CS206 Tutorial No. #2

Date: Jan 27, 2006

1. Let ϕ_1, ϕ_2 and ϕ_3 be prop. logic formulas. You are told that $(x \rightarrow \phi_1), (\phi_1 \rightarrow x), (y \rightarrow \phi_2), (\phi_2 \rightarrow y), (z \rightarrow \phi_3), (\phi_3 \rightarrow z) \vdash \bot$
   Show that $\phi_1, \phi_2, \phi_3, x, y, z \vdash \bot$ using natural deduction.

2. Let $phi_1 = \neg(x \wedge \neg(y \vee \neg(z \wedge \neg(w))))$
   (a) Give an equivalent formula in negation normal form.
   (b) Give an equivalent formula of the form $\phi_2 \wedge \phi_3$, where ϕ_2 is in CNF and ϕ_3 is in DNF. ϕ_2 should involve only x and y, ϕ_3 should involve only z and w.

   Question to ponder about: Is it always possible to split an arbitrary prop formula ϕ in $2n$ variables into two parts ϕ_2 and ϕ_3, such that ϕ_2 is a CNF formula in the first n variables, and ϕ_3 is a DNF formula in the remaining n variables, and $\phi = \phi_2 \wedge \phi_3$ or $\phi = \phi_2 \vee \phi_3$

3. A student has given the following proof of $\top \vdash x \rightarrow \neg x$ What are the sources of problem in this proof (else we would be in serious trouble with true being equivalent to false).

```
1. top
     ---------------------------------------------------------
2.   | x                assumption                           |
     |  ---------------------------------------------------  |
3.   |  | neg x         assumption                        |  |
4.   |  | bot           bot introduction rule on 2 and 3  |  |
     |  ---------------------------------------------------  |
5.   | neg x            bot elimination rule on 4            |
6.   | bot              bot intro rule on 2 and 5            |
     ---------------------------------------------------------
7. neg x                neg intro rule on 2 -- 6
     ----------------------------------------------
8.   | x                assumption                |
9.   | bot              bot intro rule on 7 and 8 |
10.  | neg x            bot elim rule on 9        |
     ----------------------------------------------
11.x -> neg x           impl intro rule on 8 to 10
```

   If we correct the mistake in the above proof, what does the proof give us, that is what is the sequent that is proved by the corrected sequence of application of proof rules?

4. Let $\phi = (a \vee b \vee c) \wedge (\neg c \vee \vee a) \wedge (\neg b) \wedge (a \vee b)$. Use DPLL kind of reasoning to show that without drawing the truth table, we can argue that no row of the truth table of ϕ will have true value of ϕ.