Dept. of Compter Sc. & Engineering, Indian Institute of Technology Bombay
CS 350 **End-Semester Examination** ***SOLUTION*** **Roll_No.: sak** **/50**
20.4.05. 0930-1230 hrs.

Instructions:- 1. QuestionPaper-cum-Answer Sheet : Answer only in the space provided;
2. You may refer to notes in your own handwriting;
3. In True(T)/False(F) questions, *cancel* out by a / what you consider inadmissible.

Q 1. Short Questions. Negative mark (- 0.5) for each incorrect answer. **/11**

(a) A problem with the "Big-M" method for solving a linear program could be numerical instability **/1**

(b) A stopping criterion in the Interior Point Method is based on duality gap. **/1**

(c) If a totally unimodular square matrix A is invertible, its inverse is unimodular: T / ~~F~~ **/3**
Why? Because (i) Each element of A_{inv} is 0, $\pm$ 1 because |cofactors| are 0, $\pm$ 1
(ii) $|A_{inv}| = \pm 1$, since $|A| = \pm 1$

(d) Saddle point always exists for a two-person zero-sum game: ~~T~~/F **/1**

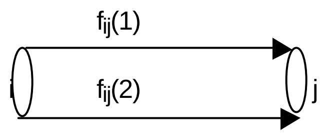
(e) The dual to the LP: {Min z| cx – z $\le$ 0, A x = b, x $\ge$ 0} is : {Max p.b | p e R^n ,q e R, pA + q c $\le$0, -q = 1) **/2**

which is the same as the dual of : {Min cx | A x = b, x $\ge$ 0 }

(f) Prove that if an (m x n) matrix A with m < n has full row rank, then $\text{rank}(AA^T) = m$ **/3**
Suppose that $\text{rank}(AA^T) < m$ => There exists a non-zero vector say y^T s.t. $AA^Ty^T = 0$
Therefore, $yAA^Ty^T = 0$ which is a contradiction since yA can not be zero (*why?*)
Hence, $\text{rank}(AA^T) = m$

Q 2. **/13**

(a) A fragment of a network <N,A | 0, k, c> is given below. Let $f^*_{ij}(1)$ and $f^*_{ij}(2)$ be an optimal set of flow assignments on the upper and lower (i,j) arcs whose capacities are $k_{ij}(1)$ and $k_{ij}(2)$. Assume that $C_{ij}(1) < C_{ij}(2)$. Then fill in the blanks with justification:- *an optimal assignment on the (i,j) arcs has the property:-* **/4**

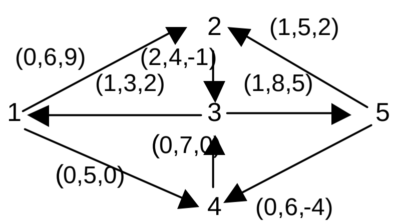


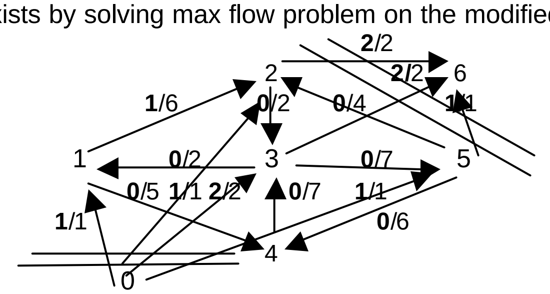
$f^*_{ij}(1) < k_{ij}(1)$ => $\underline{f^*_{ij}(2) = 0}$

$f^*_{ij}(2) > 0$ => $\underline{f^*_{ij}(1) = k_{ij}(1)}$

Justification (you can show by flow adjustment):- Suppose the * solution does not satisfy the above conditions; then make flow adjustment of ? at node i, maintaining conservation and decreasing the cost of pushing flow from i to j, contradiction as the flow assignment had been assumed to be optimal.

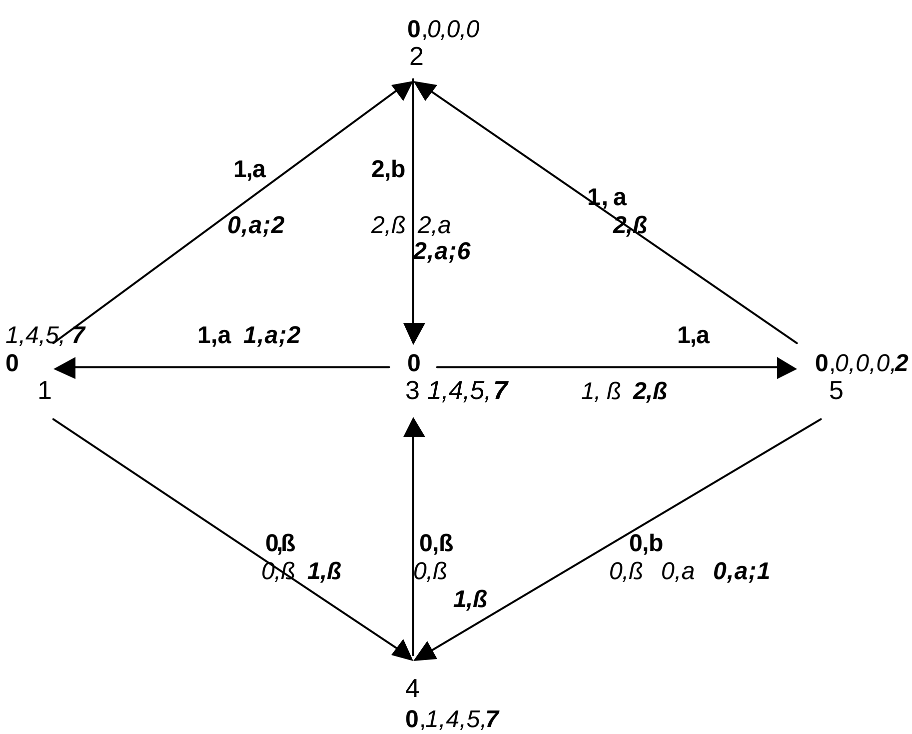
(b) Given the closed network below, find an optimal flow assignment **/5+4**
(Note:- obtain a feasible flow assignment if it exists by solving max flow problem on the modified network)





Transformed network with zero lower bounds, flow assignments and sat cut-sets

In the diagram below, the **init labels in bold** are the pi's at nodes and states on the edges.



Select the OK arc (1,2) and try flow re-routing; cut-set:X={1,3,4} gives d=1**;** *new labels in italics* at nodes and where changed on edges. Again no-rerouting possible, X={1,3,4} with d=3**;** (2,3) and (5,4) **changed labels** given in **ordinary bold.** Again no flow readjustment possible, X same with d=1. New *changed labels* are given in *italics*. One arc (1,2) ok and again flow re-routing not possible with X={1,3,4,5} with d=2. aThe new node labels are shown in ***bold italics*** and the edge labels after the flow-adjustment now possible shown also in bold italics with updated flows.
All arcs in kilter and optimum reached. ***Check***:- Is the duality gap=0? Primal Obj = 14; Dual Obj (determine the phi values, psi being zero, *why?*) = 14 (phi values given on the edges adj. to the final edge state after the semi-colon for non-beta states).

Q 3. Roll_No:______________ **/13**

(a) Show that in an (n x n) optimal assignment problem, at least one candidate is assigned to the job for which he is best qualified. / **4**

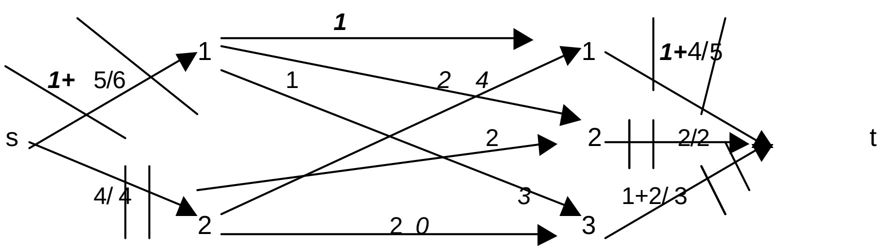
Suppose not. (Note that considering it as a LP, there are (2n-1) constraints and n^2 variables with a high degree of degeneracy.) Let, as usual, u be the dual variable vector corresponding to the "persons" and v the dual variable vector corresponding to the m/cs. Clearly there do not exist (u.v) such that the dual constraints are: $u + v \leq c$ for this solution, CS conditions do not hold, and hence the solution non-optimal.

(b) Solve the following transportation problem: **/9**

			$\{a_i\}$		
$\{c_{ij}\}$	6 2 4	6		0 2 4 5 6
	3 0 2	4		0 2 2 3 3
$\{b_j\}$	5 2 3			

		v	u
Iteration: 0		0 0 0	
1	I={1,2},J={2}, delta= 2	0 -2 0	
2	I={1}, J=Null, delta=2	0 -2 0	
3	I={1,2}, J={2,3},delta =1	0 -3 -1 (new flow in italics i->j)	
4	I={1}, J=(2,3}, delta=1	0 -4 -2 **Optimum reached** *(check duality gap=0)*	

(The last flow assignmen shown in ***bold italics.*** *Shipping Cost = 34)*



Q 4. Consider a two-person zero sum game with `m' actions for the row player R, `n' actions for the column player C, the (mxn) pay-off matrix A =(a_{ij}) represents the pay-off to R by C, when R chooses action action i and C chooses action j, and the two players play conservatively. (You may assume that A has all non- negative entries if required.) **/13**

We say that for C, action `k' dominates action `j' if a $a_{ik} \leq a_{ij}$ for all i; i.e., C will not be worse off whatever strategy R employs. Similarly, for R, action `r' dominates action `i' if $a_{rj} \geq a_{ij}$ for all j.

(a) Show that there exist optimal actions of the row and column players of an original game A which is the same as the game A' in which the dominated columns and rows (as defined) are deleted. ***/ 4***

Let D_R and D_C be the sets of dominated rows and columns, respectively. Then, consider the expected pay-off expressions as in the Von Neumann's Theorem discussed in the class:-

$$\max_{x} \min_{y|x} \sum_{j} \sum_{i} a_{ij} x_{ij} y_{ij} = \max_{x} \min_{j} \sum_{i} a_{ij} x_{ij} = \max_{x} \min_{j\_not\_in\_D_C} \sum_{i} a_{ij} x_{ij} \ \textit{(why?)} = \min_{y} \max_{i\ not\_in\_D_R} \sum_{j} a_{ij} y_{ij}$$

Similar to the preceding

Note:- Check with the selection of pure strategies

(b) Given **/ 9**

$$A = \begin{vmatrix} 4 & 6 & 6 \\ 3 & 5 & 5 \\ 6 & 4 & 4 \\ 6 & 4 & 0 \end{vmatrix} \qquad D_C = \{2\} \quad D_R = \{2,4\}$$

(i) obtain the undominated matrix $A' = \begin{vmatrix} 4 & 6 \\ 6 & 4 \end{vmatrix}$

(ii) Solve the game on A', first verifying whether or not saddle point exists for A'.

Saddle point does NOT exist – *check*.

It can be easily seen that for the prime problem: x' = (½,½), y' = (½,½) is optimal to R' and C'
From the preceding, x = (½,0,½,0), y = (½,0,½) is optimal to R and C with value = 5
Note:- Check the two LPs