CS 344
Artificial Intelligence

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Deduction Theorem: Application and Proof

- Shortens the proof procedure.
- Statement:

  If $A_1, A_2, A_3, \ldots, A_n$ |-- B then

  $A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow B$ (|-- means derives)
- We will try to prove the following:

  1. $p \rightarrow p$
  2. $p \rightarrow p \vee q$
  3. A) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

     B) $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$
  4. $(p \rightarrow q) \rightarrow ((\sim p \rightarrow q) \rightarrow q)$

  3 is called the *law of composition*.

Example proofs

- Proofs for axioms are tough to obtain from 1st principles.
- Proof for example 3A: $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

$(p \rightarrow q), \sim q, p$	|--	$\mathcal{F}$
$(p \rightarrow q), \sim q$	|--	$(p \rightarrow \mathcal{F})$
$(p \rightarrow q)$	|--	$(\sim q \rightarrow \sim p)$
	|--	$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

Example proofs (contd.)

- Proof of example 3B: $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$

$(\sim q \rightarrow \sim p), p, (q \rightarrow \mathcal{F})$	|--	$\mathcal{F}$
$(\sim q \rightarrow \sim p), p$	|--	$((q \rightarrow \mathcal{F}) \rightarrow \mathcal{F})$
$(\sim q \rightarrow \sim p), p$	|--	q
$(\sim q \rightarrow \sim p)$	|--	$(p \rightarrow q)$
	|--	$(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$

Example proofs: uses a previous theorem

- Proof of example 4:

$$(p \to q) \to ((\sim p \to q) \to q)$$

$(p \to q), (\sim p \to q), (q \to \mathcal{F})$

|--

$(p \to q) \to (\sim q \to \sim p)$ *(prev theorem)*

$\mathcal{F}$

Now repeated application of DT will get the result

Exercise

- Prove the laws of Propositional Calculus using Deduction Theorem
  - Associativity
  - Commutativity
  - Distributivity
  - De Morgan’s Laws