CS 344
Artificial Intelligence

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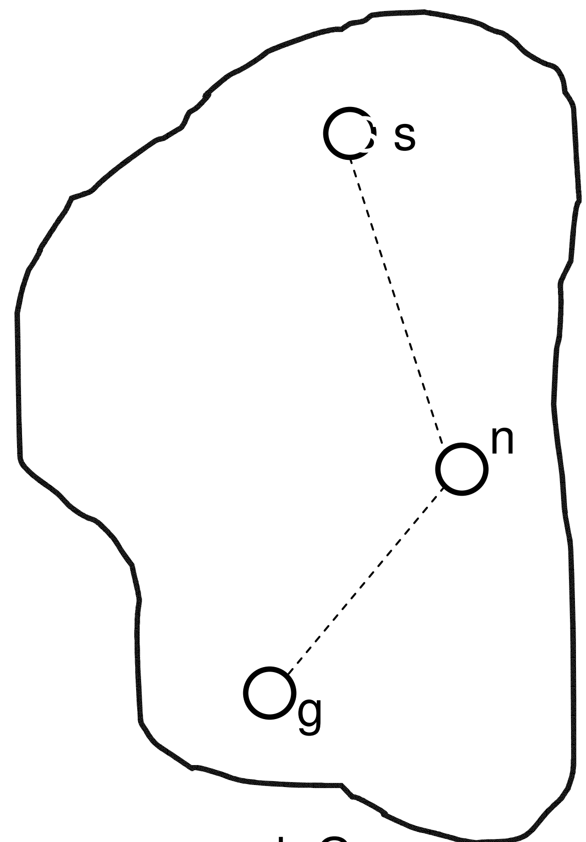
A* Algorithm – Definition and Properties

- $f(n) = g(n) + h(n)$
- The node with the least value of *f* is chosen from the *OL*.
- $f^*(n) = g^*(n) + h^*(n)$, where,

  $g^*(n)$ = actual cost of the optimal path *(s, n)*

  $h^*(n)$ = actual cost of optimal path *(n, g)*
- $g(n) \geq g^*(n)$, since the attempt is always to "maintain" the least cost path *(s, n)*
- By definition, $h(n) \leq h^*(n)$ where $h(n)$ is obtained from problem knowledge



State space graph G

8-puzzle: heuristics

Example: 8 puzzle

2	1	4
7	8	3
5	6	

s

1	6	7
4	3	2
5		8

n

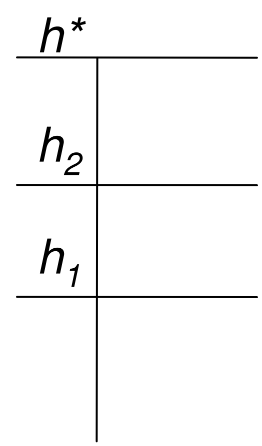
1	2	3
4	5	6
7	8	

g

$h^*(n)$ = actual no. of moves to transform n to g

1. $h_1(n)$ = no. of tiles displaced from their destined position.
2. $h_2(n)$ = sum of Manhattan distances of tiles from their destined position.

$h_1(n) \leq h^*(n)$ and $h_1(n) \leq h^*(n)$



Comparison

Missionaries and Cannibals Problem

- 3 missionaries (*m*) and 3 cannibals (*c*) on the left side of the river and only one boat is available for crossing over to the right side. At any time the boat can carry at most 2 persons and under no circumstance the number of cannibals can be more than the number of missionaries on any bank

Missionaries and Cannibals Problem: heuristics

- Start state: *<3, 3, L>*
- Goal state: *<0, 0, R>*
- $h_1(n) = (\text{no. of } m + \text{no. of } c) / 2$, on the left side
- $h_2(n) = \text{no. of } m + \text{no. of } c - 1$
- $h_1(n) \leq h^*(n)$ and $h_1(n) \leq h^*(n)$

A* Algorithm- Properties

- **Admissibility**: An algorithm is called admissible if it always terminates and terminates in optimal path
- **Theorem**: A* is admissible.
- **Lemma**: Any time before A* terminates there exists on *OL* a node *n* such that $f(n) < f^*(s)$
- **Observation**: For optimal path $s \rightarrow n_1 \rightarrow n_2 \rightarrow \ldots \rightarrow g$,
  1. $h^*(g) = 0$, $g^*(s)=0$ and
  2. $f^*(s) = f^*(n_1) = f^*(n_2) = f^*(n_3) = f^*(g)$

A* Algorithm - Definition and Properties

$$f^*(n_i) = f^*(s), \qquad n_i \neq s \text{ and } n_i \neq g$$

Following set of equations show the above equality:

$$f^*(n_i) = g^*(n_i) + h^*(n_i)$$

$$f^*(n_{i+1}) = g^*(n_{i+1}) + h^*(n_{i+1})$$

$$g^*(n_{i+1}) = g^*(n_i) + c(n_i, n_{i+1})$$

$$h^*(n_{i+1}) = h^*(n_i) - c(n_i, n_{i+1})$$

Above equations hold since the path is optimal.

Lab assignment

- Implement A* algorithm for the following problems:
  - 8 puzzle
  - Missionaries and Cannibals
  - Black and White tiles
    - 3 black and 3 white tiles are given in some initial configuration. Aim is to arrange the tiles such that all the white tiles are to the left of all the black tiles. The cost of a translation is 1 and cost of a jump is 2.
- Specifications:
  - Try different heuristics and compare with baseline case, *i.e.,* the breadth first search.
  - Violate the condition $h \leq h^*$. See if the optimal path is still found. Observe the speedup.