CS 344
Artificial Intelligence

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Proof of Deduction Theorem (D. T)

- Statement:

  If $A_1, A_2, A_3, \ldots, A_n$ |-- B then

  $A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow B$

- Proof:

  Case 1: *B* is an axiom

  $A_1, A_2, A_3, \ldots, A_{n-1}, B$

  $B \rightarrow (A_n \rightarrow B)$ … Axiom 1

  $A_n \rightarrow B$ *by MP* … QED

Proof of Deduction Theorem

<u>Case 2</u>: B is one of $A_1, A_2, A_3, \ldots, A_n$

Part (a): B is A_n

$A_1, A_2, A_3, \ldots, A_{n-1}, B$

$A_n \rightarrow A_n$ … theorem

i.e., $A_n \rightarrow B$ … QED

Part (b): B is **not** A_n

$A_1, A_2, A_3, \ldots, A_{n-1}, A_i$ … $i = 1, \ldots, (n-1)$

$A_i \rightarrow (A_n \rightarrow A_i)$ … by *Axiom1*

$A_n \rightarrow A_i$ *i.e.* B … QED

Proof of Deduction Theorem

Case 3: B is the result of MP on E_i and E_j where E_i and E_j come before B in the proof sequence and E_j is $E_i \rightarrow B$

$A_1, A_2, A_3, \ldots, A_{n-1}, E_i, \ldots, E_j, \ldots, B$

Now if

$A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow E_i$ is true

and

$A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow E_j$ is true, *i.e.*,

$(A_n \rightarrow (E_i \rightarrow B))$ is true

then by Axiom-2 and MP we have

$A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow B$

Proof of Deduction Theorem (contd.)

Showing $A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow E_i$ depends on proving DT using a shorter segment of $A_1, A_2, A_3, \ldots, A_n$ |-- B

Similarly for $A_1, A_2, A_3, \ldots, A_{n-1}$ |-- $A_n \rightarrow E_j$

As this continues we come to a situation where E_i (E_j) must be a hypothesis or axiom

These cases have already been proved. QED

Fundamental Issues

- Soundness
- Completeness
- Consistency
- Decidability
- Complexity

Introducing Semantics into the Formal System

- *Valuation function* in Propositional Calculus (PC)
  - The *valuation function* is defined as:

    $$V(E) \rightarrow \{T, F\}$$

    where *E* is a well formed formula and *T*, *F* are symbols called `true' and `false' respectively.
- Definition:
  1. $V(\mathcal{F}) = F$
  2. $V(P) = T$ or F where *P* is a proposition.
  3. $V(P \rightarrow Q)$ is given by the table called `truth table'.

Introducing Semantics into the Formal System

V(P)	*V(Q)*	*V(P → Q)*
T	T	T
F	T	T
T	F	F
F	F	T

- Definition:
  - A tautology is a well formed formula whose valuation is always *T* whatever be the valuation of its constituent propositions
  - Such formulae are also called valid formulae. A formula which is true for some valuation is called satisfiable.
  - An unsatisfiable formula is also called invalid.
- Soundness question:
  - Is every theorem in the system a tautology?
- Completeness question:
  - Is every tautology a theorem of the system?