CS 344
Artificial Intelligence

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Fuzzy sets to fuzzy logic

Forms the formulation fuzzy rule based system or fuzzy expert system

Expert System

Rules are of the form

If

$C_1 \wedge C_2 \wedge .....C_n$

then

A_i

Where C_is are conditions

Eg: C_1=Colour of the eye yellow

C_2= has fever

C_3=high bilurubin

A = hepatitis

In fuzzy logic we have fuzzy predicates

Classical logic

$P(x_1,x_2,x_3.....x_n) = 0/1$

Fuzzy Logic

$P(x_1,x_2,x_3.....x_n) = [0,1]$

Fuzzy OR

$$P(x) \vee Q(y) = \max(P(x), Q(y))$$

Fuzzy AND

$$P(x) \wedge Q(y) = \min(P(x), Q(y))$$

Fuzzy NOT

$$\sim P(x) = 1 - P(x)$$

How do we define $P(x) \rightarrow Q(y)$

Is it $\sim P(x) \vee Q(y)$?

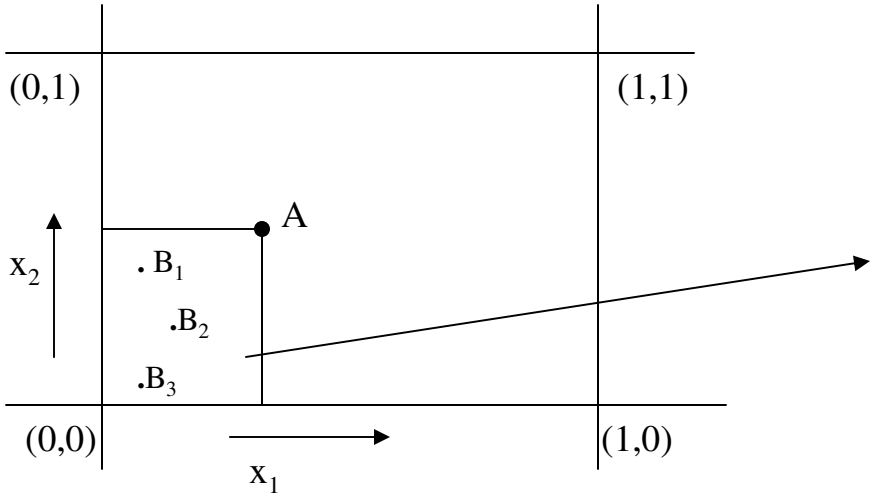
i.e. $\max(1 - P(x), Q(y))$?

<u>Meaning of fuzzy subset</u>

Suppose, following classical set theory we say

$A \subset B$ if $\mu_A(x) \leq \mu_B(x) \forall x$

Consider the n-hyperspace representation of A and B



Region where $\mu_A(x) \leq \mu_B(x)$

This effectively means

$B \in P(A)$ CRISPLY

$P(A)$ = Power set of A

Eg: Suppose

A = {0,1,0,1,0,1,……,0,1} – 10^4 elements

B = {0,0,0,1,0,1,……,0,1} – 10^4 elements

Isn't $B \subset A$ with a degree? (only differs in the 2nd element)

Fuzzy definition of subset

Measured in terms of "fit violation", i.e. violating the condition $\mu_A(x) \leq \mu_B(x)$

Degree of subset hood = 1- degree of superset hood

$$= 1 - \frac{\sum_x \max(0, \mu_A(x) - \mu_B(x))}{m(A)}$$

m(A) = cardinality of A

$$= \sum_x \mu_A(x)$$

We can show that $E(A) = S(A \cup A^c, A \cap A^c)$

Exercise 1:

Show the relationship between entropy and subset hood

Exercise 2:

Prove that

$$S(B, A) = m(A \cap B) / m(A)$$

Subset hood of B in A

Fuzzy Implication

- Many theories have advanced and many expressions exist
- The most used is Lukasiewitz formula
- t(P) = truth value of a proposition/predicate. In fuzzy logic t(P) = [0,1]
- $t(P \rightarrow Q) = \min[1,1 - t(P)+t(Q)]$

Lukasiewitz definition of implication

$$t(P \wedge Q) = \min(t(P), t(Q))$$

Eg: If pressure is high then Volume is low

$$t(high(\,pressure) \rightarrow low(volume))$$