CS 344
Artificial Intelligence

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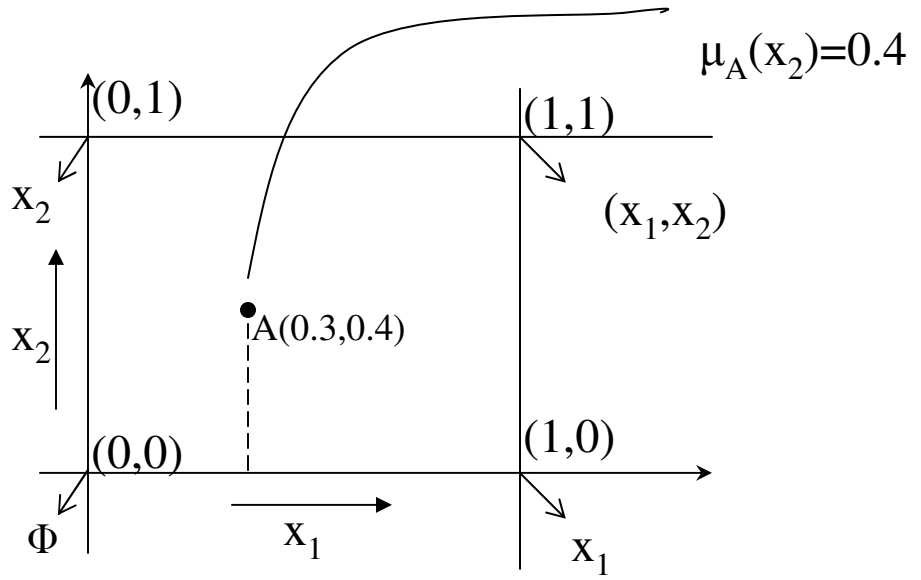
Representation of Fuzzy sets

Let $U = \{x_1, x_2, \ldots\ldots, x_n\}$

$|U| = n$

The various sets composed of elements from U are presented as points on and inside the n-dimensional hypercube. The crisp sets are the corners of the hypercube.

$U = \{x_1, x_2\}$



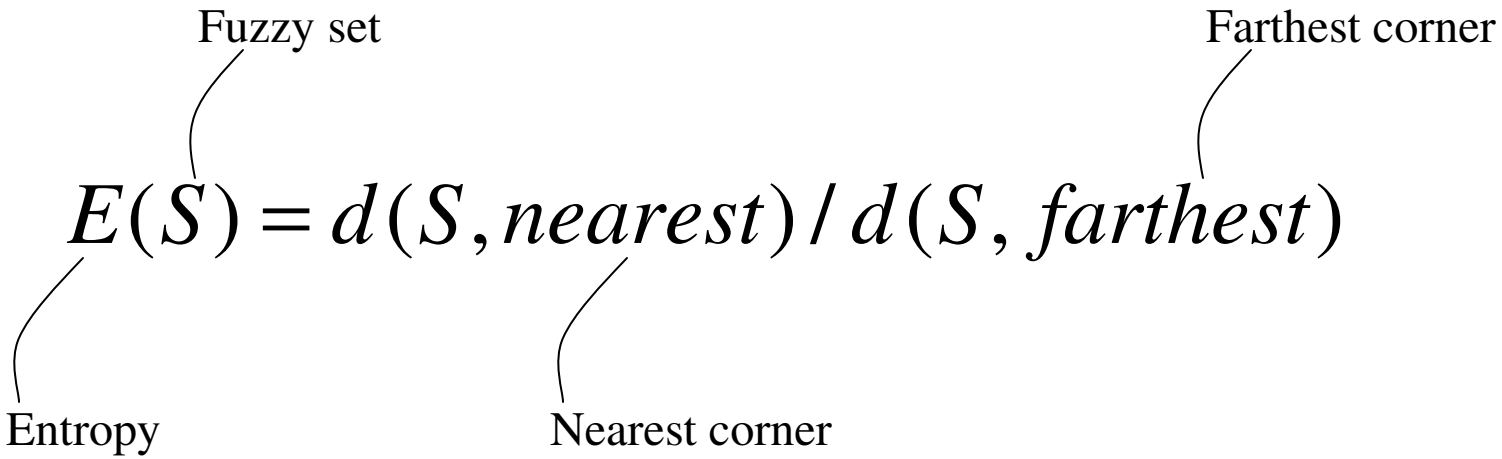
A fuzzy set A is represented by a point in the n-dimensional space as the point $\{\mu_A(x_1), \mu_A(x_2), \ldots\ldots \mu_A(x_n)\}$

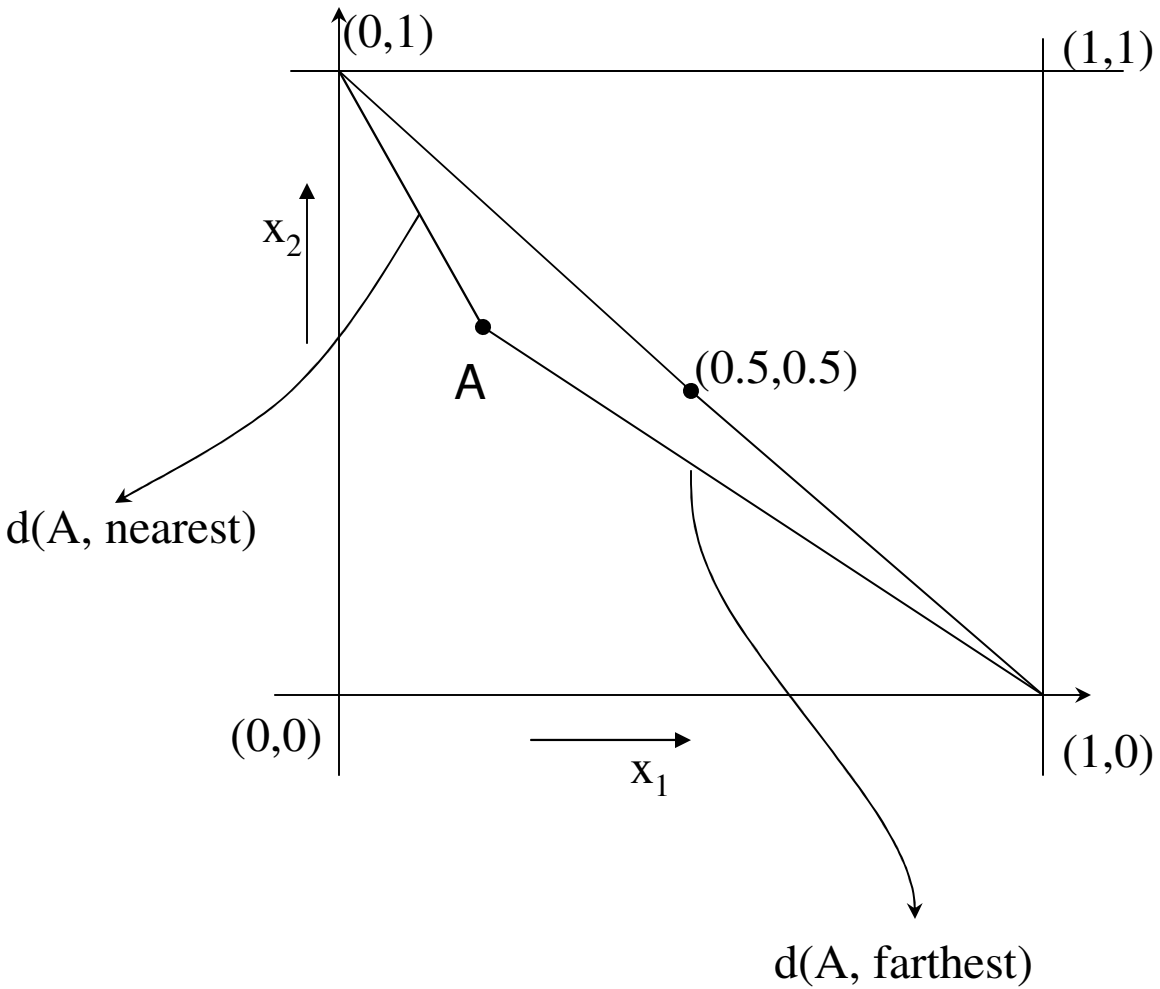
Degree of fuzziness

The centre of the hypercube is the "most fuzzy" set. Fuzziness decreases as one nears the corners

Measure of fuzziness

Called the entropy of a fuzzy set




(0,1)
(1,1)
x2
A
(0.5,0.5)
d(A, nearest)
(0,0)
x1
(1,0)
d(A, farthest)

Definition

Distance between two fuzzy sets

$$d(S_1, S_2) = \sum_{i=1}^{n} | \mu_{s_1}(x_i) - \mu_{s_2}(x_i) |$$

L_1 - norm

Let C = fuzzy set represented by the centre point

d(c,nearest) = |0.5-1.0| + |0.5 – 0.0|

= 1

= d(C,farthest)

=> E(C) = 1

Definition

Cardinality of a fuzzy set

$$m(s) = \sum_{i=1}^{n} \mu_s(x_i)$$ [generalization of cardinality of classical sets]

Union, Intersection, complementation, subset hood

$$\mu_{s_1 \cup s_2}(x) = \max[\mu_{s_1}(x), \mu_{s_2}(x)] \forall x \in U$$

$$\mu_{s_1 \cap s_2}(x) = \min[\mu_{s_1}(x), \mu_{s_2}(x)] \forall x \in U$$

$$\mu_{s^c}(x) = 1 - \mu_s(x)$$

Note on definition by extension and intension

$S_1 = \{x_i | x_i \bmod 2 = 0\}$ – Intension

$S_2 = \{0,2,4,6,8,10,\ldots\ldots\ldots\}$ – extension

How to define subset hood?