

- *Please write your Roll No. on the top right of each sheet.*
- *You must write your answers only in the spaces provided.*
- *The exam is open book and notes.*
- *Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.*
- *Unnecessarily lengthy solutions will be penalized.*
- *If you need to make any assumptions, state them clearly.*
- ***Do not copy solutions from others or indulge in unfair means.***

1. [10 marks] Consider the following sentence in predicate logic:

$$\phi_1 : \forall x \exists y (P(x, y) \wedge \forall z (P(x, z) \rightarrow (z = y)))$$

Let ϕ_2 be an unspecified predicate logic sentence in SNF that has k nullary functions, has no function of arity greater than 0, and has no occurrence of the predicate P .

A logician now claims that she can check the satisfiability of $\phi_1 \wedge \phi_2$ for any ϕ_2 satisfying the above conditions, by considering models having finite universe of cardinality at most k .

If you think the logician is correct, indicate why models with universe of cardinality k suffice to check satisfiability. Otherwise, give justification why the logician is incorrect.

2. [4+3+3 marks] In this question, we wish to state certain properties of natural numbers in predicate logic. You may use the predicates $<$ and $=$, and the functions $*$ and $+$ on natural numbers with the usual interpretation. You may also use $one()$ as a nullary function that returns the value 1.

Give predicate logic sentences expressing the following properties:

- There are natural numbers that cannot be expressed as one natural number raised to the power of another natural number distinct from 1.
- There are natural numbers that cannot be expressed as the product of distinct natural numbers, none of which is 1.
- There are infinitely many natural numbers that have only one way of factorizing them as the product of two natural numbers.