CS206 Quiz 2

Max marks: 55 **Time: 1.5 hours**

- *Be brief, complete and stick to what has been asked.*
- *If needed, you may cite results/proofs covered in class without reproducing them.*
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others.***

1. Let ϕ_1 be $\forall x \neg S(x,x)$, ϕ_2 be $\exists x P(x)$, ϕ_3 be $\forall x \exists y S(x,y)$, ϕ_4 be $\forall x(P(x) \rightarrow \exists y S(y,x))$.
   (a) *[5 marks]* Give a model M such that $M \models \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \neg\phi_4$.
   (b) *[5 marks]* Give a model M' such that $M' \models \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4$.

   Domain in your model must not have more than three elements for each subpart.

2. Prove using natural deduction.
   (a) *[10 marks]* $\forall x \exists y(P(x) \rightarrow Q(y)) \vdash \exists y \forall x(P(x) \rightarrow Q(y))$
   (b) *[10 marks]* $\exists x \exists y(H(x,y) \vee H(y,x)), \neg\exists x H(x,x) \vdash \exists x \exists y \neg(x = y)$

3. Let $\phi(x,y,z)$ and $\psi(x,y,z)$ be predicate logic formulae with free variables x, y, z and containing the binary function symbol $+$ and $*$ and the binary predicate symbols $=$ and $>$. Let M be a model in which the domain is a set of integers, the predicates $=$ and $>$ are interpreted as the usual "equal to" and "greater than" predicates over pairs of integers and functions $+$ and $*$ are interpreted as usual "addition" and "multiplication" functions over pairs of integers.

   Now consider the following program fragment in C or C++ or Java, where x, y and z are variables of type integer.

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   x =  x + y + (z*x*y);
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   Thus if the values of x, y and z prior to the execution of this statement were 1, 2 and 3 respectively, their values after execution of this statement would be 9, 2 and 3 respectively.

   (a) *[10 marks]* You are told that $\phi(x,y,z)$ evaluates to True in the model M described above if and only if the values of x, y and z **prior to** the execution of program statement are used for the free variables x, y and z in $\phi(x,y,z)$. Give a predicate logic formula $\psi(x,y,z)$ that evaluates to True in the same model M described above if and only if the values of x, y and z **after** the execution of program statement are used for the free variables x, y and z in $\psi(x,y,z)$.
   Your formula $\psi(x,y,z)$ must necessarily depend on $\phi(x,y,z)$ since the values of x, y and z prior to the execution of program statement determine the values of the variables after the execution of program statement.

(b) *[10 marks]* Now consider the inverse problem. You are told that $\psi^{'}(x, y, z)$ evaluates to True in the model M described above if and only if the values of x, y and z **after** the execution of program statement are used for the free variables x, y and z in $\psi^{'}(x, y, z)$. Give a predicate logic formula $\phi^{'}(x, y, z)$ that evaluates to True in the same model M described above if and only if the values of x, y and z **prior to** the execution of program statement are used for the free variables x, y and z in $\phi^{'}(x, y, z)$.
Your formula $\phi^{'}(x, y, z)$ must necessarily depend on $\psi^{'}(x, y, z)$ for the same reason as in the previous subquestion.

(c) *[5 marks]*Starting from $\phi(x, y, z)$, suppose we derive $\psi(x, y, z)$ as in subpart (a). Now letting $\psi(x, y, z) \equiv \psi^{'}(x, y, z)$, suppose we derive $\phi^{'}(x, y, z)$ as in subpart (b). Indicate which of the following relations hold and briefly explain why.

- $\phi(x, y, z) \equiv \phi^{'}(x, y, z)$
- $\phi(x, y, z) \rightarrow \phi^{'}(x, y, z)$
- $\phi^{'}(x, y, z) \rightarrow \phi(x, y, z)$
- None of the above.