CS 344
Artificial Intelligence

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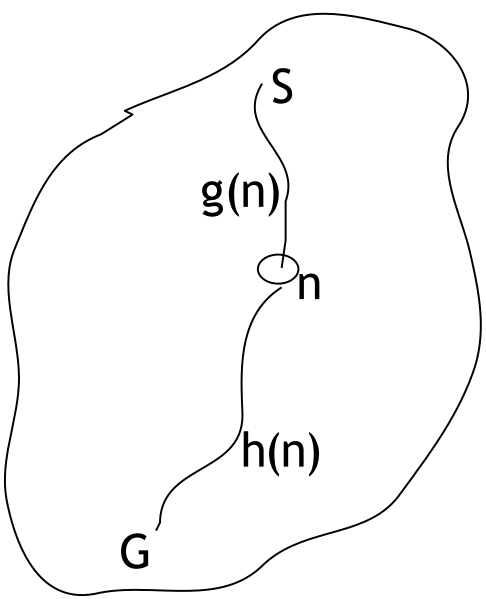
Admissibility of A*

A* always terminates finding an optimal path to the goal if such a path exists.

Intuition



1) In the open list there always exists a node n' such that $f(n) <= f^*(S)$.

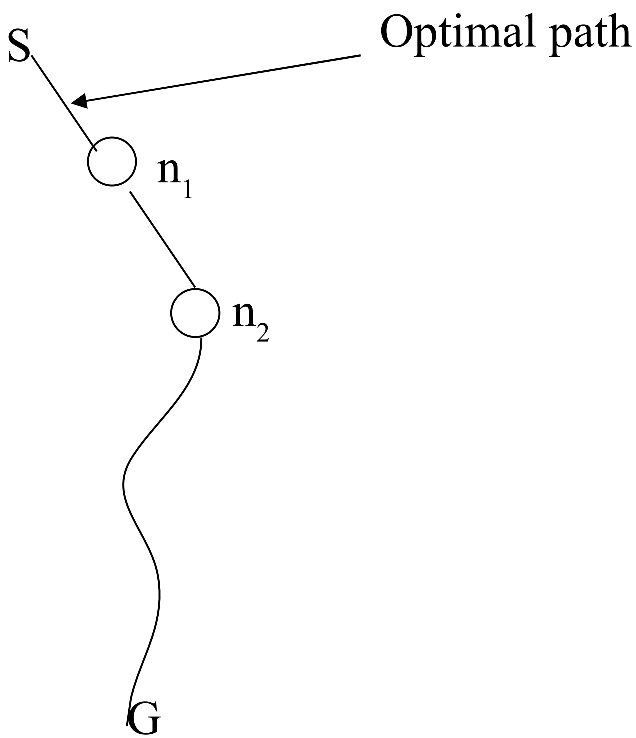
2) If A* does not terminate, the *f* value of the nodes expanded become unbounded.

1) and 2) are together inconsistent

Hence A* must terminate

Lemma

Any time before A* terminates there exists in the open list a node n' such that $f(n') <= f^*(S)$



For any node n_i on optimal path,

$f(n_i) = g(n_i) + h(n_i)$ Also $f^*(n_i) = f^*(S)$

$<= g(n_i) + h^*(n_i)$

Let n' be the fist node in the optimal path that is in OL. Since all parents of n' have gone to CL,

$g(n') = g^*(n')$

$f(n') <= f^*(S)$

If A* does not terminate

Let e be the least cost of all arcs in the search graph.

Then $g(n) >= e.l(n)$ where $l(n)$ = # of arcs in the path from S to n found so far. If A* does not terminate, $g(n)$ and hence $f(n) = g(n) + h(n)$ $[h(n) >= 0]$ will become unbounded.

This is not consistent with the lemma. So A* has to terminate.

2nd part of admissibility of A*

The path formed by A* is optimal when it has terminated

Proof

Suppose the path formed is not optimal

Let G be expanded in a non-optimal path.

At the point of expansion of G,

$$
\begin{aligned}
f(G) &= g(G) + h(G) \\
&= g(G) + 0 \\
&> g^*(G) = g^*(S) + h^*(S) \\
&\qquad = f^*(S) \; [f^*(S) = \text{cost of optimal path}]
\end{aligned}
$$

This is a contradiction

So path should be optimal

Theorem

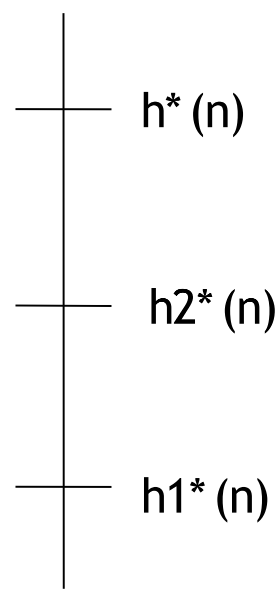
A version A_2^* of A^* that has a "better" heuristic than another version A_1^* of A^* performs at least "as well as" A_1^*

Meaning of "better"

$h2(n) > h1(n)$ for all n

Meaning of "as well as"

A_1^* expands at least all the nodes of A_2^*



For all nodes n, except the goal node

Proof by induction on the search tree of A_2*.

A* on termination carves out a tree out of *G*

Induction

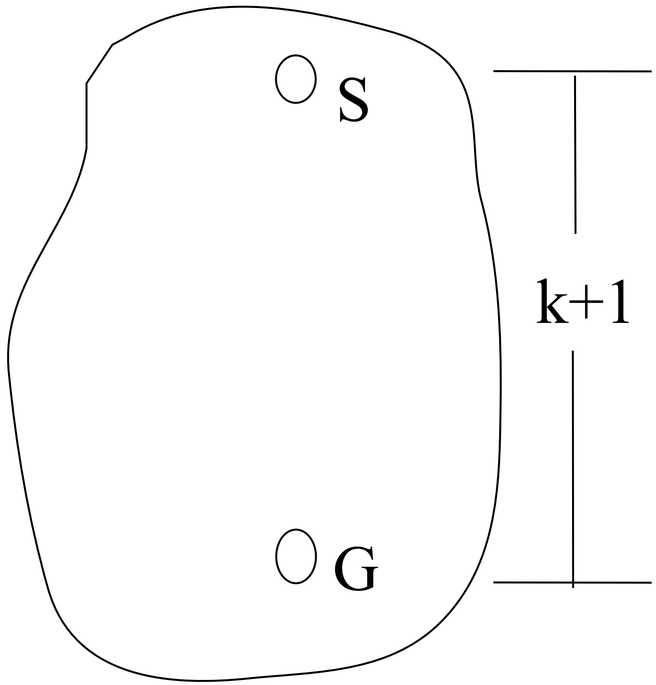
on the depth *k* of the search tree of A_2*. A_1* before termination expands all the nodes of depth *k* in the search tree of A_2*.

k=0. True since start node *S* is expanded by both

Suppose A_1* terminates without expanding a node *n* at depth *(k+1)* of A_2* search tree.

Since A_1* has seen all the parents of *n* seen by A_2*

$g1(n) <= g2(n) \qquad (1)$



Since A_1* has terminated without expanding *n*,

$f1(n) >= f^*(S)$ (2)

Any node whose *f* value is strictly less than $f^*(S)$ has to be expanded.

Since A_2* has expanded *n*

$f2(n) <= f^*(S)$ (3)

From (1), (2), and (3)

$h1(n) >= h2(n)$ which is a contradiction. Therefore, A_1* has to expand all nodes that A_2* has expanded.

<u>Exercise</u>

If better means $h2(n) > h1(n)$ for some *n* and $h2(n) = h1(n)$ for others, then Can you prove the result ?