CS 344
Artificial Intelligence

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Formal Systems

- Rule governed
- Strict description of structure and rule application
- Well formed formulae
- Inference rules
- Assignment of semantics
- Notion of proof
- Notion of soundness, completeness, consistency, decidability etc.

An example System

Hilbert's formalization of propositional calculus

1. Elements are propositions : Capital letters

2. Operator is only one : $\rightarrow$ (called implies)

3. Special symbol ϕ (called 'false')

4. Two other symbols : $'('$ and $')'$

5. Well formed formula is constructed according to the grammar

   $WFF \rightarrow P \mid \phi \mid (WFF \rightarrow WFF)$

6. Inference rule : only one

   Given $A \rightarrow B$ and A write B - known as MODUS PONENS

7. Axioms : Starting structures

A1: $(A \to (B \to A))$

A2: $((A \to (B \to C)) \to ((A \to B) \to (A \to C)))$

A3 $(((A \to F) \to F) \to A)$

This formal system defines the propositional calculus

Notion of proof

1. Sequence of well formed formulae
2. Start with a set of hypotheses
3. The expression to be proved should be the last line in the sequence
4. Each intermediate expression is either one of the hypotheses or one of the axioms or the result of modus ponens
5. An expression which is proved only from the axioms and inference rules is called a THEOREM within the system

Example of proof

From P and $P \rightarrow Q$ and $Q \rightarrow R$, prove R

H1: P

H2: $P \rightarrow Q$

H3: $Q \rightarrow R$

i)	P	H1
ii)	$P \rightarrow Q$	H2
iii)	Q	MP, (i), (ii)
iv)	$Q \rightarrow R$	H3
v)	R	MP, (iii), (iv)

Prove that $(P \to P)$ is a THEOREM

i) $P \to (P \to P)$ A1 : P for A and B

ii) $P \to ((P \to P) \to P)$ A1: P for A and $(P \to P)$ for B

iii) $[(P \to ((P \to P) \to P)) \to ((P \to (P \to P)) \to (P \to P))]$

A2: with P for A, $(P \to P)$ for B and P for C

iv) $(P \to (P \to P) \to (P \to P))$ MP, (ii), (iii)

v) $(P \to P)$ MP, (i), (iv)