CS350 **Mid-semester Examination** **Roll No. ______________** **Marks**: **/40**

Solution

23.2.2005, 1430-1700 Hrs. *This is a Question_Paper-cum_Answer Sheet: Write only in the space provided*

1. *Instructions*:- (i) Fill in the blanks most appropriately where "___" given. (ii) where choice (e.g., T[*True*]/F[*False*] is given, specify your choice by deleting what you consider as inadmissible; *wrong choice* would receive – 0.5 marks. (iii) All notations, unless stated otherwise would be as used in the class. Each *correct choice* – 1 mark. ***/12***

(a) A subset S of R^n may be either convex or concave: T/ ~~F~~

(b) The normal to the hyperplane $\{x | x \varepsilon R^n,\ c.\ x = \alpha\}$ is given by c and points in the direction of increase/~~decrease~~ of the function c.x

(c) A convex function, not necessarily differentiable, can have discontinuities: T / ~~F~~

(d) An ellipsoid in R^n is convex and has infinite number of extreme points.

(e) Augmenting a system of equations with an equation that is a linear combination of others *reduces* the solution set: ~~T~~ / F

(f) In the application of the Simplex Algorithm to: $\{\min z = c.x \mid x \varepsilon R^n,\ Ax = b,\ x \geq 0\}$,

suppose that at some iteration: the (feasible) basis B; its inverse B^{-1}; the cost coefficients corresponding to the basic columns of B is c_B.

Then:

(i) the relative cost factor $\bar{c}_s$ for some column `s' in terms of c_s, c_B, B^{-1} and P_s is $c_s - c_B B^{-1} P_s$

(ii) if min z -> - infinity, then $\bar{c}_s (= c_s - c_B B^{-1} P_s)$ < 0 and $B^{-1} P_s$ ≤ 0

(iii) there exists a solution x^0 such that $x^0=(x_B, 0,.,0)' + \lambda\ (-B^{-1} P_s ,0, ..,1,0,\ldots, 0)$ $(\lambda > 0)$ satisfies $Ax = b, x \geq 0, c.x < 0$

(g) Degeneracy in linear programming means that at some iteration, there is one or more zeroes on the rhs and this may lead to circling.

Common Mistakes:- gradient of fn; reasoning to obtain the condition when min z tends to – infinity; Fn convexity; ext point concept; confusing degeneracy with tie for blocking row

2. A BPO unit runs four shifts and has the following personnel requirements (Req.):-

	Shift-1: 0000 – 0600 hrs.	Shift-2: 0600 – 1200 hrs.	Shift-3: 1200 – 1800 hrs.	Shift-4: 1800 – 2400 hrs.
Req.:	10	14	8	10

Note that personnel work two continuous shifts and could start work from the beginning of any of the four shifts, each day.

Let x_j be the number of personnel scheduled to work from the start of the shift-j, j=1,2,3,4. We wish to minimize the number of people over the day. **/16**

(a) Write a linear program to meet the requirement of each shift with minimum total personnel. **/4**

$$x_1 + x_4 \geq 10$$
$$x_1 + x_2 \geq 14$$
$$x_2 + x_3 \geq 8$$
$$x_3 + x_4 \geq 10$$
$$(x_1, x_2, x_3, x_4) \geq 0$$
$$x_1 + x_2 + x_3 + x_4 = z \text{ (min.)}$$

Grading Notes:- Constraints:4 x ½=2; non-neg: 1; obj.: 1
Common Mistakes: Meaning of each day => operation repeats

(b) Bring the above system into *standard* form by introducing surplus variables {x_j , j = 5,6,7,8}, respectively for the periods 1, 2, 3 and 4. Represent this standard in tableau form. **/2**

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	(-z)		rhs
1	0	0	1	-1	0	0	0	0	=	10
1	1	0	0	0	-1	0	0	0	=	14
0	1	1	0	0	0	-1	0	0	=	8
0	0	1	1	0	0	0	-1	0	=	10
1	1	1	1	0	0	0	0	1	=	0

Grading Notes:- Constraints: 4 x ½ = 2
Common Mistakes:- not understanding "surplus" var to bring to standard form; unnec. multipliplying both sides by minus sign to "get" identity matrix

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(c) Bring the system into canonical form w.r.t. x_5, x_2 , x_7 , x_4 and (-z). Give values of π and B^{-1} on the tableau itself (with any required annotation). Comment on this solution. **/6**

Basic Var.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	(-z)		rhs
x_5	-1	0	1	0	1	0	0	-1	0	=	0
x_2	1	1	0	0	0	-1	0	0	0	=	14
x_7	1	0	-1	0	0	-1	1	0	0	=	6
x_4	0	0	1	1	0	0	0	-1	0	=	10
-z	0	0	0	0	0	1	0	1	1	=	24

- B_{inv} (boxed columns x_5–x_8, rows x_5–x_4)

π (boxed -z row, columns x_5–x_8)

Comment:- The solution is degenerate.

Grading Notes:- 5 rows (B_{inv} and rhs) x 1 = 5; show B_{inv} , π : 2 x ½ = 1; Comment (No or wrong)= - 1/2
Common Mistakes:- No comment on the soln; not using the given data of B to get B_{inv}; not giving right - B_{inv} , π

(d) Find an alternate optimal solution if it exists. *(Bring non-basic with rel.cost factor = 0)* **/4**

Basic Var.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	-z		rhs
x_3	-1	0	1	0	1	0	0	-1	0	=	0
x_2	1	1	0	0	-1	0	0	0	0	=	14
x_7	0	0	0	0	1	-1	1	-1	0	=	6
x_4	1	0	0	1	-1	0	0	0	0	=	10
-z	0	0	0	0	0	1	0	1	1	=	24

Grading Notes:- Introduce x_1 or x_3 : pivot col: 1; pivot row: 1; 2 rows to bring to can form: 2)
Common Mistakes:- Not stating the criterion (viz. that the entering var must have its rel. cost factor 0) to get alternate soln and the step to get the same; just writing the answer though the tableau is given

3. Suppose that the $S = \{x| x \varepsilon R^n, A\, x = b, x \geq 0 >$ is non-empty, but the point $p = (p_1, p_2, \ldots, p_n)'$ in not in S; i.e., it does satisfy the system $< A\, x = b, x \geq 0 >$.
Using Phase-I of the Simplex Algorithm, show that there exists a hyperplane that separates S and p.
Note:- First specify which system does *not* have a solution. Then, deduce the condition that end of Phase-I yields. **/12**

The system : $< A\, x = b,\ I\, x = p,\ x \geq 0 >$ has no solution.

By Phase-I, there exist multipliers γ and σ in R^n s.t.

$$\gamma A + \sigma \leq 0 \quad \text{and} \quad \gamma b + \sigma p > 0 \text{ (i.e, } \sigma p > -\gamma b) \qquad (1)$$

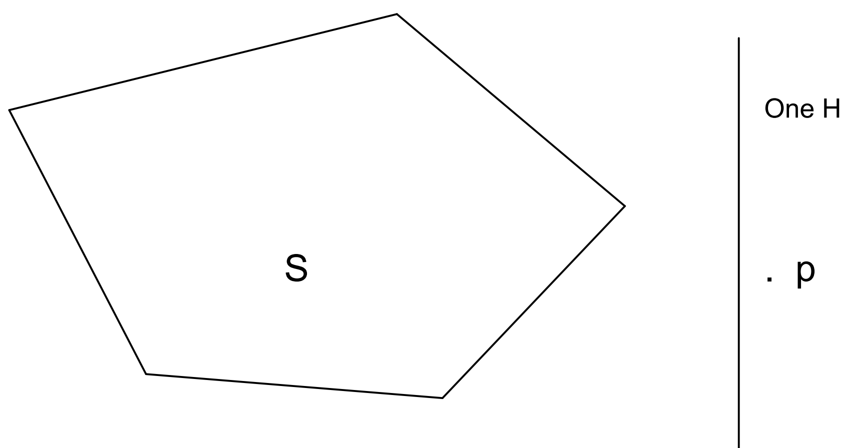
Multiplying both sides of the 1st inequality by $x \geq 0$, we get :

$$\gamma \; A\, x + \sigma\, x \leq 0$$

For x in S, the 1st inequality: $\gamma b + \sigma x \leq 0$; i.e., $\sigma x < -\gamma b$ (2)

The required hyperplane is : $\sigma x = \gamma b$, and the result follows.

Note:- This is called the Separating Hyperplane Theorem.



Grading Notes:- Writing correctly which sysem has no soln: 3; Writing the two Phase-I conditions: 2 + 1 = 3 and simplifying the latter 1; Using the 1st condition – 2 and simplifying it 1; identifying the condition for x in S: 1; The separating H:1 …. Totaling to 12

Common Mistakes:- Not understanding the problem defn; not proceeding step-by-step; Not understanding the Phase-I infeasibility condn.(or optimality at end of Phase-I); Wasting time by not concentrating on what was asked