CS344: Introduction to Artificial Intelligence

(associated lab: CS386)

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Lecture–3: Fuzzy Inferencing: Inverted Pendulum
Inferencing

- Two methods of inferencing in classical logic
  - Modus Ponens
    - Given $p$ and $p \rightarrow q$, infer $q$
  - Modus Tolens
    - Given $\sim q$ and $p \rightarrow q$, infer $\sim p$

- How is fuzzy inferencing done?
A look at reasoning

- Deduction: \( p, p \rightarrow q \mid - q \)
- Induction: \( p_1, p_2, p_3, \ldots \mid - \text{for_all } p \)
- Abduction: \( q, p \rightarrow q \mid - p \)
- Default reasoning: Non-monotonic reasoning: Negation by failure
  - If something cannot be proven, its negation is asserted to be true
  - E.g., in Prolog
Completeness and Soundness

- Completeness question
  - Provability - Is the machine powerful enough to establish a fact?

- Soundness – Anything that is proved to be true is indeed true
  - Truth - Is the fact true?
Fuzzy Modus Ponens in terms of truth values

- Given \( t(p)=1 \) and \( t(p \rightarrow q)=1 \), infer \( t(q)=1 \)
- In fuzzy logic,
  - given \( t(p) \geq a, \ 0 \leq a \leq 1 \)
  - and \( t(p \rightarrow >q)=c, \ 0 \leq c \leq 1 \)
- What is \( t(q) \)
- How much of truth is transferred over the channel

\[
p \quad \rightarrow \quad q
\]
Lukasiewitz formula for Fuzzy Implication

- \( t(P) \) = truth value of a proposition/predicate. In fuzzy logic \( t(P) = [0,1] \)
- \( t(P \rightarrow Q) = \min[1, 1 - t(P) + t(Q)] \)

Lukasiewitz definition of implication
Use Lukasiewicz definition

- $t(p \rightarrow q) = \min[1, 1 - t(p) + t(q)]$
- We have $t(p \rightarrow q) = c$, i.e., $\min[1, 1 - t(p) + t(q)] = c$
- Case 1:
  - $c = 1$ gives $1 - t(p) + t(q) \geq 1$, i.e., $t(q) \geq a$
  - Otherwise, $1 - t(p) + t(q) = c$, i.e., $t(q) \geq c + a - 1$
- Combining, $t(q) = \max(0, a + c - 1)$
- This is the amount of truth transferred over the channel $p \rightarrow q$
**ANDING of Clauses on the LHS of implication**

\[ t(P \land Q) = \min(t(P), t(Q)) \]

Eg: If pressure is high then Volume is low

\[ t(\text{high}(\text{pressure}) \rightarrow \text{low}(\text{volume})) \]
Fuzzy Inferencing

Core

The Lukasiewicz rule

\[ t( P \rightarrow Q ) = \min[1, 1 + t(P) - t(Q)] \]

An example

Controlling an inverted pendulum

\[ \dot{\theta} = \frac{d\theta}{dt} = \text{angular velocity} \]

[Diagram of an inverted pendulum with a motor labeled as 'Motor' and current 'i' indicated]
The goal: To keep the pendulum in vertical position ($\theta=0$) in dynamic equilibrium. Whenever the pendulum departs from vertical, a torque is produced by sending a current ‘$i$’

Controlling factors for appropriate current

Angle $\theta$, Angular velocity $\theta^\prime$

Some intuitive rules

If $\theta$ is +ve small and $\theta^\prime$ is –ve small
then current is zero

If $\theta$ is +ve small and $\theta^\prime$ is +ve small
then current is –ve medium
## Control Matrix

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<th></th>
<th>-ve med</th>
<th>-ve small</th>
<th>Zero</th>
<th>+ve small</th>
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</table>
Each cell is a rule of the form

If \( \theta \) is \( <> \) and \( \theta' \) is \( <> \)

then \( i \) is \( <> \)

4 “Centre rules”

1. if \( \theta = Zero \) and \( \theta' = Zero \) then \( i = Zero \)
2. if \( \theta \) is +ve small and \( \theta' = Zero \) then \( i \) is –ve small
3. if \( \theta \) is –ve small and \( \theta' = Zero \) then \( i \) is +ve small
4. if \( \theta = Zero \) and \( \theta' \) is +ve small then \( i \) is –ve small
5. if \( \theta = Zero \) and \( \theta' \) is –ve small then \( i \) is +ve small
Linguistic variables

1. Zero
2. +ve small
3. -ve small

Profiles
Inference procedure

1. Read actual numerical values of $\theta$ and $\theta^\prime$.
2. Get the corresponding $\mu$ values $\mu_{\text{Zero}}$, $\mu_{(\text{+ve small)}}$, $\mu_{(\text{-ve small)}}$. This is called FUZZIFICATION.
3. For different rules, get the fuzzy $i$ values from the R.H.S of the rules.
4. “Collate” by some method and get ONE current value. This is called DEFUZZIFICATION.
5. Result is one numerical value of $i$. 
if $\theta$ is Zero and $d\theta/dt$ is Zero then $i$ is Zero
if $\theta$ is Zero and $d\theta/dt$ is +ve small then $i$ is –ve small
if $\theta$ is +ve small and $d\theta/dt$ is Zero then $i$ is –ve small
if $\theta$ +ve small and $d\theta/dt$ is +ve small then $i$ is -ve medium

**Rules Involved**
Suppose $\theta$ is 1 radian and $d\theta/dt$ is 1 rad/sec

- $\mu_{\text{zero}}(\theta = 1) = 0.8$ (say)
- $\mu_{\text{+ve-small}}(\theta = 1) = 0.4$ (say)
- $\mu_{\text{zero}}(d\theta/dt = 1) = 0.3$ (say)
- $\mu_{\text{+ve-small}}(d\theta/dt = 1) = 0.7$ (say)
Fuzzification

Suppose \( \theta \) is 1 radian and \( \frac{d\theta}{dt} \) is 1 rad/sec

- \( \mu_{\text{zero}}(\theta = 1) = 0.8 \) (say)
- \( \mu_{\text{+ve-small}}(\theta = 1) = 0.4 \) (say)
- \( \mu_{\text{zero}}(\frac{d\theta}{dt} = 1) = 0.3 \) (say)
- \( \mu_{\text{+ve-small}}(\frac{d\theta}{dt} = 1) = 0.7 \) (say)

if \( \theta \) is Zero and \( \frac{d\theta}{dt} \) is Zero then i is Zero

\[ \min(0.8, 0.3) = 0.3 \]

hence \( \mu_{\text{zero}}(i) = 0.3 \)

if \( \theta \) is Zero and \( \frac{d\theta}{dt} \) is +ve small then i is –ve small

\[ \min(0.8, 0.7) = 0.7 \]

hence \( \mu_{\text{-ve-small}}(i) = 0.7 \)

if \( \theta \) is +ve small and \( \frac{d\theta}{dt} \) is Zero then i is –ve small

\[ \min(0.4, 0.3) = 0.3 \]

hence \( \mu_{\text{-ve-small}}(i) = 0.3 \)

if \( \theta \) +ve small and \( \frac{d\theta}{dt} \) is +ve small then i is -ve medium

\[ \min(0.4, 0.7) = 0.4 \]

hence \( \mu_{\text{-ve-medium}}(i) = 0.4 \)
Finding $i$

Possible candidates:
- $i=0.5$ and $-0.5$ from the "zero" profile and $\mu=0.3$
- $i=-0.1$ and $-2.5$ from the "-ve-small" profile and $\mu=0.3$
- $i=-1.7$ and $-4.1$ from the "-ve-small" profile and $\mu=0.3$
Defuzzification: Finding \( i \) by the *centroid* method

Possible candidates:

\( i \) is the x-coord of the centroid of the areas given by the *blue trapezium*, the *green trapeziums* and the *black trapezium*.