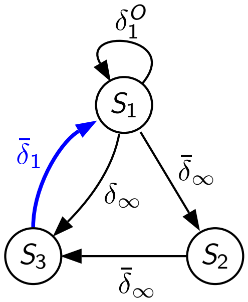
Loop independent flow dependence due to `T`

Example 4: Dependence

Program to swap arrays

```
for (i=0; i<N; i++)
{
    T = A[i];        /* S1 */
    A[i] = B[i];     /* S2 */
    B[i] = T;        /* S3 */
}
```

Dependence Graph

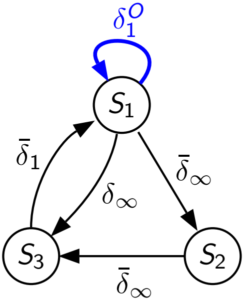


Loop carried anti dependence due to T

Example 4: Dependence

Program to swap arrays	Dependence Graph

```
for (i=0; i<N; i++)
{
    T = A[i];          /* S1 */
    A[i] = B[i];       /* S2 */
    B[i] = T;          /* S3 */
}
```

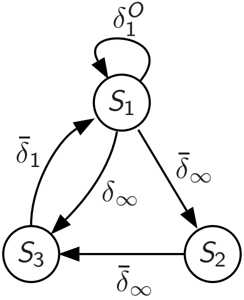


Loop carried output dependence due to `T`

Example 4: Dependence

Program to swap arrays	Dependence Graph
see code below	see graph below

```
for (i=0; i<N; i++)
{
    T = A[i];          /* S1 */
    A[i] = B[i];       /* S2 */
    B[i] = T;          /* S3 */
}
```



Tutorial Problem for Discovering Dependence

Draw the dependence graph for the following program
(Earlier program modified to swap 2-dimensional arrays)

```
for (i=0; i<N; i++)
{
    for (j=0; j<N; j++)
    {   T = A[i][j];          /* S1 */
        A[i][j] = B[i][j]; /* S2 */
        B[i][j] = T;          /* S3 */
    }
}
```

Data Dependence Theorem

There exists a dependence from statement S_1 to statement S_2 in common nest of loops if and only if there exist two iteration vectors **i** and **j** for the nest, such that

1. $\mathbf{i} < \mathbf{j}$ or $\mathbf{i} = \mathbf{j}$ and there exists a path from S_1 to S_2 in the body of the loop,
2. statement S_1 accesses memory location M on iteration **i** and statement S_2 accesses location M on iteration **j**, and
3. one of these accesses is a write access.

Anti Dependence and Vectorization

Read lexicographically precedes Write

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   C[i] = A[i+2];
   A[i] = B[i];
}
```

Anti Dependence and Vectorization

Read lexicographically precedes Write

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   C[i] = A[i+2];
   A[i] = B[i];
}
```

⇒

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i=i+4) {
   C[i:i+3] = A[i+2:i+5];
   A[i:i+3] = B[i:i+3];
}
```

Anti Dependence and Vectorization

Write lexicographically precedes Read

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   C[i] = A[i+2];
}
```

Anti Dependence and Vectorization

Write lexicographically precedes Read

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   C[i] = A[i+2];
}
```

→

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   C[i] = A[i+2];
   A[i] = B[i];
}
```

Anti Dependence and Vectorization

Write lexicographically precedes Read

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   C[i] = A[i+2];
}
```

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   C[i] = A[i+2];
   A[i] = B[i];
}
```

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i=i+4) {
   C[i:i+3] = A[i+2:i+5];
   A[i:i+3] = B[i:i+3];
}
```

True Dependence and Vectorization

Write lexicographically precedes Read

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   A[i+2] = C[i];
   B[i] = A[i];
}
```

True Dependence and Vectorization

Write lexicographically precedes Read

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i++) {
   A[i+2] = C[i];
   B[i] = A[i];
}
```

→

```
int A[N], B[N], C[N], i;
for (i=0; i<N; i=i+4) {
   A[i+2:i+5] = C[i:i+3];
   B[i:i+3] = A[i:i+3];
}
```

Conjunction of Dependences and Vectorization

Anti Dependence and True Dependence

```
int A[N], i;
for (i=0; i<N; i++) {
   A[i] = A[i+2];
}
```

Conjunction of Dependences and Vectorization

Anti Dependence and True Dependence

```
int A[N], i;
for (i=0; i<N; i++) {
   A[i] = A[i+2];
}
```

→

```
int A[N], i, temp;
for (i=0; i<N; i++) {
   temp = A[i+2];
   A[i] = temp;
}
```

Conjunction of Dependences and Vectorization

Anti Dependence and True Dependence

```
int A[N], i;
for (i=0; i<N; i++) {
    A[i] = A[i+2];
}
```

→

```
int A[N], i, temp;
for (i=0; i<N; i++) {
    temp = A[i+2];
    A[i] = temp;
}
```

↓

```
int A[N], T[N], i;
for (i=0; i<N; i++) {
    T[i] = A[i+2];
    A[i] = T[i];
}
```

Conjunction of Dependences and Vectorization

Anti Dependence and True Dependence

```
int A[N], i;
for (i=0; i<N; i++) {
   A[i] = A[i+2];
}
```

```
int A[N], i, temp;
for (i=0; i<N; i++) {
   temp = A[i+2];
   A[i] = temp;
}
```

```
int A[N], T[N], i;
for (i=0; i<N; i++) {
   T[i] = A[i+2];
   A[i] = T[i];
}
```

```
int A[N], T[N], i;
for (i=0; i<N; i=i+4) {
   T[i:i+3] = A[i+2:i+5];
   A[i:i+3] = T[i:i+3];
}
```

Conjunction of Dependences and Vectorization

True Dependence and Anti Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   B[i+2] = A[i+1];
}
```

Conjunction of Dependences and Vectorization

True Dependence and Anti Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   B[i+2] = A[i+1];
}
```

→

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i+2] = A[i+1];
   A[i] = B[i];
}
```

Conjunction of Dependences and Vectorization

True Dependence and Anti Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   A[i] = B[i];
   B[i+2] = A[i+1];
}
```

→

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i+2] = A[i+1];
   A[i] = B[i];
}
```

↓

```
int A[N], B[N], i;
for (i=0; i<N; i=i+4) {
   B[i+2:i+5] = A[i+1:i+4];
   A[i:i+3] = B[i:i+3];
}
```

Cyclic Dependency and Vectorization

Cyclic True Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i+2] = A[i];
   A[i+1] = B[i];
}
```

Cyclic Dependency and Vectorization

Cyclic True Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i+2] = A[i];
   A[i+1] = B[i];
}
```

Cyclic Anti Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i] = A[i+1];
   A[i] = B[i+2];
}
```

Cyclic Dependency and Vectorization

Cyclic True Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i+2] = A[i];
   A[i+1] = B[i];
}
```

Cyclic Anti Dependence

```
int A[N], B[N], i;
for (i=0; i<N; i++) {
   B[i] = A[i+1];
   A[i] = B[i+2];
}
```

Rescheduling of statements will not break the cyclic dependency - cannot vectorize

Last but not the least . . .

Thank You!