CS344: Introduction to Artificial Intelligence
(associated lab: CS386)

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Lecture–5 and 6: Propositional Calculus and Co-operative Puzzle Solving
Propositional Calculus and Puzzles
Propositions

- Stand for facts/assertions
- Declarative statements
  - As opposed to interrogative statements (questions) or imperative statements (request, order)

Operators

\[
\text{AND}(\wedge), \text{OR}(\lor), \text{NOT}(\neg), \text{IMPLICATION}(\Rightarrow)
\]

\(\Rightarrow\) and \(\neg\) form a minimal set (can express other operations)
  - Prove it.

Tautologies are formulae whose truth value is always T, whatever the assignment is
**Model**

In propositional calculus any formula with $n$ propositions has $2^n$ models (assignments).
- Tautologies evaluate to $T$ in all models.

Examples:

1) $P \lor \neg P$

2) $\neg(P \land Q) \iff (\neg P \lor \neg Q)$

- e Morgan with AND
Semantic Tree/Tableau method of proving tautology

Start with the negation of the formula

$$\neg[-(P \land Q) \Rightarrow (\neg P \lor \neg Q)] \quad \text{- $\alpha$ - formula}$$

$$\neg (P \land Q) \quad \text{β - formula}$$

$$\neg (\neg P \lor \neg Q) \quad \text{- $\alpha$ - formula}$$

$p$

$q$

$p$

$q$
Example 2:

\[ \neg[ A \land (B \lor C) \Rightarrow (A \land B) \lor (A \land C)] \]

(\(\alpha\) - formula)

\[ A \land (B \lor C) \]

(\(\alpha\) - formulae)

\[ \neg((A \land B) \lor (A \land C)) \]

(\(\beta\) - formulae)

\[ \neg(A \land B) \]

\[ \neg(A \land C) \]

Contradictions in all paths
A puzzle
(Zohar Manna, Mathematical Theory of Computation, 1974)

From Propositional Calculus
Tourist in a country of truth-sayers and liers

- Facts and Rules: In a certain country, people either always speak the truth or always lie. A tourist T comes to a junction in the country and finds an inhabitant S of the country standing there. One of the roads at the junction leads to the capital of the country and the other does not. S can be asked only yes/no questions.

- Question: What single yes/no question can T ask of S, so that the direction of the capital is revealed?
Diagrammatic representation

Capital

S (either always says the truth
Or always lies)

T (tourist)
Deciding the Propositions: a very difficult step- needs human intelligence

- P: Left road leads to capital
- Q: S always speaks the truth
Meta Question: What question should the tourist ask

- The form of the question
- Very difficult: needs human intelligence
- The tourist should ask
  - *Is R true?*
  - *The answer is “yes” if and only if the left road leads to the capital*
  - *The structure of R to be found as a function of P and Q*
A more mechanical part: use of truth table

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>S’s Answer</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>Yes</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>Yes</td>
<td>F</td>
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<td>F</td>
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<td>No</td>
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<td>F</td>
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<td>T</td>
</tr>
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</table>
Get form of $R$: quite mechanical

- From the truth table
  - $R$ is of the form $(P \ x\text{-nor} \ Q)$ or $(P \equiv \ Q)$
Get $R$ in English/Hindi/Hebrew...

- Natural Language Generation: non-trivial
- The question the tourist will ask is
  - *Is it true that the left road leads to the capital if and only if you speak the truth?*
- Exercise: A more well known form of this question asked by the tourist uses the X-OR operator instead of the X-Nor. What changes do you have to incorporate to the solution, to get that answer?
Another Similar Problem

*From Propositional Calculus*
Another tourist example: this time in a restaurant setting in a different country (Manna, 1974)

- **Facts:** A tourist is in a restaurant in a country when the waiter tells him:
  - “do you see the three men in the table yonder? One of them is X who always speaks the truth, another is Y who always lies and the third is Z who sometimes speaks the truth and sometimes lies, i.e., answers yes/no randomly without regard to the question.

- **Question:** Can you (the tourist) ask three yes/no questions to these men, always indicating who should answer the question, and determine who of them is X, who y and who Z?
Solution: Most of the steps are doable by humans only

- Number the persons: 1, 2, 3
  - 1 can be X/Y/Z
  - 2 can be X/Y/Z
  - 3 can be X/Y/Z

- Let the first question be to 1
- One of 2 and 3 has to be NOT Z.
- Critical step in the solution: only humans can do?
Now cast the problem in the same setting as the tourist and the capital example

- Solving by analogy
- Use of previously solved problems
- Hallmark of intelligence
Analogy with the tourist and the capital problem

- Find the direction to the capital
  - Find Z; who amongst 1, 2 and 3 is Z?
- Ask a single yes/no question to S (the person standing at the junction)
  - Ask a single yes/no question to 1
- Answer forced to reveal the direction of the capital
  - Answer forced to reveal who from 1, 2, 3 is Z
Question to 1

- Ask “Is R true” and the answer is yes if and only if 2 is not Z.

- Propositions
  - P: 2 is not Z
  - Q: 1 always speaks the truth, i.e., 1 is X
Use of truth table as before

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>Yes</td>
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Question to 1: the first question

- *Is it true that 2 is not Z if and only if you are X?*
Analysis of 1’s answer

- **Ans= yes**
  - Case 1: 1 is X/Y (always speaks the truth or always lies)
    - 2 is indeed not Z (we can trust 1’s answer)
  - Case 2: 1 is Z
    - 2 is indeed not Z (we cannot trust 1’s answer; but that does not affect us)
Analysis of 1’s answer (contd)

- Ans = no
  - Case 1: 1 is X/Y (always speaks the truth or always lies)
    - 2 is Z; hence 3 is not Z
  - Case 2: 1 is Z
    - 3 is not Z

Note carefully: how cleverly Z is identified. Can a machine do it?
Next steps: ask the 2\textsuperscript{nd} question to determine $X/Y$

- Once “Not Z” is identified- say 2, ask him a tautology
  - Is $P \equiv P$
    - If yes, 2 is $X$
    - If no, 2 is $Y$
Ask the 3rd Question

- Ask 2 “is 1 Z”
- If 2 is X
  - Ans=yes, 1 is Z
  - Ans=no, 1 is Y
- If 2 is Y (always lies)
  - Ans=yes, 1 is X
  - Ans=no, 1 is Z
- 3 is the remaining person
What do these examples show?

- Logic systematizes the reasoning process
- Helps identify what is mechanical/routine/automatable
- Brings to light the steps that only human intelligence can perform
  - These are especially of foundational and structural nature (e.g., deciding *what propositions to start with*)
- Algorithmizing reasoning is not trivial