CS206 Spring 2006 Quiz 1

Time: 60 mins   Total marks: 30

- ***The exam is open book and notes.***

- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***

- ***If you need to make any assumptions, state them clearly.***

- ***Do not copy solutions from others or indulge in unfair means.***

1. *[10 marks]* Use natural deduction to prove the following sequent, popularly known in logic as the *Resolution Rule*: $\phi \vee \psi,\ \neg\phi \vee \psi\ \vdash\ \psi$.

   To score marks, your proof must have no more than 15 steps, and **must not** use *Law of the Excluded Middle (LEM)*.

2. *[10 marks]* Consider the following set of atomic propositions, representing English language sentences:

   (a) Proposition p_0: Shyam is a liar.

   (b) Proposition p_1: The universe started with a soft meow.

   In addition, consider the set of atomic propositions $\{q_0, q_1\}$, where q_i represents the sentence "Shyam said that the sentence corresponding to proposition p_i is true".

   For purposes of this question, we will assume a simple binary world, where a person either always speaks the truth or always lies. It then follows that $p_0 \wedge q_i\ \rightarrow\ \neg p_i$ and $\neg p_0 \wedge q_i\ \rightarrow\ p_i$ for all $i \in \{0, 1\}$. These four formulae can therefore be used as premises for purposes of this question.

   Show using natural deduction that "If Shyam said that the sentence corresponding to p_0 is true, then the universe started with a soft meow".

   To score marks, you may use LEM **at most once** in your proof, and your proof must have no more than 20 steps.

3. *[5+5 marks]* Consider the formula $\phi = \neg((x \wedge \neg(y \wedge z)) \vee (x \wedge y \wedge z))$, where x, y, z are atomic propositions.

   (a) Convert ϕ to negation-normal form (NNF).

   (b) Convert ϕ to conjunctive normal form (CNF) with the *minimum number of clauses*. You may use commonly ised propositional logic identities (e.g., distributive laws, $\top \vee \psi = \top$, $\psi \wedge \neg\psi = \bot$, etc.) to minimize the number of clauses. To score marks, you **must mention** the identities you are using.