CS 344
Artificial Intelligence

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Formalization of propositional logic (review)

Axioms :

$(A \to (B \to A))$ A1

$((A \to (B \to C)) \to ((A \to B) \to (A \to C)))$ A2

$(((A \to F) \to F) \to A)$ A3

Inference rule:

Given $(A \to B)$ and A, write B

A Proof is:

A sequence of

i) Hypotheses

ii) Axioms

iii) Results of MP

A Theorem is an

Expression proved from axioms and inference rules

Example: To prove $(P \rightarrow P)$

i) $P \rightarrow (P \rightarrow P)$ A1 : P for A and B

ii) $P \rightarrow ((P \rightarrow P) \rightarrow P)$ A1: P for A and $(P \rightarrow P)$ for B

iii) $[(P \rightarrow ((P \rightarrow P) \rightarrow P)) \rightarrow ((P \rightarrow (P \rightarrow P)) \rightarrow (P \rightarrow P))]$

A2: with P for A, $(P \rightarrow P)$ for B and P for C

iv) $(P \rightarrow (P \rightarrow P) \rightarrow (P \rightarrow P))$ MP, (ii), (iii)

v) $(P \rightarrow P)$ MP, (i), (iv)

Shorthand

1. $\neg P$ is written as $P \to F$ and called '*NOT P*'

2. $((P \to F) \to Q)$ is written as $(P \vee Q)$ and called

'*P OR Q*'

3. $((P \to (Q \to F)) \to F)$ is written as $(P \wedge Q)$ and called

'*P AND Q*'

Exercise: (Challenge)

- Prove that $A \to \neg(\neg(A))$

A very useful theorem (Actually a meta theorem, called deduction theorem)

<u>Statement</u>

If

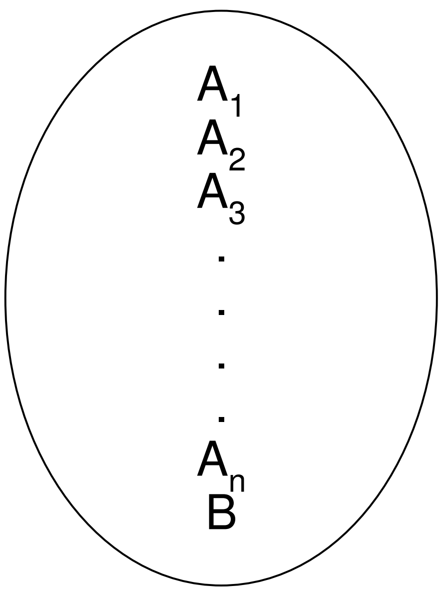
$$A_1, A_2, A_3 \ldots\ldots\ldots\ldots A_n \vdash B$$

then

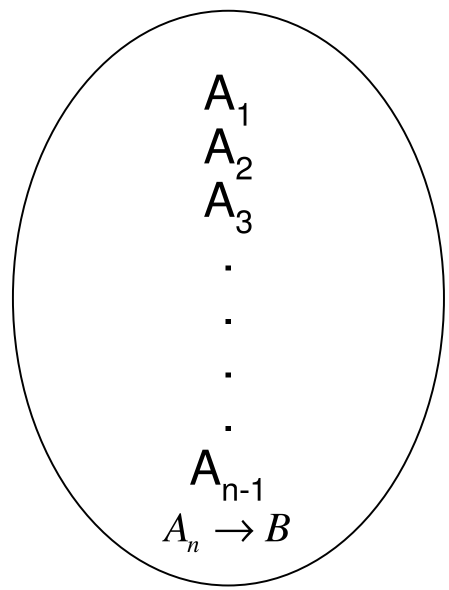
$$A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots A_{n-1} \vdash A_n \rightarrow B$$

$\vdash$ is read as 'derives'

Given



Picture 1



Picture 2

Use of Deduction Theorem

Prove

$$A \to \neg(\neg(A))$$

i.e., $$A \to ((A \to F) \to F)$$

$A, A \to F \vdash$ F (M.P)

A $\vdash (A \to F) \to F$ (D.T)

$\vdash A \to ((A \to F) \to F)$ (D.T)

Very difficult to prove from first principles, *i.e.*, using axioms and inference rules only

Prove $P \to (P \vee Q)$

i.e. $P \to ((P \to F) \to Q)$

$P, P \to F, Q \to F \vdash F$

$P, P \to F \vdash (Q \to F) \to F$ (D.T)

$\vdash Q$ (M.P with A3)

P $\vdash (P \to F) \to Q$

$\vdash P \to ((P \to F) \to Q)$