CS344
Artificial Intelligence
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Inferencing in Fuzzy Logic

• Solving the Fuzzy Rule based system.
• Steps
  1. Read values
  2. Fuzzify
  3. Fire rules
  4. Collate truth values
  5. Defuzzify
  6. Obtain answer
• Lukasiewitz Rule (L Rule)
  \[ t(p \rightarrow q) = \min(1, 1 - t(p) + t(q)) \]
Modus Ponens in Fuzzy System

if \( t(p) \geq a \) -- low water mark

and \( t(p \rightarrow q) = c, \ (0 < a, c \leq 1) \)

\[ t(q) = ? \]

- **Given**: \( t(p \rightarrow q) = \min(1, 1 - t(p) + t(q)) \), \( t(p) \geq a \)
  
  - **Case 1**:
    
    \( c = 1 \)
    
    \[ 1 - t(p) + t(q) \geq 1 \]
    
    i.e. \( t(q) \geq t(p) \geq a \)
  
  - **Case 2**:
    
    \( c < 1 \)
    
    \[ 1 - t(p) + t(q) = c \]
    
    i.e. \( t(q) = c + t(p) - 1 \geq c + a - 1 \)
  
  - From case 1 and 2, \( 0 \leq t(q) \leq 1 \)

- \( t(q) \geq \max(0, c + a - 1) \) -- General Expression
Modus Tolens in Fuzzy System

\[ \text{if } (p \rightarrow q) = c, \quad (0 < c \leq 1) \]

and \[ t(q) \leq b \quad \text{-- high water mark} \]

\[ t(q) = ? \]

- **Given:** \[ t(p \rightarrow q) = \min(1, 1-t(p) + t(q)) = c, \quad t(q) \leq b \]
  - Case 1:
    \[ c = 1 \]
    \[ 1 - t(p) + t(q) \geq 1 \]
    \[ i.e. \quad t(p) \leq t(q) \leq b \]
  - Case 2:
    \[ c < 1 \]
    \[ 1 - t(p) + t(q) = c \]
    \[ i.e. \quad t(p) = 1 + t(q) - c \leq 1 + b - c \]

From case 1 and 2, \[ 0 \leq t(p) \leq 1 \]

- \[ t(p) \leq \min(1, 1 + b - c) \]
- **Note - Crisp Modus Tolens:** Given \( p \rightarrow q \) and \( \sim q \) infer \( \sim p \)
Revisiting the Pendulum Problem

Let \( \theta = 1^\circ \), \( \theta' = 0.05^\circ/\text{sec} \)

1. If \( \theta \) is zero and \( \theta' \) is zero then \( i \) is zero.
2. If \( \theta \) is zero and \( \theta' \) is positive small then \( i \) is negative small.
3. If \( \theta \) is positive small and \( \theta' \) is zero then \( i \) is negative small.
4. If \( \theta \) is zero and \( \theta' \) is negative small then \( i \) is positive small.
5. If \( \theta \) is negative small and \( \theta' \) is zero then \( i \) is positive small.
6. If \( \theta \) is positive small and \( \theta' \) is negative small then \( i \) is zero.
7. If \( \theta \) is negative small and \( \theta' \) is positive small then \( i \) is zero.
8. If \( \theta \) is positive small and \( \theta' \) is positive small then \( i \) is negative medium.
9. If \( \theta \) is negative small and \( \theta' \) is negative small then \( i \) is positive medium.
Pendulum Problem

- Fuzzify (get $\mu$ values from profiles)
  $\mu_{\text{zero}}(1^\circ) = 0.7$
  $\mu_{\text{+ve small}}(1^\circ) = 0.3$
  $\mu_{\text{zero}}(0.5^\circ/\text{sec}) = 0.8$
  $\mu_{\text{+ve small}}(0.5^\circ/\text{sec}) = 0.4$

- **Rule 1**
  LHS = $\mu_{\text{zero}}(1^\circ)$ and $\mu_{\text{zero}}(0.5^\circ/\text{sec})$
  $= \min(1^\circ) = 0.7$
  RHS = $\mu_{\text{zero}}(i) = 0.7$ (assuming $c = 1$ in L. Rule)

- **Rule 2**
  RHS = $\mu_{\text{-ve small}}(i) = 0.3$

- **Rule 3**
  RHS = $\mu_{\text{-ve small}}(i) = 0.4$

- **Rule 4**
  RHS = $\mu_{\text{-ve medium}}(i) = 0.3$
Defuzzification

• To get crisp values
  1. Read values off the Y-axis
  2. Project onto X-axis
  3. Decide the values
• Step 1 and 2 above give rise to two values
• Two strategies to choose values for this step
  a) Max of min
  b) Centroid method
• Centroid method:
  – $\alpha$-cut is the area below the curve cut at the $\mu$ value
  – Required current = X-axis value of centroids of areas $A_1, A_2, ...$
  – Required $(X_c, Y_c)$ is summation of values of centroid
    \[
    X_c = \frac{\sum(x.A)}{\sum A} \quad \text{and} \quad Y_c = \frac{\sum(y.A)}{\sum A}
    \]