An Efficient Data Dependence Analysis for Parallelizing Compilers

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Agenda

- Background
- Basic Concepts
- Algorithm
- Examples
- Conclusion
- References

Approaches in Dependence Testing

- Based on Numerical Methods, Inaccurate
- Multiple Dimensions simultaneously, Time Consuming
- λ test, Efficient and Accurate

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Numerical Methods

- Based on solving Diophantine Equations and the Bounds on real functions
- Data areas accessed by two array references are examined Dimension by Dimension
- Inaccurate

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Example

$$
\begin{array}{ll}
 & N = 50 \\
 & DO\ I = 1, N \\
 & \quad DO\ J = 2, N \\
S1: & \qquad A[2*I+3*J+N, 3*I+J+N-1] = ...(r1) \\
S2: & \qquad .... = A[I-J+N+1, 2*I-J+N-2] \quad (r2) \\
 & \quad ENDO \\
 & ENDDO
\end{array}
$$

- Let x_1 = I and x_2 = J for reference r1
- Let x_3 = I and x_4 = J for reference r2
- Diaphontine Equations are

  $2x_1 + 3x_2 - x_3 + x_4 = 1........(1)$
  $3x_1 + x_2 - 2x_3 + x_4 = -1........(2)$
  $where\ 1 \leqslant x_1, x_3 \leqslant 50, 2 \leqslant x_2, x_4 \leqslant 50$

Example (Cont...)

- Solution by previous Numerical Methods :-
    - Each Dimension is treated seperately
    - Any equation has no solution within loop bounds, no dependence
    - If each equation has solution independently, dependence has to be assumed
    - $(x_1, x_2, x_3, x_4) = (1,2,9,2)$ - solution to (1)
    - $(x_1, x_2, x_3, x_4) = (1,2,4,2)$ - solution to (2)
- So we assume dependence exists

Example (Cont...)

- Actual solution $\quad (1) \times 3 - (2) \times 2$
- Reduced Equation $\quad 7x_2 + x_3 + x_4 = 5$
- No solution
- Previous Numerical Methods failed because of presence **coupled subscripts**

Effect of *coupled subscripts* on determination of *dependence directions*

$$
\begin{array}{lll}
 & DO\ I = 1, 100 & \\
 & DO\ J = 1, 100 & \\
S1: & \quad A[I, J] = ...(3) & \\
S2: & \quad A[J, I] = ...(4) & \\
 & ENDO & \\
 & ENDO &
\end{array}
$$

$$\Downarrow$$

$$i_1 = j_2 ....(5)$$
$$j_1 = i_2 ....(6)$$
$$1 \leq i_1, i_2, j_1, j_2 \leq 100 ....(7)$$
$$i_1 = i_2, j_1 < j_2 ....(8)$$

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Effect of *coupled subscripts* on determination of *dependence directions (Cont...)*

- Previous Methods treat each dimension separately
- (5), (7), (8) is considered, solution is $i_1 = j_2 = 100$
- (6), (7), (8) is considered, solution is $j_1 = i_2 = 1$
- Actually there is no solution, if both equations are considered simultaneously
- Previous Methods failed because of presence of *coupled subscripts*

Notations used

1. Reference pair $(r1, r2)$
2. $r1$ is $A(f_1(i_1, i_2, ..., i_{l_1}), f_2(i_1, i_2, ..., i_{l_1}), ..., f_m(i_1, i_2, ..., i_{l_1}))$
3. $r2$ is $A(g_1(j_1, j_2, ..., j_{l_2}), g_2(j_1, j_2, ..., j_{l_2}), ..., g_m(j_1, j_2, ..., j_{l_2}))$

Basic Concepts

$$
\begin{aligned}
&f_1(i_1, i_2, \ldots, i_{l_1}) = g_1(j_1, j_2, \ldots, j_{l_2}) \\
&f_2(i_1, i_2, \ldots, i_{l_1}) = g_2(j_1, j_2, \ldots, j_{l_2}) \\
&\vdots \\
&f_m(i_1, i_2, \ldots, i_{l_1}) = g_2(j_1, j_2, \ldots, j_{l_2})
\end{aligned}
$$

$$
\begin{aligned}
&a_1^{(1)} v^{(1)} + a_1^{(2)} v^{(2)} + \cdots + a_1^{(n)} v^{(n)} + c_1 = 0 \\
&a_2^{(1)} v^{(1)} + a_2^{(2)} v^{(2)} + \cdots + a_2^{(n)} v^{(n)} + c_2 = 0 \\
&\vdots \\
&a_m^{(1)} v^{(1)} + a_m^{(2)} v^{(2)} + \cdots + a_m^{(n)} v^{(n)} + c_m = 0
\end{aligned}
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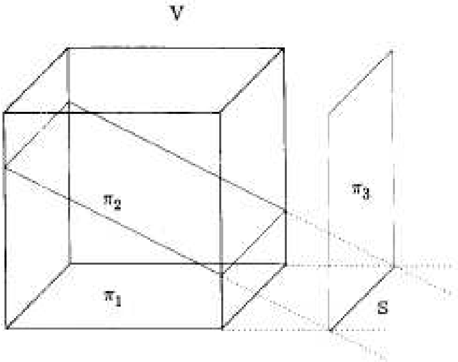
Geometrical illustrations



Case for Two-dimensional array reference

Our equations are: $f_1 = 0$ and $f_2 = 0$
where $f_i = a_i^{(1)} v^{(1)} + a_i^{(2)} v^{(2)} + \cdots + a_i^{(n)} v^{(n)} + c_i$

Linear combination is: $f_{\lambda_1,\lambda_2} = \lambda_1 f_1 + \lambda_2 f_2$
in expanded form
$f_{\lambda_1,\lambda_2} = (\lambda_1 a1^{(1)} + \lambda_2 a_2^{(1)}) v^{(1)} + (\lambda_1 a1^{(2)} + \lambda_2 a_2^{(2)}) v^{(2)} + \cdots + (\lambda_1 a1^{(n)} + \lambda_2 a_2^{(n)}) v^{(n)}$

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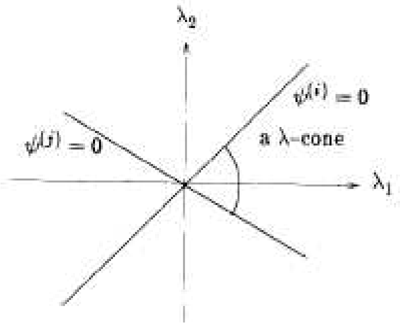
$(\lambda_1 a1^{(n)} + \lambda_2 a_2^{(n)}) v^{(n)}$

Definition

$$\psi^{(i)} = \lambda_1 a_1^{(i)} + \lambda_2 a_2^{(i)}$$

$\phi^{(i,j)} = \psi^{(i)} + \psi^{(j)}$ where $v^{(i)}$ and $v^{(j)}$ are related by a dependance direction



Canonical Solution

$$a\lambda_1 + b\lambda_2 = 0$$

$$(\lambda_1, \lambda_2) = (1, 0),\ \ if\ \ a\ \ =\ \ 0$$

$$(\lambda_1, \lambda_2) = (0, 1),\ \ if\ \ b\ \ =\ \ 0$$

$$(\lambda_1, \lambda_2) = (b, -a),\ \ if\ \ a, b \neq 0\ and\ \ b > 0$$

$$(\lambda_1, \lambda_2) = (-b, a),\ \ if\ \ a, b \neq 0\ and\ \ b < 0$$

Λ *Set* : Set of all canonical solutions to the ψ equations and ϕ equations.

Every *canonical solution* determines a λ plane

Thoerem : *S* intersects *V* if and only if every λ plane intersects *V*

Algorithm

- Determine the ψ equations and ϕ equations
- Determine the Λ set
- Each element of Λ set determines a λ plane
- Each λ plane is tested to see if intersects V by checking its max and min values
- If any one of the λ planes does not intersect V we stop

λ – *test*

$$f_{\lambda_1,\lambda_2} = (2\lambda_1 + 3\lambda_2)x_1 + (3\lambda_1 + \lambda_2)x_2$$
$$+(-\lambda_1 - 2\lambda_2)x_3 + (\lambda_1 + \lambda_2)x_4 = 0$$

ψ Equations :

$$2\lambda_1 + 3\lambda_2 = 0$$
$$3\lambda_1 + \lambda_2 = 0$$
$$-\lambda_1 - 2\lambda_2 = 0$$
$$\lambda_1 + \lambda_2 = 0$$

λ — *test*

$\Lambda = (3, -2), (1, -3), (2, -1), (1, -1)$
Consider the loop bounds:
$1 \leq x_1, x_3 \leq 50,\ \ 2 \leq x_2, x_4 \leq 50$

Here, (3, -2) shows the absence of data dependence and hence we stop and do not consider the next λ planes

λ — *test*

$$i_1 = j_2$$
$$j_1 = i_2$$
$$1 \leq i_1, i_2, j_1, j_2 \leq 100$$
$$i_1 = i_2,\ \ j_1 < j_2$$
$$f_{\lambda_1,\lambda_2} = (\lambda_1 + 0\lambda_2)i_1 + (0\lambda_1 + \lambda_2)j_1$$
$$+(-\lambda_1 + 0\lambda_2)j_2 + (0\lambda_1 - \lambda_2)i_2 = 0$$

ψ Equations :

$$\lambda_1 + 0\lambda_2 = 0$$
$$0\lambda_1 + \lambda_2 = 0$$
$$-\lambda_1 + 0\lambda_2 = 0$$
$$0\lambda_1 - \lambda_2 = 0$$

λ — *test*

ϕ Equations :
$\lambda_1 - \lambda_2 = 0$
$-\lambda_1 + \lambda_2 = 0$

$\Lambda = (1,0), (0,1), (1,1)$
Consider the loop bounds:
$1 \leq x_1, x_3 \leq 50, \;\; 2 \leq x_2, x_4 \leq 50$

Here, (1,1) shows the absence of data dependence and hence we stop

Conclusion

Discussed λ test and examined that it is better than previous Numerical methods

References

[1] An Efficient Data Dependence Analysis for Parallelizing Compilers, Zhiyuan Li and Pen-Chung Yew and Chuan-Qi Zhu IEEE Transactions on Parallel and Distributed Systems, 1990