CS206 Tutorial No. #1

Date: Jan 18, 2006

1. Using the $\wedge_i$, $\wedge_e$, $\vee_i$, $\vee_e$, $\rightarrow_i$, $\rightarrow_e$, $\bot_i$(also called $\neg_e$), $\bot_e$, $\neg_i$, $\neg\neg_e$, $\neg\neg_i$ rules, Modus Tollens and **zero applications** of LEM, prove the following sequents.

   (a) $\phi_1 \rightarrow (\phi_2 \rightarrow \neg\phi_1) \vdash \phi_2 \rightarrow \neg\phi_1$

   (b) Distributive Laws:

      i. $(\phi_1 \wedge \phi_2) \vee (\phi_1 \wedge \phi_3) \vdash \phi_1 \wedge (\phi_2 \vee \phi_3)$

      ii. $\phi_1 \wedge (\phi_2 \vee \phi_3) \vdash (\phi_1 \wedge \phi_2) \vee (\phi_1 \wedge \phi_3)$

      iii. $(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3) \vdash \phi_1 \vee (\phi_2 \wedge \phi_3)$

      iv. $\phi_1 \vee (\phi_2 \wedge \phi_3) \vdash (\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$

   (c) De Morgan's Laws:

      i. $\neg(\phi_1 \vee \phi_2) \vdash \neg\phi_1 \wedge \neg\phi_2$

      ii. $\neg\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \vee \phi_2)$

      iii. $\neg(\phi_1 \wedge \phi_2) \vdash \neg\phi_1 \vee \neg\phi_2$

      iv. $\neg\phi_1 \vee \neg\phi_2 \vdash \neg(\phi_1 \wedge \phi_2)$

   (d) $\vdash \phi_1 \rightarrow (\phi_2 \rightarrow (\neg\phi_3 \vee (((\phi_4 \vee \phi_5 \rightarrow \phi_6) \rightarrow (\phi_4 \rightarrow \phi_6) \wedge (\phi_5 \rightarrow \phi_6))) \vee ((\phi_7 \rightarrow (\phi_8 \rightarrow \phi_9)) \rightarrow ((\phi_7 \rightarrow \phi_8) \rightarrow \phi_9))))$

2. Let p and q be atomic propositions that take values from the set $\{True, False\}$. Consider the following two formulae: $\phi_1 = (p \rightarrow \neg\phi_2)$, and $\phi_2 = (q \rightarrow \neg\phi_1)$.

   (a) Show using natural deduction that $\vdash \phi_1 \vee \phi_2$.

   Note: You may use LEM *atmost once* in the proof. Other than this, you must use only the basic introduction and elimination rules of natural deduction.

   (b) Show that there are exactly *two pairs* of propositional logic formulae (ϕ_1, ϕ_2) that satisfy the above definitions. Also, give justification for your answer.