Efficient Algorithms for Reachability in Pushdown Timed Automata

S. Akshay

Dept of CSE, Indian Institute of Technology Bombay, India

Joint work with Paul Gastin, Karthik R. Prakash

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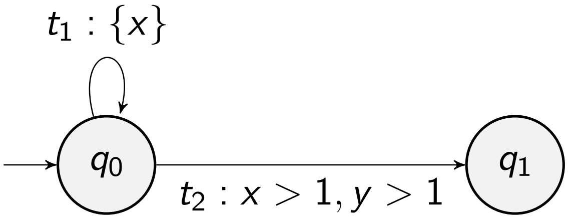
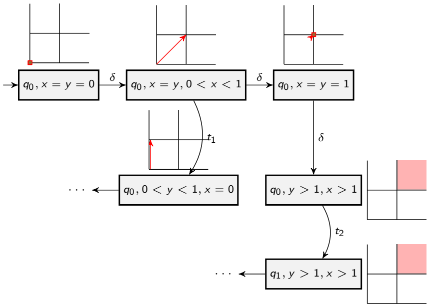
Modeling Timed Systems using Automata

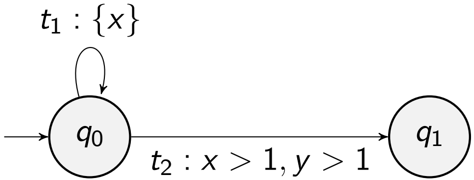


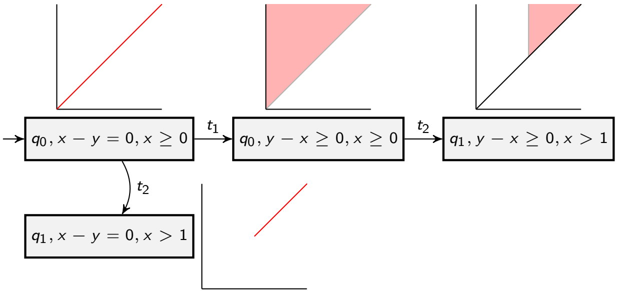


The timed automaton model

- Introduced by Alur & Dill in 1990 [AD90]
- Clocks as variables, guards on transitions and resets.
- Reachability is PSPACE-complete – Region Abstraction
  - Exploration of regions: always finite but often large.
- Well studied model with many extensions.

Big leap forward: Making Timed Automata Practical (Previous talk!)





Zone based abstractions of Timed automata

- Zones: union of regions, "better" abstractions of constraints
  - Exploration of zone graph: Can be infinite but often small.
  - Simulation/subsumption or extrapolation guarantees finiteness.
- UPPAAL [BLL+95, LPY97, PL00, BDL+06], TChecker [HP19], many tools use this!
- Widely used as feasible in practice for many benchmarks...

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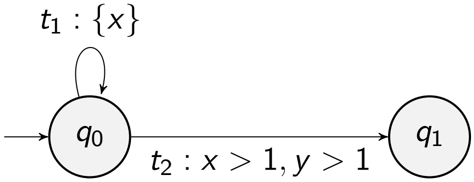

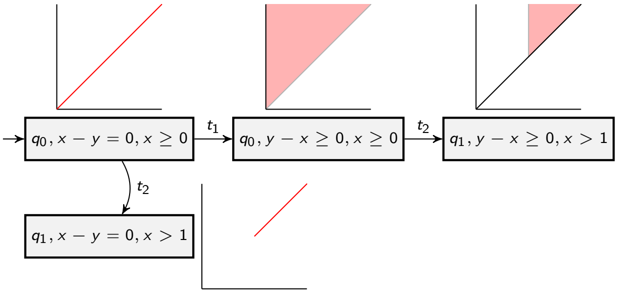

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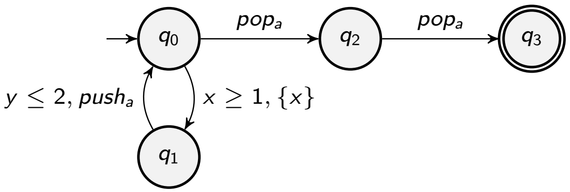
Does the "Zone approach" work for extensions of TA?

Pushdown timed automata (PDTA)



A natural extension combining Time and Recursion

- Introduced in [BER94], just after Timed automata [AD90].
- PDTA = Timed automata + (pushdown) stack!

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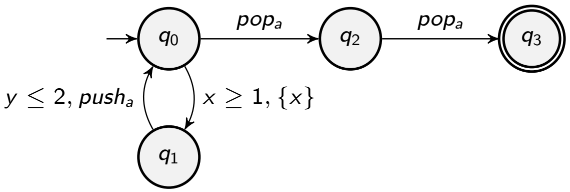

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- For instance, [TW10, AAS12, CL15, AGK18, CLLM17, AGJK19, CL21]

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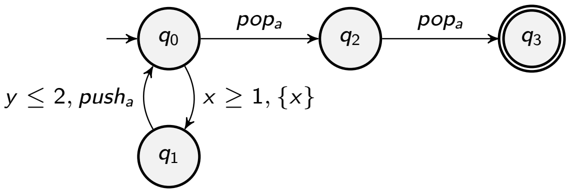

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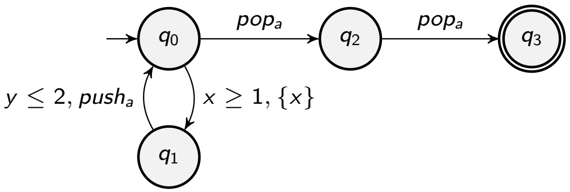

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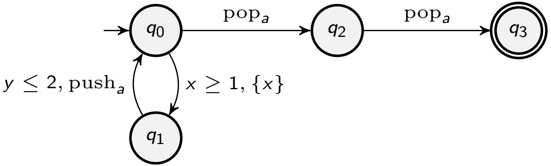
No known zone based approach... Why?!

Our problem statement



The well-nested control-state reachability problem for PDTA

- Is there a run in PDTA, from initial state to target state s.t.,
  - at initial and target states, the stack is empty.
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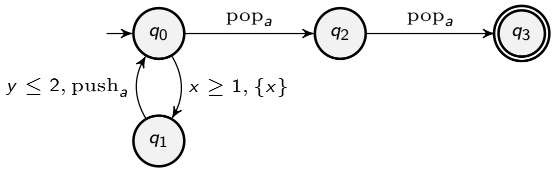

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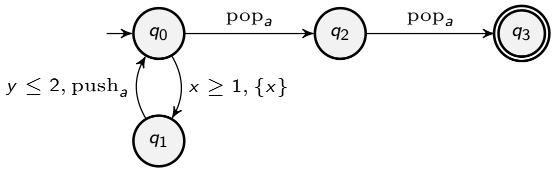

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Main Challenge

- Each recursive call starts a new exploration of zone graph.
- Can we still use simulations to prune and obtain finiteness?

Outline of the talk

1. Re-look at zone algorithms for TA, using re-write rules.
   - Strategies to prune: Simulations and equivalences

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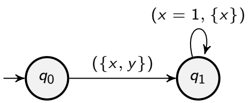
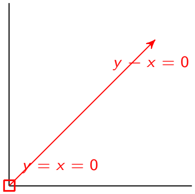
5. Experimental results and comparisons.

Recall: Zones in Timed automata





- Initial clock valuation: $(x = y = 0)$.
- Allowing time elapse: $(y - x = 0, x \geq 0)$
  - $\overrightarrow{(x = y = 0)} = (y - x = 0 \wedge x \geq 0)$ is the initial zone, Z_0

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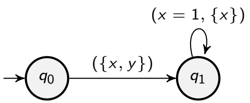

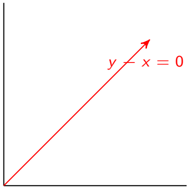

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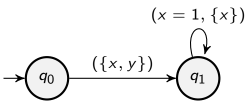

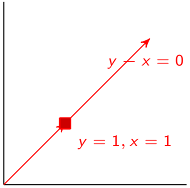

$y - x = 0 \wedge x \geq 0 \wedge x = 1$

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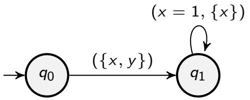

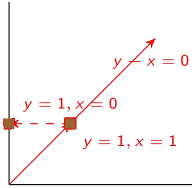

$[\{x\}](x = 1, y = 1)$

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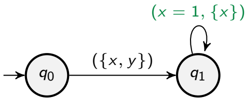

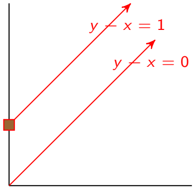

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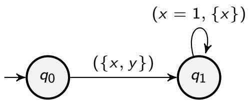
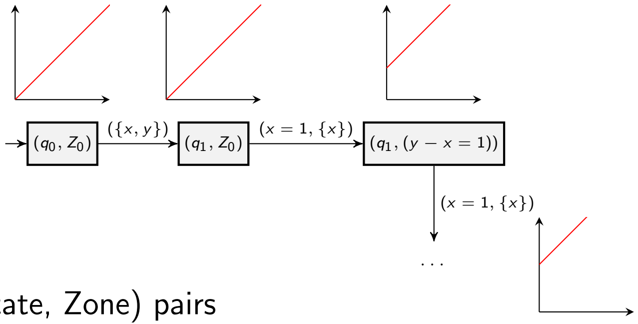
$$Z' = \overrightarrow{[R](Z \wedge g)}$$

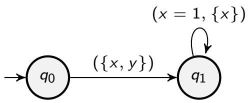
Recall: Zone based Reachability in Timed Automata

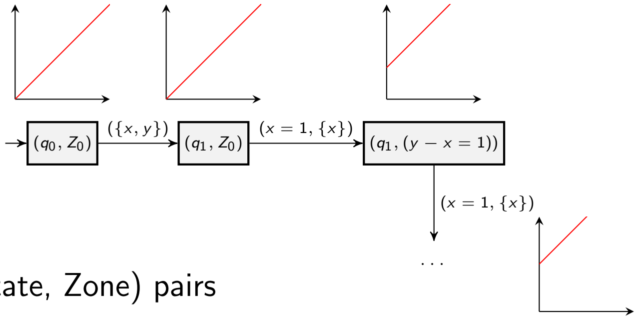




- Zone graph is defined on nodes, i.e., (state, Zone) pairs

$$(q, Z) \xrightarrow{t} (q', Z') \text{ if } t = (q, g, R, q'), Z' = \overrightarrow{[R](Z \wedge g)}$$





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First re-look: We view this as a fix pt computation

$$\frac{}{S := \{(q_0, Z_0)\}}\ \text{start}$$

$$\frac{(q, Z) \in S \qquad q \xrightarrow{g,R} q' \qquad Z' = \overrightarrow{R(g \wedge Z)} \neq \emptyset}{S := S \cup \{(q', Z')\}}\ \text{Trans}$$

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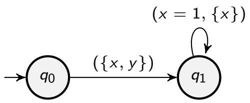

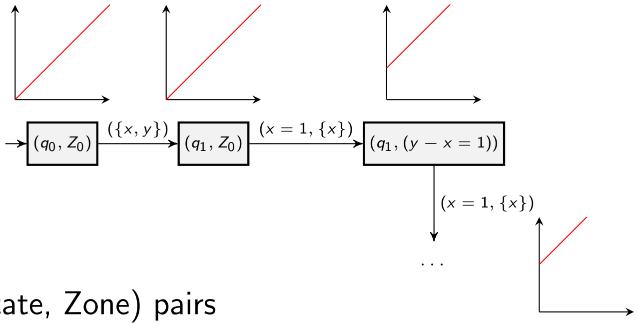

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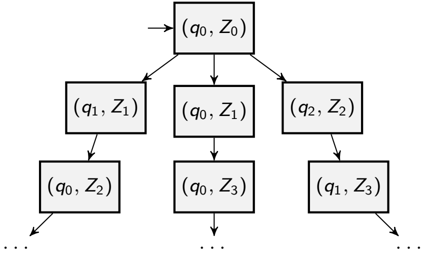
- Reachability using Zone graph construction is sound, and complete, but non-terminating.

Recall: Getting a finite Zone graph using simulations



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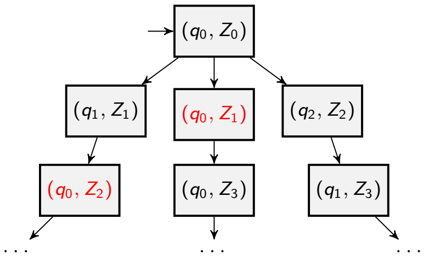

Simulation

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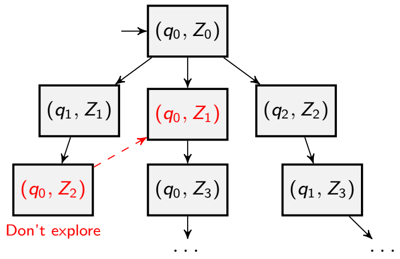

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(q_0, Z_2) \arrow[d, "*"'] \arrow[r, phantom, "\preceq_{q_0}"] & (q_0, Z_1) \arrow[d, "*"'] \\
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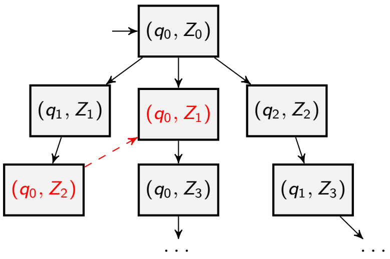

Strongly Finite Simulation

- $(q_0, Z_2) \preceq_{q_0} (q_0, Z_1)$ (Behaviour of Z_2 captured by Z_1 at q_0).
- In any infinite sequence of nodes $(q_0, Z_0), (q_1, Z_1), \cdots$, there must exist $j < i$, s.t., $q_i = q_j$ and $(q_i, Z_i) \preceq_{q_i} (q_j, Z_j), (q_j, Z_j) \preceq_{q_i} (q_i, Z_i)$

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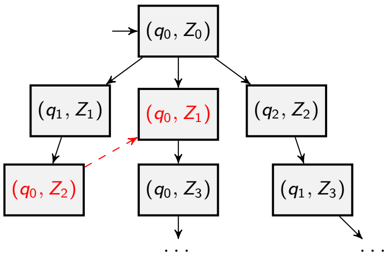

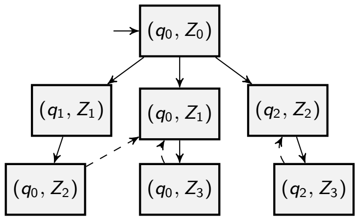

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Strongly finite simulations guarantee finite zone graph preserving soundness, completeness!

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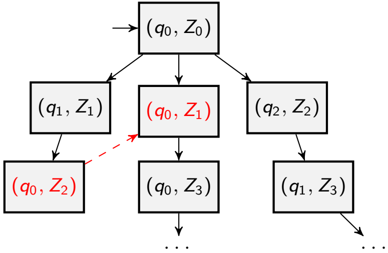

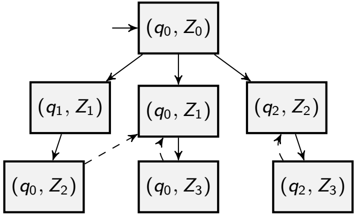

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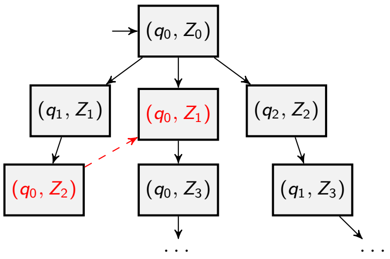

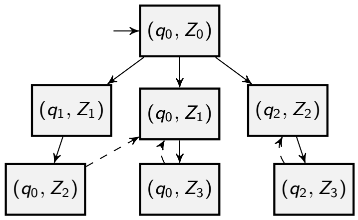

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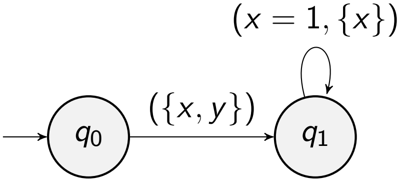
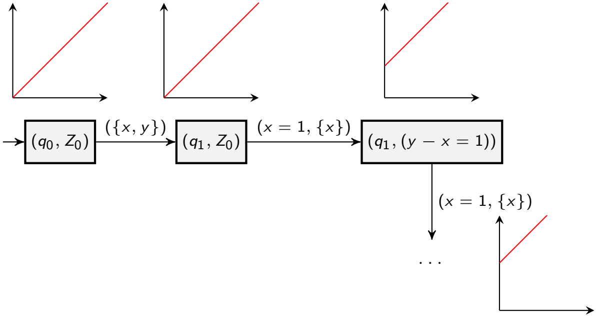
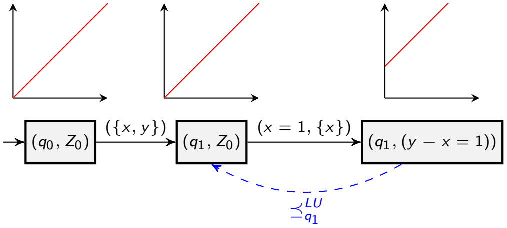
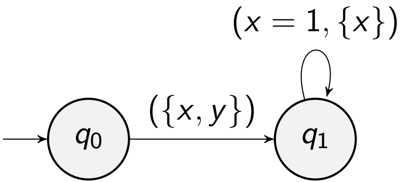
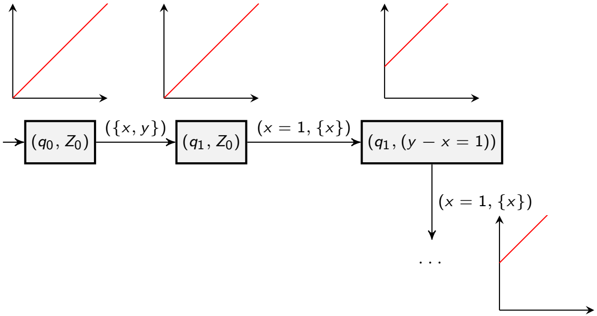


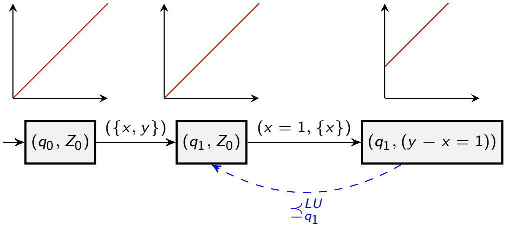




Recall: Getting a finite Zone graph using simulations

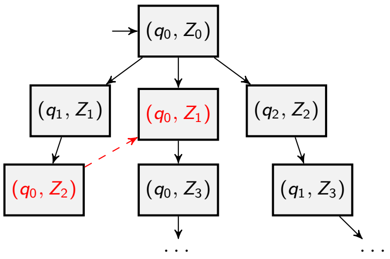


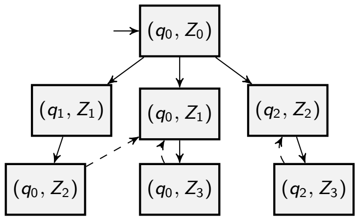




We don't need to know any more details.
We only care that such simulations exist!

Recall: Getting a finite Zone graph using simulations





Modify the re-write rule based saturation algorithm

$$\frac{}{S := \{(q_0, Z_0)\}}\ \text{start}$$

$$\frac{(q, Z) \in S \qquad q \xrightarrow{g,R} q' \qquad Z' = \overrightarrow{R(g \wedge Z)} \neq \emptyset}{S := S \cup \{(q', Z')\},\ \text{unless}\ \exists (q', Z'') \in S,\ Z' \preceq_{q'} Z''}\ \text{Trans}$$

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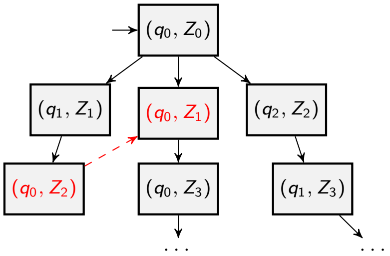

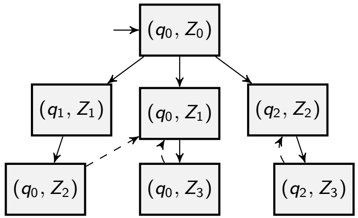

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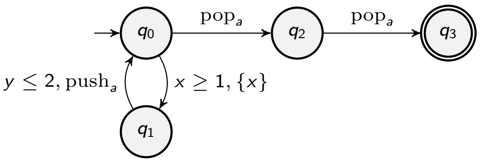
This algorithm is sound, complete and terminating for computing set of reachable nodes in TA.

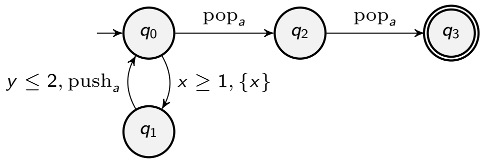
From TA to PDTA



The well-nested control-state reachability problem for PDTA

- Given PDTA A, an initial state q_0 and a target state q_f, is there a run of A from q_0 to q_f s.t.,
  - at initial and target states stack is empty.
  - in between stack can grow arbitrarily.

From TA to PDTA



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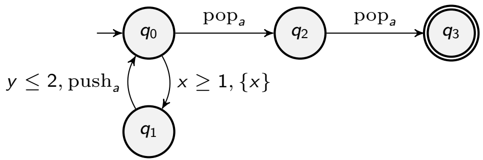

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- As in TA, we will instead compute set of all reachable nodes (with empty stack).

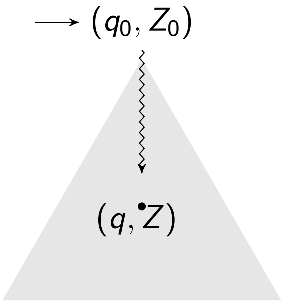
Let us try the same approach as above!

Viewing well-nested reachability in PDTA

$\longrightarrow (q_0, Z_0)$

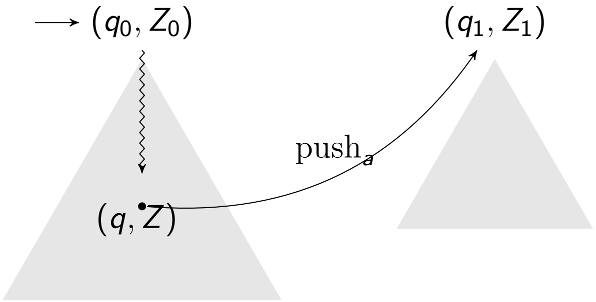
- We start with the initial node

Viewing well-nested reachability in PDTA



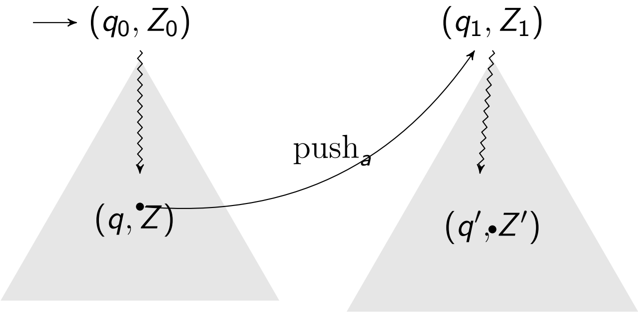
- We start with the initial node and explore as before as long as we see internal transitions (no push-pop).

Viewing well-nested reachability in PDTA



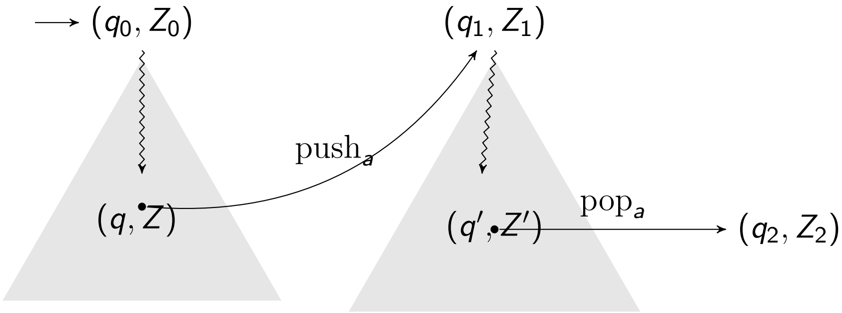
- When we see a Push, we start a new tree/context!

Viewing well-nested reachability in PDTA



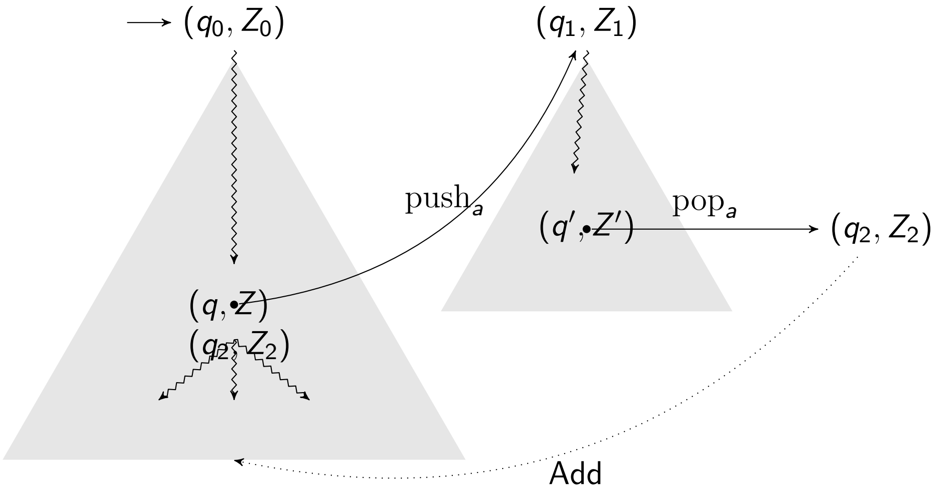
- When we see a Push, we start a new tree/context!
- Continue as long as we only see internal transitions.

Viewing well-nested reachability in PDTA



- Continue as long as we only see internal transitions.
- When we see a "matching" Pop transition,

Viewing well-nested reachability in PDTA



- When we see a "matching" Pop transition, we return to original context and continue from corresponding Push.

Reachability rules for PDTA

- We construct set of nodes explored, as in TA, but parametrized by the root $S_{(q_0,Z_0)}$.

$$\frac{}{S_{(q_0,Z_0)} := \{(q_0, Z_0)\}}\ \text{Start}$$

$$\frac{(q', Z') \in S_{(q,Z)} \qquad q' \xrightarrow{g,\mathrm{nop},R} q'' \qquad Z'' = \overrightarrow{R(g \wedge Z')} \neq \emptyset}{S_{(q,Z)} := S_{(q,Z)} \cup \{(q'', Z'')\},}\ \text{Internal}$$

Reachability rules for PDTA

- We construct set of nodes explored, as in TA, but parametrized by the root $S_{(q_0,Z_0)}$.
- In addition, we maintain the set of roots $\mathfrak{S}$!

$$\frac{}{\mathfrak{S} := \{(q_0, Z_0)\},\ S_{(q_0,Z_0)} := \{(q_0, Z_0)\}}\ \text{Start}$$

$$\frac{(q, Z) \in \mathfrak{S} \qquad (q', Z') \in S_{(q,Z)} \qquad q' \xrightarrow{g,\mathrm{nop},R} q'' \qquad Z'' = \overrightarrow{R(g \wedge Z')} \neq \emptyset}{S_{(q,Z)} := S_{(q,Z)} \cup \{(q'', Z'')\},}\ \text{Internal}$$

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- When we see a push we add it to set of roots, and start exploration from here.

$$\frac{(q,Z) \in \mathfrak{S} \qquad (q',Z') \in S_{(q,Z)} \qquad q' \xrightarrow{g,\mathrm{push}_a,R} q'' \qquad Z'' = \overrightarrow{R(g \wedge Z')} \neq \emptyset}{\mathfrak{S} := \mathfrak{S} \cup \{(q'',Z'')\},\ S_{(q'',Z'')} = \{(q'',Z'')\}}\ \text{Push}$$

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- Finally, when we see pop, we continue exploring tree where corresponding push happened.

$$\frac{\begin{array}{cccc}(q,Z) \in \mathfrak{S} & (q',Z') \in S_{(q,Z)} & q' \xrightarrow{g,\mathrm{push}_a,R} q'' & Z'' = \overrightarrow{R(g \wedge Z')} \\ (q'',Z'') \in \mathfrak{S} & (q'_1,Z'_1) \in S_{(q'',Z'')} & q'_1 \xrightarrow{g_1,\mathrm{pop}_a,R_1} q_2 & Z_2 = \overrightarrow{R_1(g_1 \wedge Z'_1)} \neq \emptyset\end{array}}{S_{(q,Z)} := S_{(q,Z)} \cup \{(q_2,Z_2)\}}\ \text{Pop}$$

Reachability rules for PDTA

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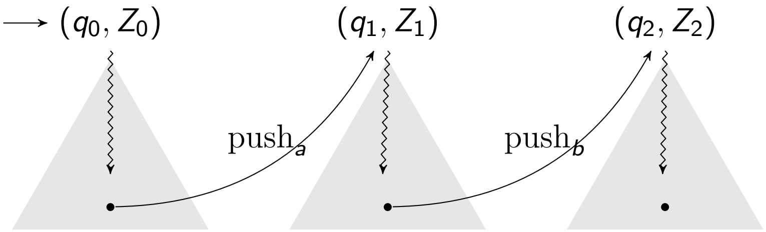
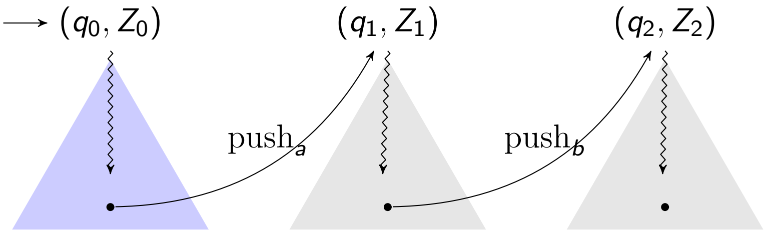
- This set of rules is sound and complete for well-nested control-state reachability in PDTA.
- Issue: But it is not terminating!

How to handle Push-Pop in the Zone graph



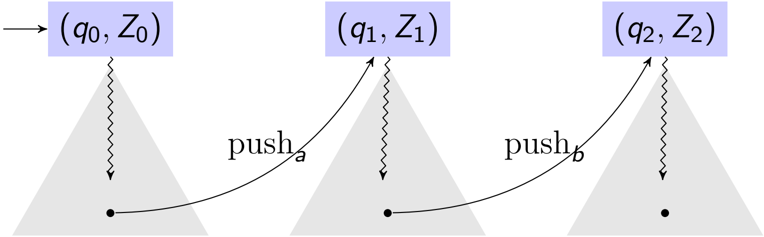
- Two sources of infinity!

How to handle Push-Pop in the Zone graph



- Two sources of infinity!
  - Number of nodes in a tree

How to handle Push-Pop in the Zone graph



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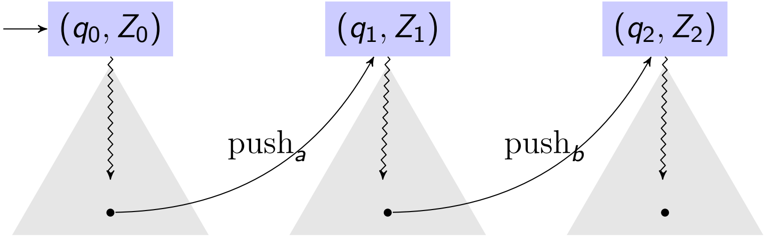
  - Number of root nodes, since each push starts tree at new root!

How to handle Push-Pop in the Zone graph



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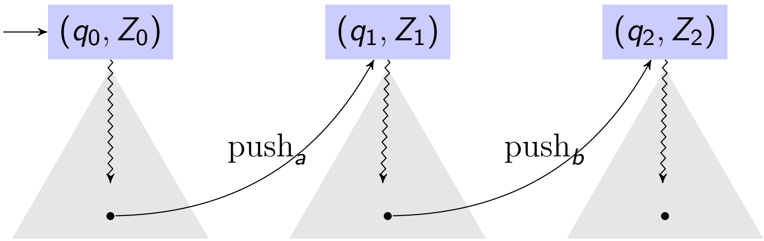
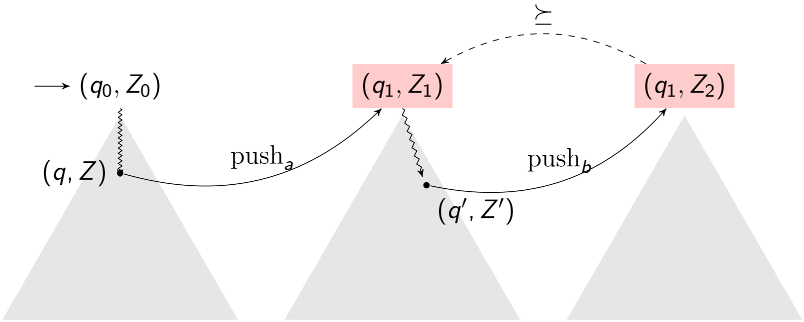


- Two sources of infinity!
  - Number of nodes in a tree
  - Number of root nodes, since each push starts tree at new root!
- Simulation inside a tree (i.e., within each tree) handles the first.
- But not the second! We lose soundness...

The problem with simulation & soundness



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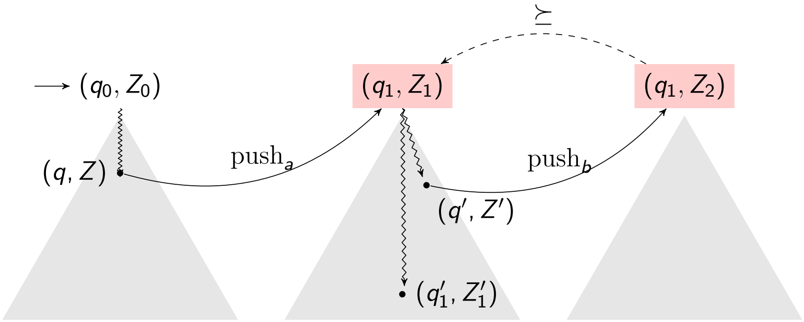

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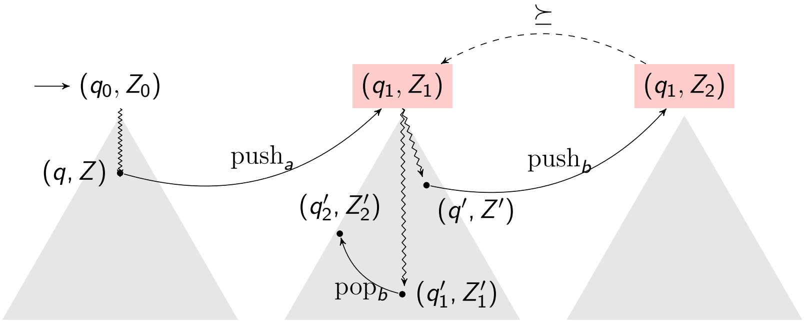

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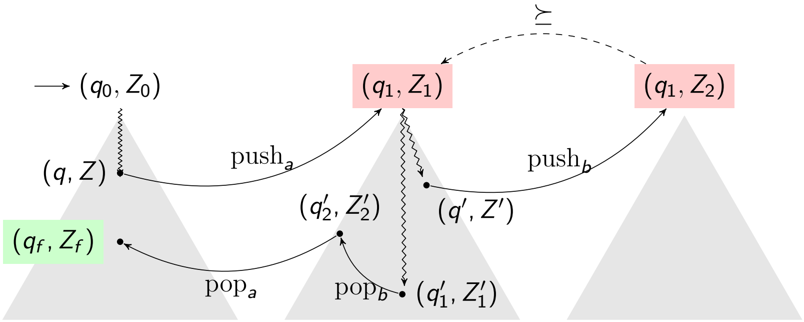

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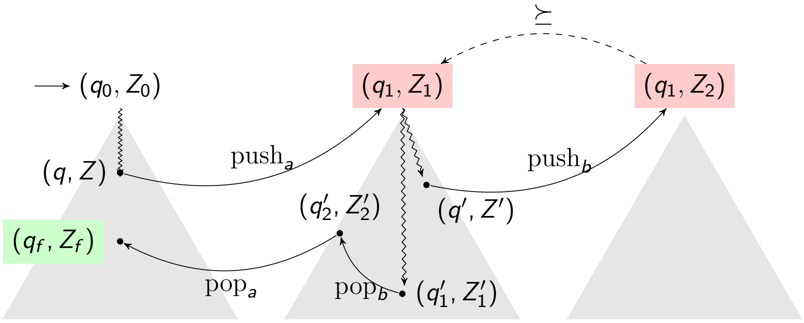
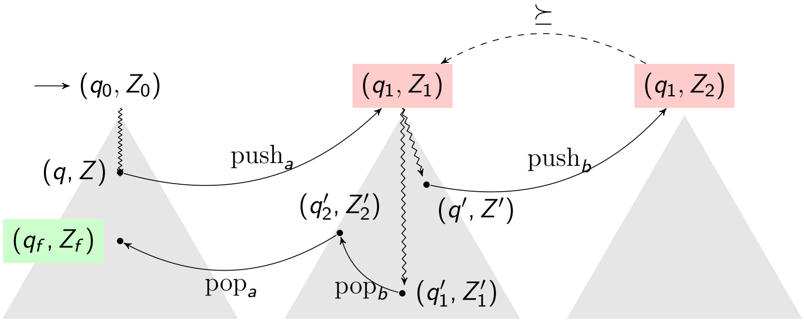


$$(q_0, Z_0) \to (q, Z) \xrightarrow{\mathrm{push}_a} (q_1, Z_1) \to (q', Z') \xrightarrow{\mathrm{push}_b} (q_1, Z_2)$$

$$\rotatebox{-90}{$\preceq$}$$

$$(q_1, Z_1) \to (q'_1, Z'_1) \xrightarrow{\mathrm{pop}_b} (q'_2, Z'_2) \xrightarrow{\mathrm{pop}_a} (q_f, Z_f)$$

The problem with simulation & soundness



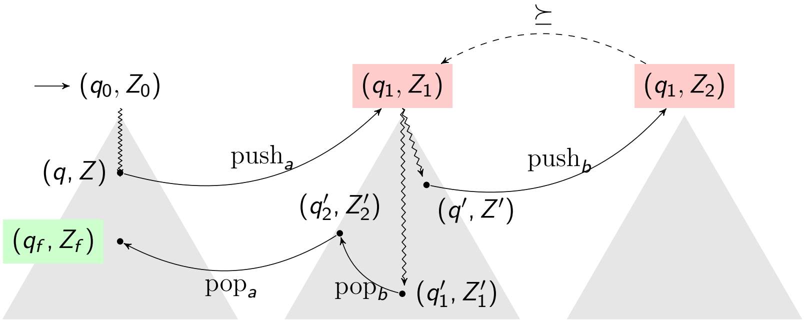
$$(q_0, Z_0) \to (q, Z) \xrightarrow{\text{push}_a} (q_1, Z_1) \to (q', Z') \xrightarrow{\text{push}_b} (q_1, Z_2) \not\to$$

Not Sound!

$$\curlyeqsucc$$

$$(q_1, Z_1) \to (q_1', Z_1') \xrightarrow{\text{pop}_b} (q_2', Z_2') \xrightarrow{\text{pop}_a} (q_f, Z_f)$$

The problem with simulation & soundness



$$(q_0, Z_0) \rightarrow (q, Z) \xrightarrow{\text{push}_a} (q_1, Z_1) \rightarrow (q', Z') \xrightarrow{\text{push}_b} (q_1, Z_2) \not\rightarrow$$

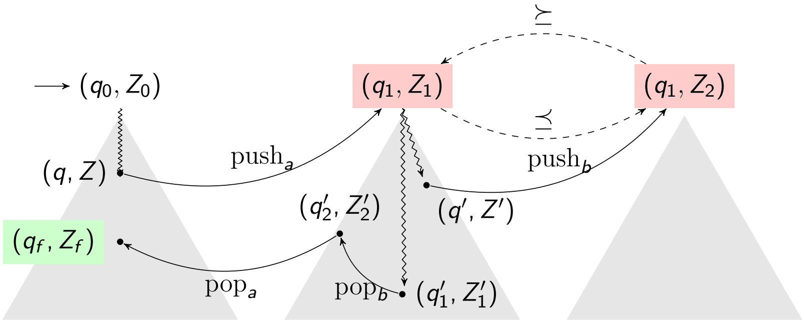
Not Sound!

$$\rotatebox{90}{$\succeq$}$$

$$(q_1, Z_1) \rightarrow (q'_1, Z'_1) \xrightarrow{\text{pop}_b} (q'_2, Z'_2) \xrightarrow{\text{pop}_a} (q_f, Z_f)$$

So how do we fix it?

The problem with simulation & soundness



$$(q_0, Z_0) \to (q, Z) \xrightarrow{\text{push}_a} (q_1, Z_1) \to (q', Z') \xrightarrow{\text{push}_b} (q_1, Z_2)$$

$$\succeq \quad \preceq$$

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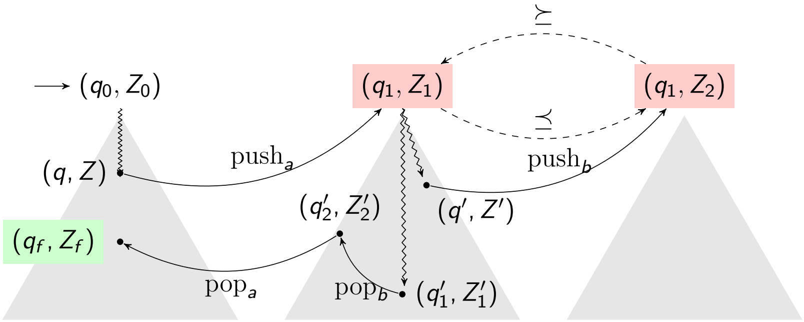
Use equivalence!

The problem with simulation & soundness



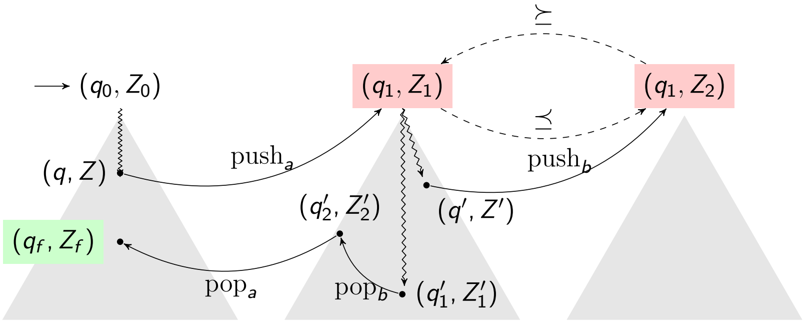
$$(q_0, Z_0) \rightarrow (q, Z) \xrightarrow{\text{push}_a} (q_1, Z_1) \rightarrow (q', Z') \xrightarrow{\text{push}_b} (q_1, Z_2) \rightarrow (q'_1, Z''_1)$$

$$\begin{array}{ccc} \preceq\ \succeq & & \preceq\ \succeq \end{array}$$

$$(q_1, Z_1) \rightarrow (q'_1, Z'_1) \xrightarrow{\text{pop}_b} (q'_2, Z'_2) \xrightarrow{\text{pop}_a} (q_f, Z_f)$$

Use equivalence!

The problem with simulation & soundness



$$(q_0, Z_0) \to (q, Z) \xrightarrow{\mathrm{push}_a} (q_1, Z_1) \to (q', Z') \xrightarrow{\mathrm{push}_b} (q_1, Z_2) \to (q_1', Z_1'')\ldots$$

$$\preceq\ \succeq \qquad \preceq\ \succeq$$

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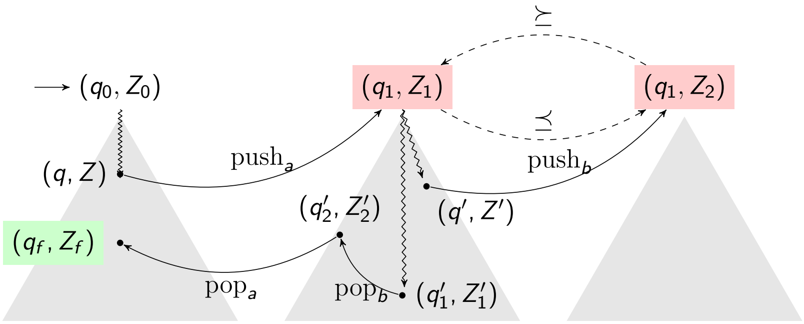
The problem with simulation & soundness



$$(q_0, Z_0) \to (q, Z) \xrightarrow{\text{push}_a} (q_1, Z_1) \to (q', Z') \xrightarrow{\text{push}_b} (q_1, Z_2) \to (q_1', Z_1'')\ldots$$

$$\rotatebox{90}{$\preceq$} \;\; \rotatebox{90}{$\succeq$} \qquad \rotatebox{90}{$\preceq$} \;\; \rotatebox{90}{$\succeq$}$$

$$(q_1, Z_1) \to (q_1', Z_1') \xrightarrow{\text{pop}_b} (q_2', Z_2') \xrightarrow{\text{pop}_a} (q_f, Z_f)$$

Use equivalence!

Thus,

- Checking equivalence to prune at roots gives a sound and complete procedure.
- The enumeration will terminate since the simulation is "strongly finite".

Rules for PDTA to regain finiteness

$$\frac{}{\mathfrak{S} := \{(q_0, Z_0)\},\ S_{(q_0,Z_0)} := \{(q_0, Z_0)\}}\ \text{Start}$$

$$\frac{(q, Z) \in \mathfrak{S} \qquad (q', Z') \in S_{(q,Z)} \qquad q' \xrightarrow{g,\mathrm{nop},R} q'' \qquad Z'' = \overrightarrow{R(g \wedge Z')} \neq \emptyset}{S_{(q,Z)} := S_{(q,Z)} \cup \{(q'', Z'')\}}\ \text{Internal}$$

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Rules for PDTA to regain finiteness

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Main Theorem

This set of rules is sound, complete & terminating for well-nested control-state reachability in PDTA.

Implementation and Experiments

Implemented[1] on top of Open Source tool TChecker

- The rules only give a fix pt saturation algorithm.
- To implement it efficiently, we needed to
  1. Come up with a good data structure.
  2. Decide on order of exploration.
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- Tried two ways of pruning
  - Simulation within trees and equivalence across roots.
  - Equivalence everywhere
- Region based approach from [AGKS17]

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B_1	**0.2**	**17**	0.2	17	235.6	4100
B_2	**20.0**	**5252**	20.7	5252	T.O.	$\geq$154700
B_3	**0.2**	**6**	0.2	6	1043.8	14300
$B_4(100, 10)$	**0.8**	**202**	5.4	2212	OoM	OoM
$B_4(100, 1000)$	**0.7**	**202**	3564.3	201202	OoM	OoM
$B_4(5000, 100)$	**23.2**	**10002**	3429.3	1010102	OoM	OoM
B_5	**38.2**	**3006**	501.0	34799	NA	NA

Time in ms, some benchmarks were custom-crafted, others from prior papers, B_5 had open guards. B_4 was a parametrized example, where first component relates to size of PDTA, second to clock constraints.

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Simulation-based Zone algorithm was always as good and often much better.

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– Thanks!

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