HomeWork-1. Formulation and Review of Complexity.

1) Suppose k trucks can be used to serve n clients from a single depot.Each client must be visited once. The time for truck k to travel from i to j is c_{ijk}. The tour of each truck cannot take longer than L_k. Formulate the problem of finding a feasible schedule.

2) 1kg Chicken tandoori contains 100 units proteins, 40 units fats, 30 units cholestrol and overall 60 energy units. 1kg Rice contains 10 units proteins, 20 units fats and 100 units energy. 1kg Curd contains 60 units proteins, 40 units fats, 20 units cholestrol and 40 units energy. In the time in consideration a person cannot eat more than 5kg of food. Also it is necessary to eat 250 units proteins and 80 units fats. It is dangerous to consume more than 100 units of cholestrol.
Formulate an LP so that the person gets as much energy as possible.

3) {ref: Nemhauser/Wolsey}
A delivery company must make deliveries to 10 customers whose respective demands are d_j for j=1,2,...10. The company has 4 trucks with capacities L_k and operation costs c_k for k=1,2,3,4. A single truck cannot deliver to more than 5 customers, and customer pairs {1,7}, {2,6} and {2,9} cannot be delivered by the same truck. Formulate a model to determine which trucks to use as to minimize the cost of delivering to all customers.

4) G = (V,E) is a CONNECTED graph. The vertices and edges both are weighted.
Formulate integer linear programs for the following:
a) Find a maximum-weighted tree.
b) Find a minimum-weighted collection of edges which cover (include) all the vertices.
c) Find a maximum-weighted collection of vertex-disjoint edges.
i.e. No two edges in the collection should be incident on the same vertex.

5) In the course cs301, you people have learnt what NP-Completeness is. Use the basic ideas to show the following:
Problem :
We consider the following integer linear program. Ax=b.
Integer linear program is one in which the solutions are all integers.
Hence x is a vector of positive integers. We have a cost function y=cx.
Minimize y. Is the value of y less than some constant k?

a) Show that this prolem is NP-complete.
note: it is ok to show that in general ILPs are NP-hard.
b) Show this using two ways (different NP problems) if you can.
c) Show that an Integer Linear Program is NP-hard using SAT.
d) Can we say the same (NP-hardness?) for a general LP?