CS206 Spring 2008 Compensation Quiz 1

Time: 60 mins Total marks: 20

- ***Please write your Roll No. on the top right of each sheet.***
- ***You must write your answers only in the spaces provided.***
- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited, unless specifically asked for.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others or indulge in unfair means.***

1. An undirected graph $G = (V, E)$ with $2n$ vertices can be represented by an $2n \times 2n$ adjacency matrix A_G, where $A_G[i][j] = 1$ iff there is an edge between the i^{th} and j^{th} vertices, and $A_G[i][j] = 0$ otherwise. Let us assume that the maximum no. of edges incident on any vertex (also called the *degree* of the graph) is d, and the total no. of edges in E is m.

   A *perfect matching* of G is a subset F of edges in E such that every vertex in V has exactly one edge in F incident on it. Thus, no vertex can be shared by two edges in F. Neither can we have any vertex that has no edges in F incident on it.

   We wish to find if there exists a perfect matching of G using propositional logic.

   (a) *[10 marks]* Since a solution to the perfect matching problem consists of choosing some edges and not choosing others, an obvious approach is to use a proposition p_e for every edge e in E with the following interpretation. Setting p_e to True would correspond to having e in the set F, while setting p_e to False would correspond to keeping e out of the set F.

   Give a strategy for constructing a propositional logic formula ϕ_G on the set of propositions $\{p_e \mid e \in E\}$ such that ϕ_G is satisfiable iff there exists a perfect matching in the graph G. What is the size of your formula (total number of occurences of propositions and operators) in terms of n, m and d?

   Your formula must be polynomial in the size of the graph. You must also clearly indicate what the various subformulas in your formula correspond to, i.e. which constraints of the original perfect matching problem are represented by which subformula.

(b) *[10 marks]* In the previous subquestion, the total number of propositions used was $|E| = m$. Note that m can be as high as $n.d$ for a graph with $2n$ vertices and degree d. We now wish to solve the same problem as above using fewer propositions.

Give a strategy for constructing a propositional logic formula ψ_G using at most $O(n. \log_2 d)$ propositions, such that ψ_G is satisfiable iff there exists a perfect matching in the graph G. Indicate clearly what the new set of propositions correspond to in the original problem, and give brief justification for your formula (what constraints are represented by which subformulas, etc.)

DO NOT WRITE BEYOND THIS LINE.