CS350 **Quiz-2** **RollNo.:______________** **/15**

4.4.05

QuestionPaper-cum-Answer Sheet : Answer only in the space provided

Solution *(Grading Algo. given in italics in each question)*

Q 1. Given the LP:- **/8**

$$\text{Min.} \quad -2x_1 - 3x_2 - x_3$$

s.t.

$$\frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 \le 1$$

$$\frac{1}{3}x_1 + \frac{4}{3}x_2 + \frac{7}{3}x_3 \le 3$$

$$x \ge 0$$

(a) The Dual LP is:- **/3**

Let $p = (p_1, p_2)$ be the vector of dual variables. Each primal variable gives rise to a dual constraint.

The Dual LP (DP) is:-

$$\text{Max.} \quad p_1 + 3p_2$$

s.t.

$$\frac{1}{3}p_1 + \frac{1}{3}p_2 \le -2$$

$$\frac{1}{3}p_1 + \frac{4}{3}p_2 \le -3$$

$$\frac{1}{3}p_1 + \frac{7}{3}p_2 \le -1$$

$$(p_1, p_2) \le 0$$

$(p_1,p_2) \le 0$ … 1

3 const.+1 obj. … 4 x ½=2

(b) Using the Weak Duality Theorem, a lower bound to (P) is: <u>-9</u> (corr. to feasible p =(-9,0)) **/1**

(c) Verify using CS conditions on (P) and (DP) that $x=(1,2,0)^T$ is optimal. **/2**

$x_1^* > 0 \Rightarrow \frac{1}{3}p_1^* + \frac{1}{3}p_2^* = -2$

$x_2^* > 0 \Rightarrow \frac{1}{3}p_1^* + \frac{4}{3}p_2^* = -3$

$\Rightarrow p^* = (-5,-1)$

Also: $\bar{c}_3 = -1 - (-5/3, -7/3) > 0 \Rightarrow x_3^* = 0$ by CS (holds)

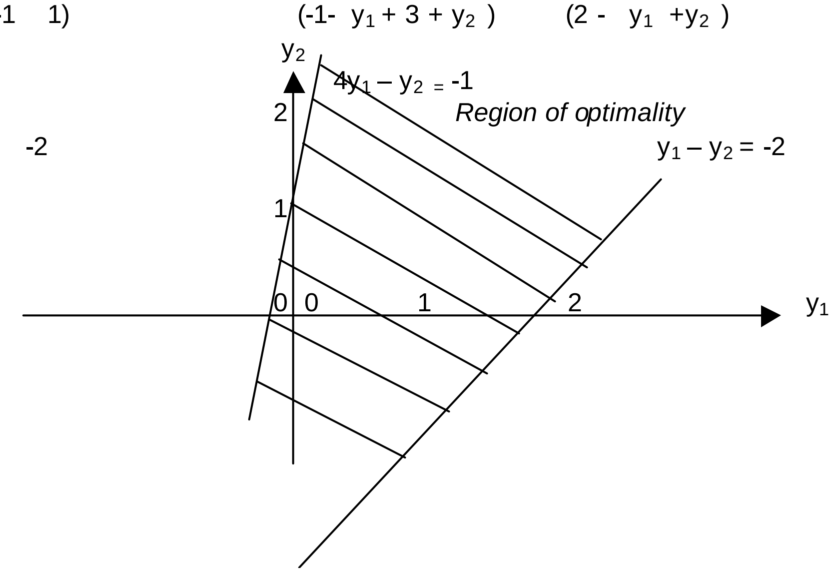
2 x ½ =1, $\bar{c}_3$ w/cs =1

$c\,x^* = -2 - 6 = -8$; $p^*b = -5 - 3 = -8$, exhibiting that for this (x^*, p^*) the resp. optimal values are equal which also demonstrates the use of the Duality Theorem.

(d) Let $b_1 = 1 + y_1$ and $b_2 = 3 + y_2$. Plot in 2 dimensions the values for which the basis optimal in part (c) is still optimal. **/2**

$B = \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 4/3 \end{pmatrix}$ $B_{inv} = \begin{pmatrix} 4 & -1 \\ -1 & 1 \end{pmatrix}$ $B_{inv}\,b = \begin{pmatrix} 4+4y_1 - 3 - y_2 \\ -1 - y_1 + 3 + y_2 \end{pmatrix} = \begin{pmatrix} 1 + 4y_1 - y_2 \\ 2 - y_1 + y_2 \end{pmatrix} \ge 0$

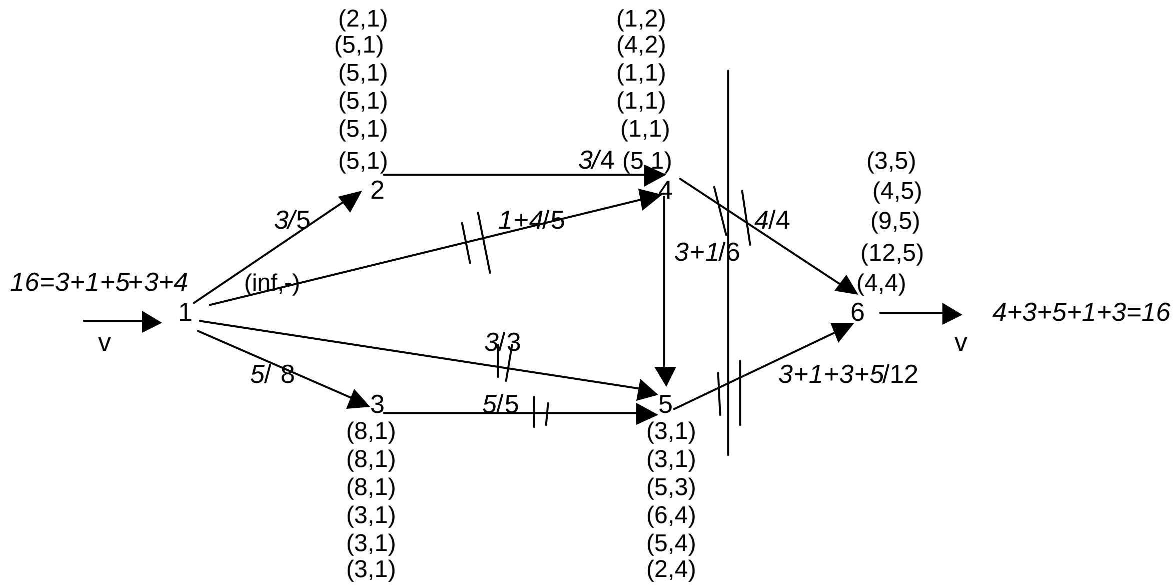
$4y_1 - y_2 \ge -1$ and $y_1 - y_2 \ge -2$



B_{inv} …1/2, $B_{inv}b$…1, plot…1/2

Q 2 Given the following network <N,A|0,k> where node 1 is the source and 6 the sink, all arc lower bounds are zero and their respective upper bounds given adjacent to the arcs, and v the flow through the network:-

/**7**[4+(1+2)]
(a)6x1/2…3+v…1/2+mincut…1/2;(b)(i)2x1/2…1;(ii)2x1=2*



(a) Find the max flow through the network using the labeling algorithm. Show the successive labels on the above diagram itself, as well as a min-cut set.

(b) Now we are permitted to increase the upper bound of (only) *one* arc by any amount so as to *increase max v the most*: (i) identify the subset of arcs and why? {(4,6),(5,6)}, because this is the saturated min cut-set

(ii) which arc and by how much? Note: You can start from the final set of labels obtained under (a).

$v^*(k_{46}=\text{inf}) - v^*(k_{46}=4) = 1$ (Note that the min cut-set is:{(2,4),(1,4),(1,5),(3,5)}

$v^*(k_{56}=\text{inf}) - v^*(k_{56}=12) = 1$ (Note that the min cut-set is again:{(2,4),(1,4),(1,5),(3,5)}

Hence, we can increase either of the two arcs (4,6) or (5,6) by 1 unit.