CS 350 Quiz 1(5%) /10

10/2/05 This is a question_paper-cum-answer-sheet; Answer only in the space provided.
In True/False, circle your choice.

1.(a) Let $P_1,\ldots,P_k$ be a collection of k non-zero vectors in R^n such that P_i is perpendicular to P_j for all i#j. Then, P_i (i arbitrary) can be expressed as a linear combination of the remaining P_j , j#i.

/0.5+1

~~True~~ / False because _if $p_i = \sum_{j \neq i} \lambda_j * p_j$, then the dot product $<p_i, p_i>$ satisfies $0 < <p_i, p_i> = <\sum_{j \neq i} \lambda_j p_j, p_i> = 0$ (why) Contradiction

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(b) Let f : R^n --> R be a convex function. Then, the set $S_b = \{x \mid x \; R^n, f(x) \leq b\}$ is convex..

/0.5+1.5

True / ~~False~~ because :- $x^1 \varepsilon S_b, x^2 \varepsilon S_b$

$f(\lambda x^1+(1-x)x^2) \leqq \lambda f(x^1) + (1-\lambda) f(x^2)$
$\leqq \lambda b+(1-\lambda)b = b$

(c) A square matrix $A = (a_{ij})$ whose elements are {0,1} and which can be transformed to the identity matrix by rearranging its row vectors is called a Permutation Matrix. Show that $AA^T = I$ /1.5

A permutation matrix has only one 1 in each row and no two 1s in the same column.

Let $A=(a_{ij})$; $A^T=(a`_{ij})=(a_{ji})$. Let $B=AA^T$

Then, $b_{ij} = \Sigma\, a_{ik} a_{kj} = \Sigma a_{ik}\; a_{jk} = \{ \begin{matrix} 1, & i=j \\ 0, & \text{otherwise} \end{matrix}$

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2. Bring the following system into canonical form with respect to (x_1, x_2) by two pivot steps. /3

$2 x_1 - 3x_2 + 4x_3 - 2x_4 = 8$
$6 x_1 +5x_2 - 7x_3 - x_4 = 4$

$x_1 - 3/2\, x_2+2x_3 - x_4 = 4$
$14x_2 - 19x_3 + 5x_4 = -20$

$x_1 - 1/28\, x_3 - 13/28\, x_4 = 13/7$
$x_2 - 19/14\, x_3 + 5/14\, x_4 = -10/7$

Now bring x_3 in place of x_1 /2

$-28 x_1 + x_3 + 13x_4 = -52$
$-38 x_1 + x_2 + 18x_4 = -72$