CS344
Artificial Intelligence

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Fuzzy Logic

- Models Human Reasoning
- Works with imprecise statements such as:

  In a process control situation, "*If* the temperature is moderate and the pressure is high, *then* turn the knob slightly right"
- The rules have "Linguistic Variables", typically adjectives qualified by adverbs (adverbs are hedges).

Underlying Theory: Theory of Fuzzy Sets

- Intimate connection between logic and set theory.
- Given any set 'S' and an element 'e', there is a very natural predicate, $\mu_s(e)$ called as the *belongingness predicate*.
- The predicate is such that,

$$\mu_s(e) = 1, \quad \text{iff } e \in S$$
$$= 0, \quad \text{otherwise}$$

- For example, $S = \{1, 2, 3, 4\}$, $\mu_s(1) = 1$ and $\mu_s(5) = 0$
- A predicate *P(x)* also defines a set naturally.

  $S = \{x \mid P(x) \text{ is } true\}$

  For example, *even(x)* defines $S = \{x \mid x \text{ is even}\}$

Fuzzy Set Theory (contd.)

- Fuzzy set theory starts by questioning the fundamental assumptions of set theory *viz.*, the belongingness predicate, μ, value is 0 or 1.
- Instead in Fuzzy theory it is assumed that,

$$\mu_s(e) = [0, 1]$$

- Fuzzy set theory is a generalization of classical set theory also called Crisp Set Theory.
- In real life belongingness is a fuzzy concept.

Example: Let, T = set of "tall" people

$$\mu_T(\text{Ram}) = 1.0$$

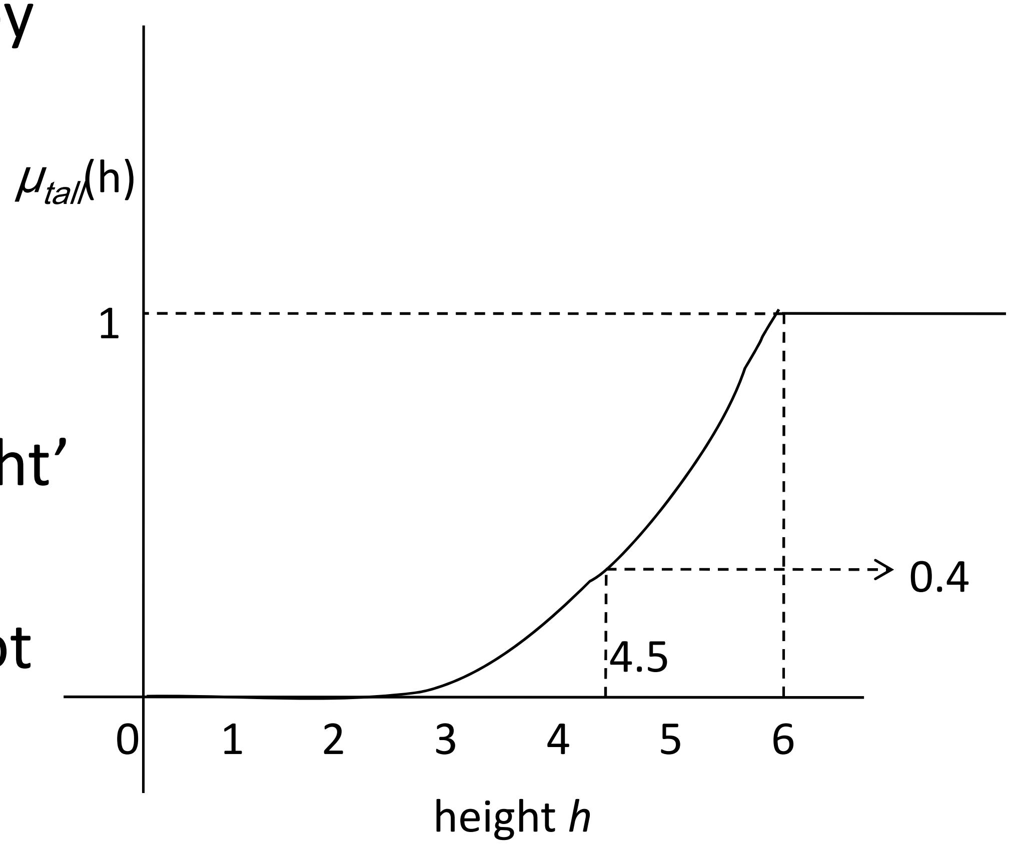
$$\mu_T(\text{Shyam}) = 0.2$$

Shyam belongs to T with degree *0.2*.

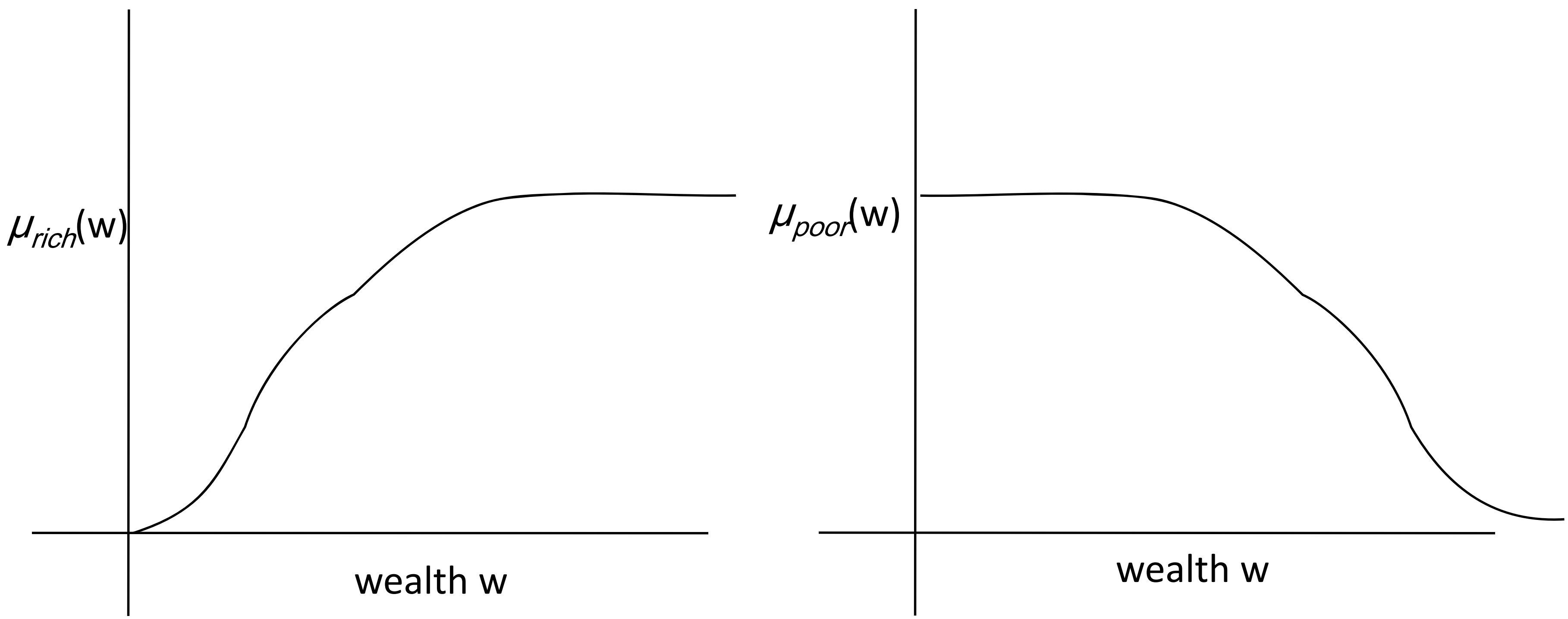
Linguistic Variables

- Fuzzy sets are named by Linguistic Variables (typically adjectives).
- Underlying the LV is a numerical quantity

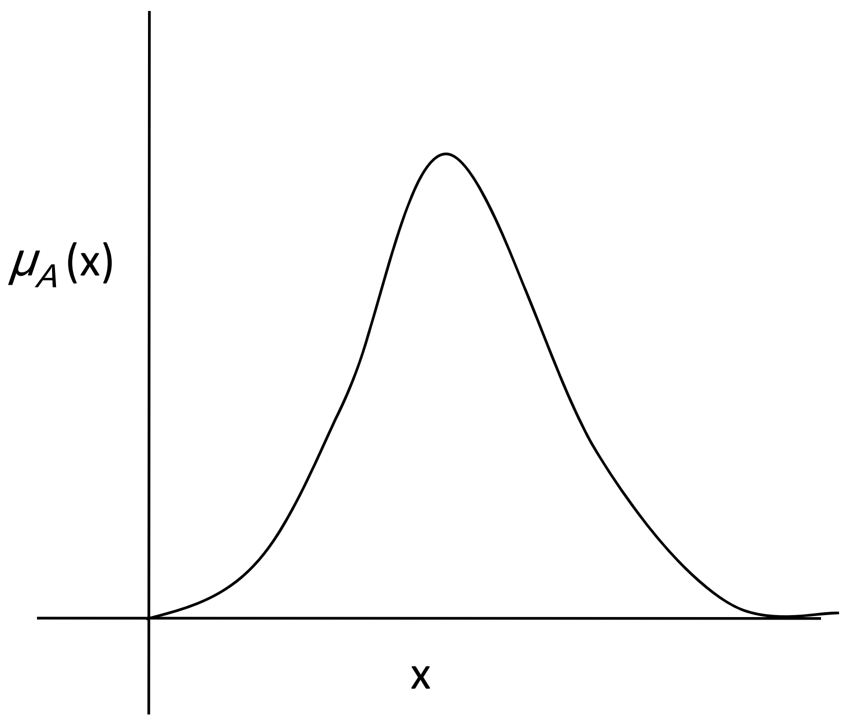
  E.g. For 'tall' (LV), 'height' is numerical quantity.
- Profile of a LV is the plot shown in the figure shown alongside.



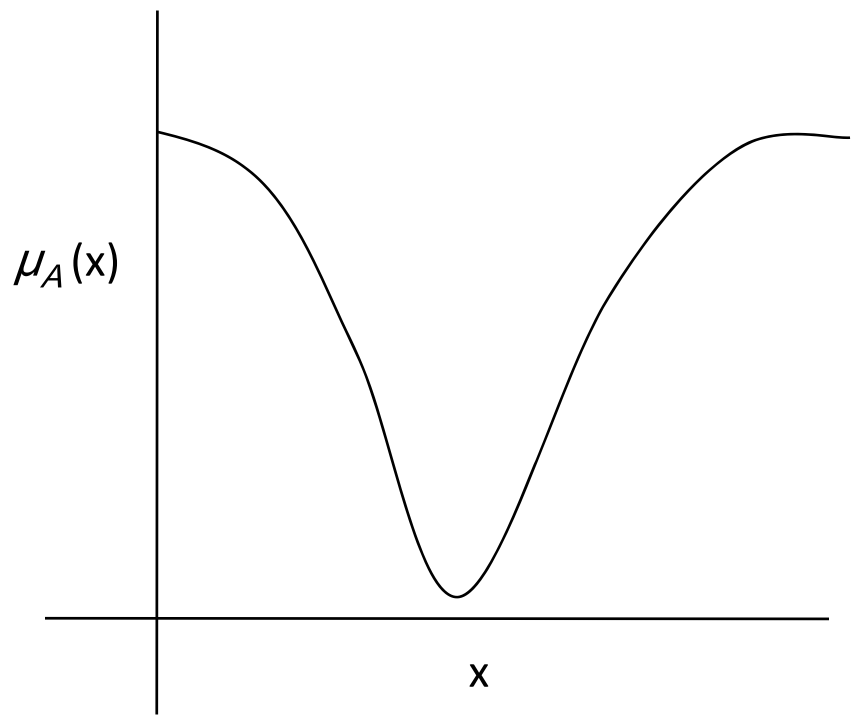
Example Profiles



Example Profiles



Profile representing
moderate (*e.g.* moderately rich)



Profile representing
extreme (*e.g.* extremely poor)

Concept of Hedge

- Hedge is an intensifier
- Example:

  LV = tall, LV_1 = very tall,
  LV_2 = somewhat tall
- 'very' operation:

  $\mu_{very\ tall}(x) = \mu^2_{tall}(x)$
- 'somewhat' operation:

  $\mu_{somewhat\ tall}(x) = \sqrt{(\mu_{tall}(x))}$