CS626: NLP, Speech and Web

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Extra Lecture (no. 20, 21): A*

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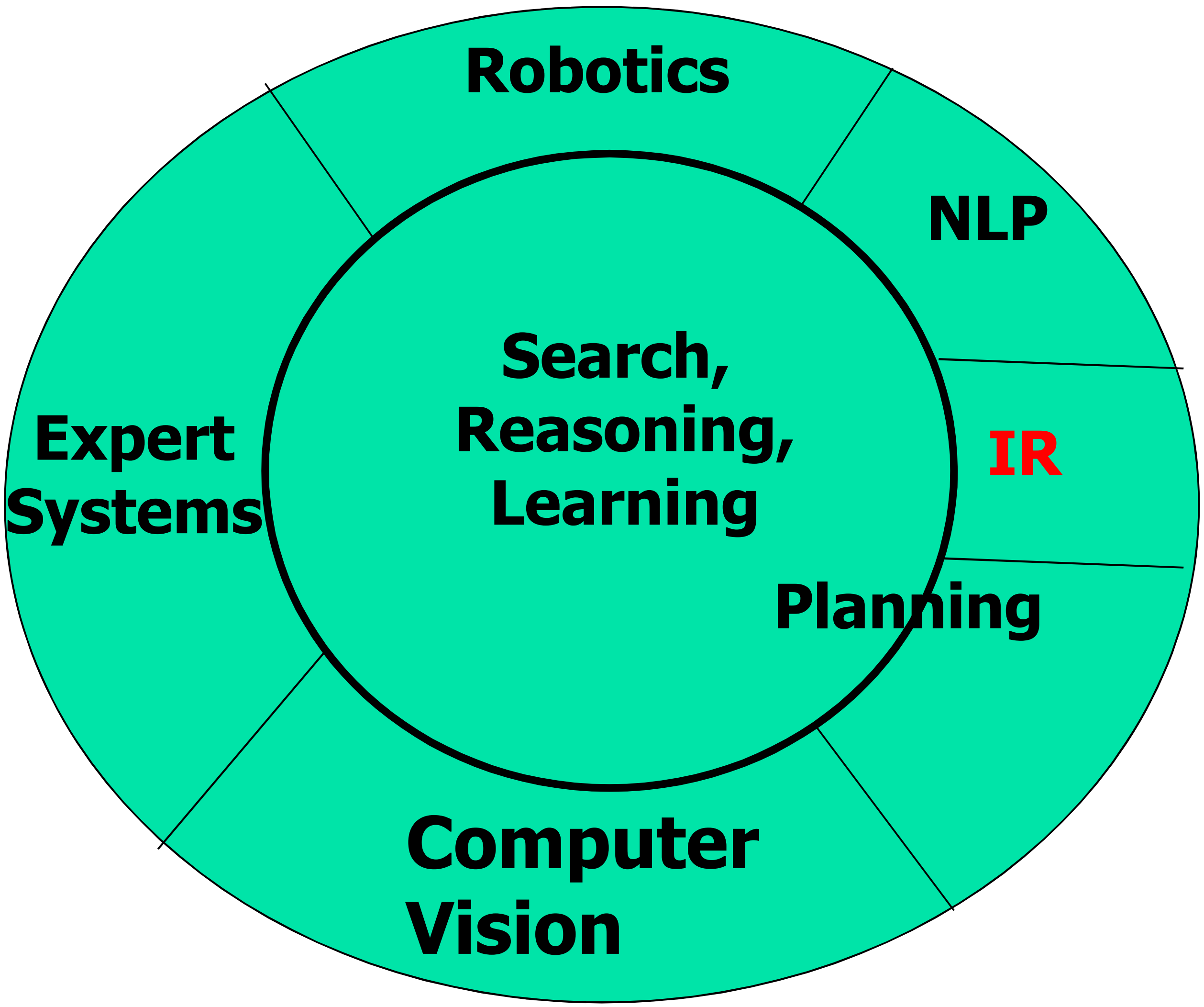
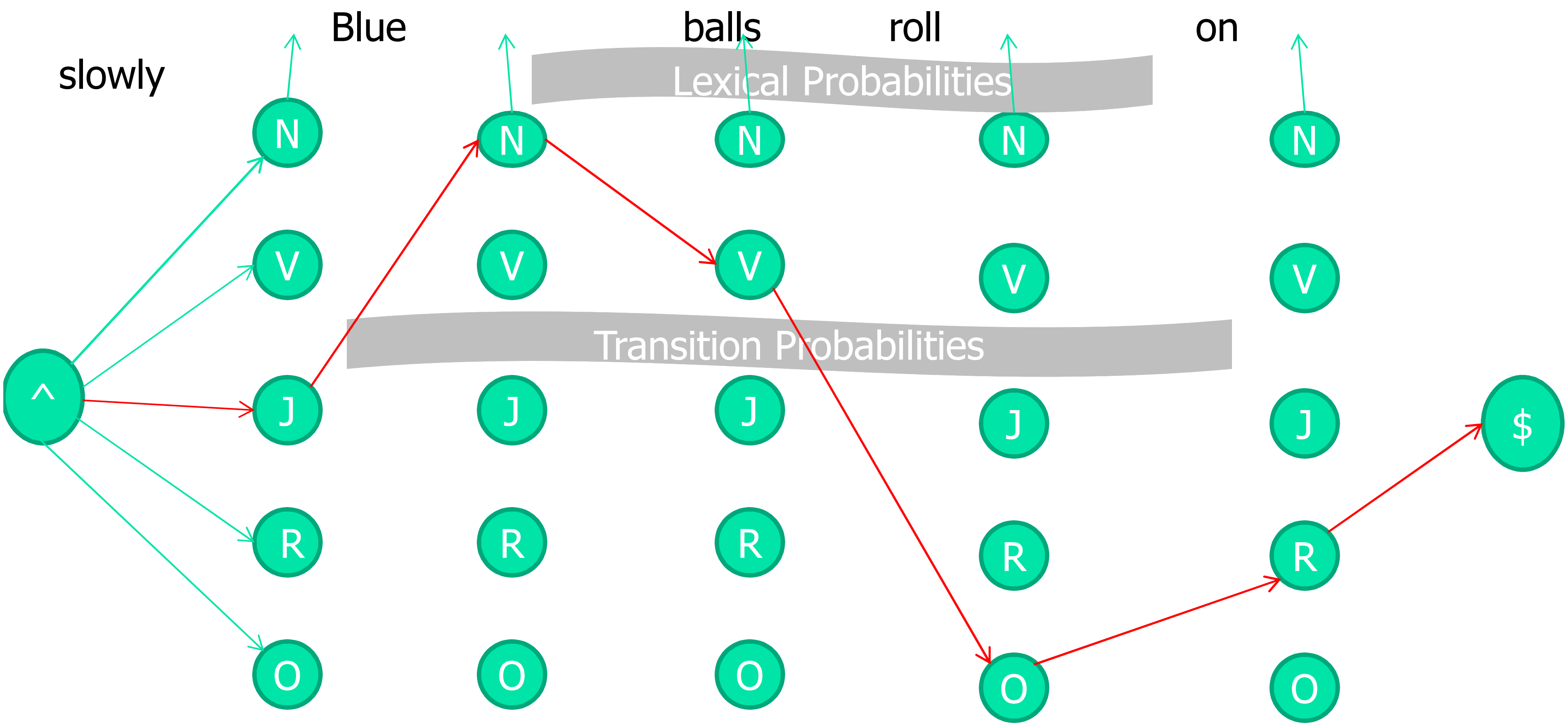
AI Perspective (post-web)



Motivation: Problem: Implement A* to get the best Tag sequence



Note: Red arrows indicate the correct tag sequence.

Search building blocks

- State Space : Graph of states (Express constraints and parameters of the problem)
- Operators : Transformations applied to the states.
- Start state : S_0 (Search starts from here)
- Goal state : $\{G\}$ - Search terminates here.
- Cost : Effort involved in using an operator.
- Optimal path : Least cost path

Examples

Problem 1 : 8 – puzzle

4	3	6
2	1	8
7		5

S

1	2	3
4	5	6
7	8	

G

Tile movement represented as the movement of the blank space.

Operators:

L : Blank moves left

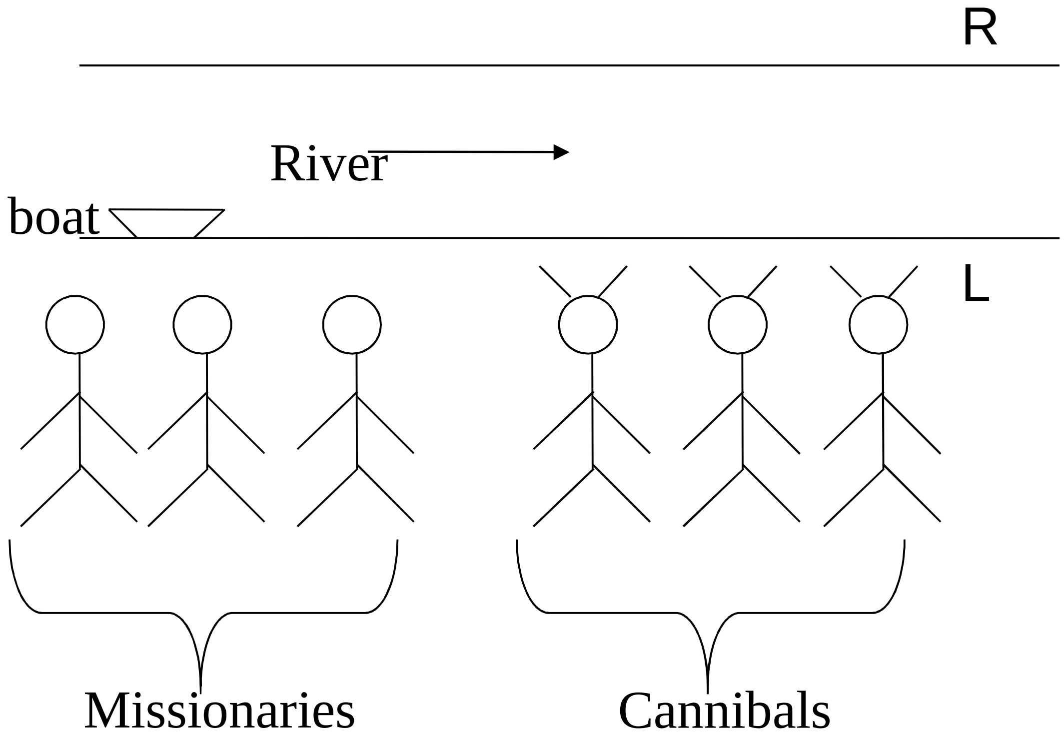
R : Blank moves right

U : Blank moves up

D : Blank moves down

$$C(L) = C(R) = C(U) = C(D) = 1$$

Problem 2: Missionaries and Cannibals



<u>Constraints</u>

- The boat can carry at most 2 people
- On no bank should the cannibals outnumber the missionaries

State : *<#M, #C, P>*

#M = Number of missionaries on bank *L*

#C = Number of cannibals on bank *L*

P = Position of the boat

S0 = <3, 3, L>

G = < 0, 0, R >

Operations

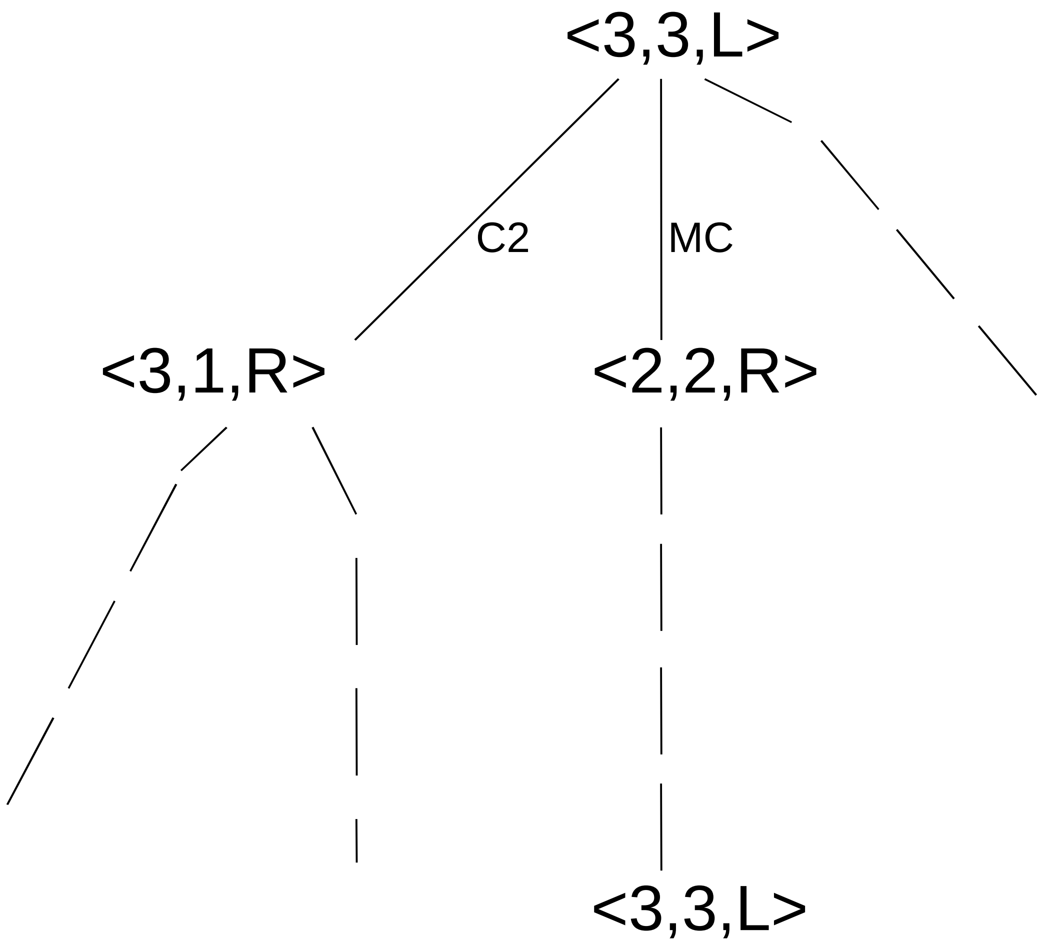
M2 = Two missionaries take boat

M1 = One missionary takes boat

C2 = Two cannibals take boat

C1 = One cannibal takes boat

MC = One missionary and one cannibal takes boat



Partial search tree

Problem 3

B	B	B	W	W	W	

G: States where no **B** is to the left of any **W**

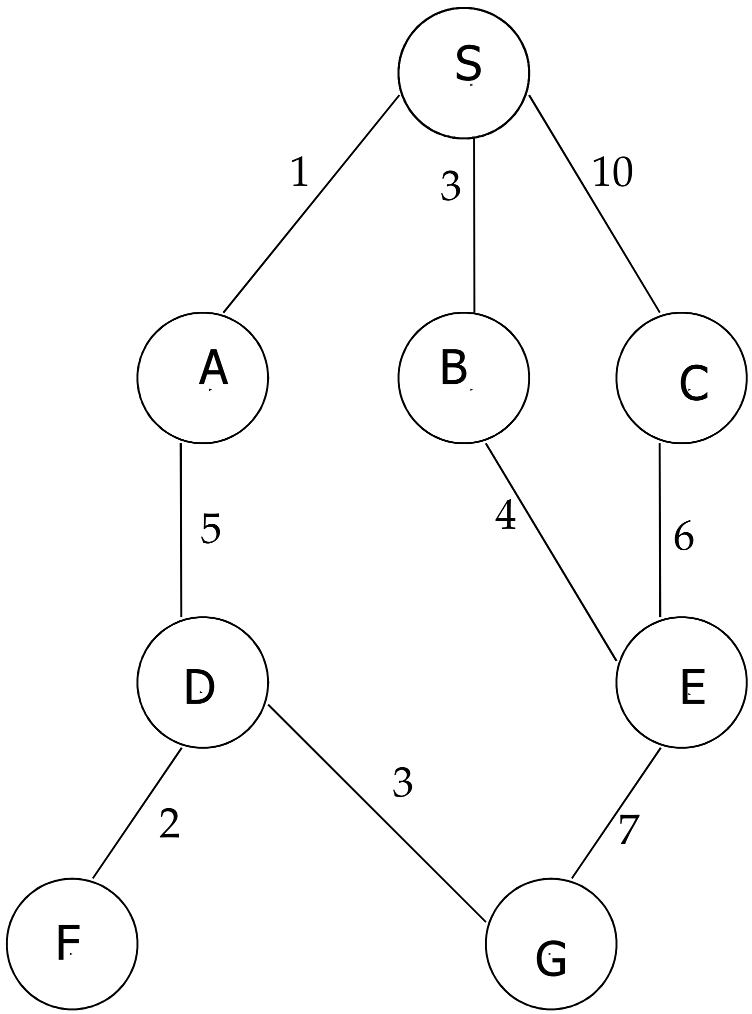
Operators:

1) A tile jumps over another tile into a blank tile with cost 2

2) A tile translates into a blank space with cost 1

Algorithmics of Search

General Graph search Algorithm



Graph G = (V,E)

1) Open List : S $^{(Ø, 0)}$
Closed list : Ø

2) OL : $A^{(S,1)}$, $B^{(S,3)}$, $C^{(S,10)}$
CL : S

3) OL : $B^{(S,3)}$, $C^{(S,10)}$, $D^{(A,6)}$
CL : S, A

4) OL : $C^{(S,10)}$, $D^{(A,6)}$, $E^{(B,7)}$
CL: S, A, B

5) OL : $D^{(A,6)}$, $E^{(B,7)}$
CL : S, A, B , C

6) OL : $E^{(B,7)}$, $F^{(D,8)}$, $G^{(D, 9)}$
CL : S, A, B, C, D

7) OL : $F^{(D,8)}$, $G^{(D,9)}$
CL : S, A, B, C, D, E

8) OL : $G^{(D,9)}$
CL : S, A, B, C, D, E, F

9) OL : Ø
CL : S, A, B, C, D, E,
F, G

Steps of GGS
(*principles of AI, Nilsson,)*

- 1. Create a search graph *G*, consisting solely of the start node *S*; put *S* on a list called *OPEN.*
- *2.* Create a list called *CLOSED* that is initially empty.
- 3. Loop: if *OPEN* is empty, exit with failure.
- 4. Select the first node on *OPEN*, remove from *OPEN* and put on *CLOSED*, call this node *n*.
- 5. if *n* is the goal node, exit with the solution obtained by tracing a path along the pointers from *n* to *s* in *G*. (ointers are established in step 7).
- 6. Expand node *n*, generating the set *M* of its successors that are not ancestors of *n*. Install these memes of *M* as successors of *n* in *G*.

GGS steps (contd.)

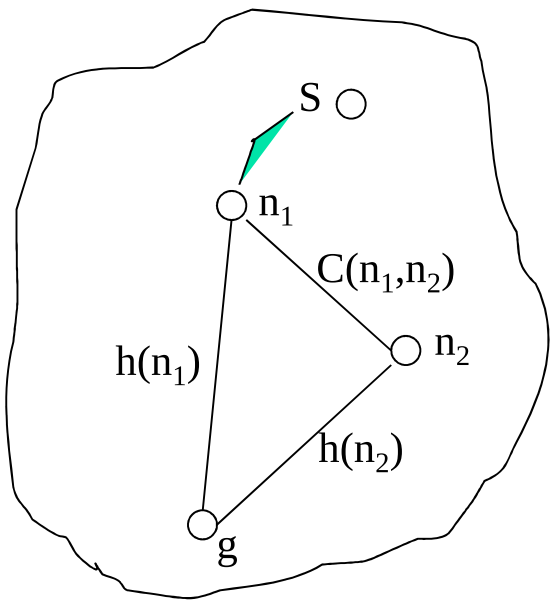
- 7. Establish a pointer to *n* from those members of *M* that were not already in *G* (*i.e.*, not already on either *OPEN* or *CLOSED*). Add these members of *M* to *OPEN*. For each member of *M* that was already on *OPEN* or *CLOSED*, decide whether or not to redirect its pointer to *n*. For each member of M already on *CLOSED*, decide for each of its descendents in *G* whether or not to redirect its pointer.
- 8. Reorder the list *OPEN* using some strategy.
- 9. Go *LOOP.*

GGS is a general umbrella

OL is a queue (BFS)

OL is stack (DFS)

OL is accessed by using a functions
f= *g*+*h*
(Algorithm A)



$$h(n_1) \leq C(n_1, n_2) + h(n_2)$$

Algorithm A

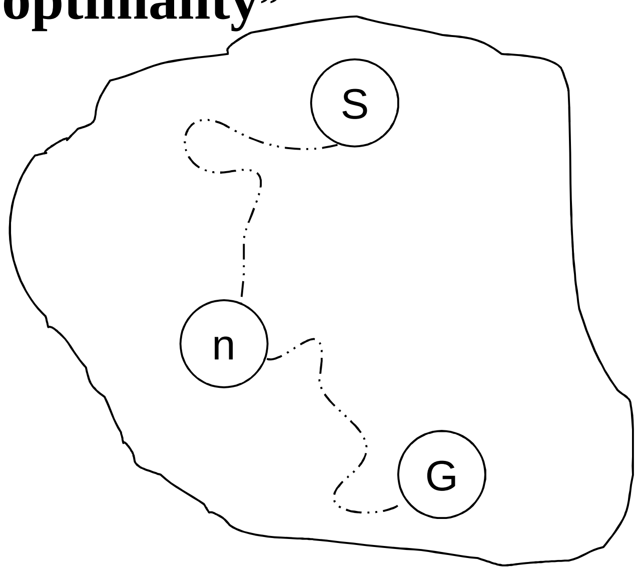
- A function f is maintained with each node

  $f(n) = g(n) + h(n)$, n is the node in the open list
- Node chosen for expansion is the one with least f value
- For BFS: $h = 0$, g = number of edges in the path to S
- For DFS: $h = 0$, $g = \frac{1}{\text{No of edges in the path to S}}$

Algorithm A*

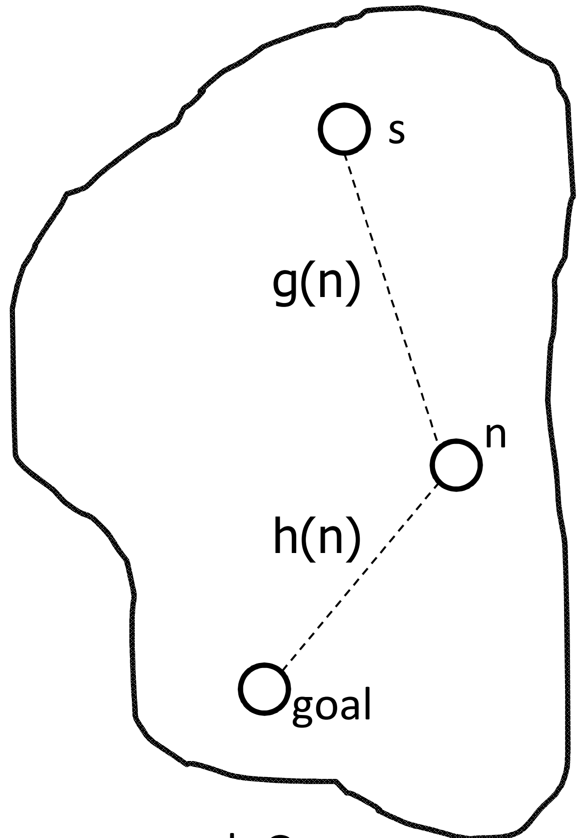
- One of the most important advances in AI
- $g(n)$ = least cost path to n from S found so far
- $h(n) <= h^*(n)$ where $h^*(n)$ is the actual cost of optimal path to G(node to be found) from n

"Optimism leads to optimality"



A* Algorithm – Definition and Properties

- $f(n) = g(n) + h(n)$
- The node with the least value of f is chosen from the *OL*.
- $f^*(n) = g^*(n) + h^*(n)$, where,
  $g^*(n)$ = actual cost of the optimal path (s, n)
  $h^*(n)$ = actual cost of optimal path (n, g)
- $g(n) \geq g^*(n)$
- By definition, $h(n) \leq h^*(n)$



State space graph G

8-puzzle: heuristics

Example: 8 puzzle

2	1	4
7	8	3
5	6	

s

1	6	7
4	3	2
5		8

n

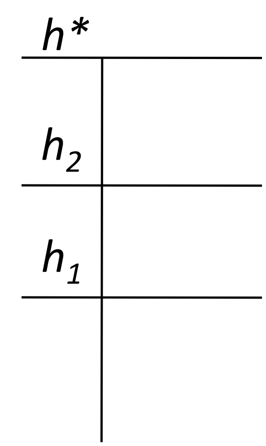
1	2	3
4	5	6
7	8	

g

$h^*(n)$ = actual no. of moves to transform *n* to *g*

1. $h_1(n)$ = no. of tiles displaced from their destined position.
2. $h_2(n)$ = sum of Manhattan distances of tiles from their destined position.

$h_1(n) \leq h^*(n)$ and $h_1(n) \leq h^*(n)$



Comparison

Eight puzzle problem

Number of Tiles displaced from their original position

Tiles:	1	2	3	4	5	6	7	8
Displacement:	1	1	1	1	1	1	1	1

***h1 = 8* (sum of the number of tiles required displacement)**

Manhattan displacement Required in tiles to get destined position(Manhattan Distances of tiles from goal)

Tiles:	1	2	3	4	5	6	7	8
Displacement:	1	1	1	3	2	2	1	1

***h2 = 12* (sum of the tile's manhatten disptances from goal)**

h* = Actual displacement from goal.

h1 <= h* and h2 <= h*

A* critical points

- **Goal**
  1. Do we know the goal?
  2. Is the distance to the goal known?
  3. Is there a path (known?) to the goal?

A* critical points

- **About the path**

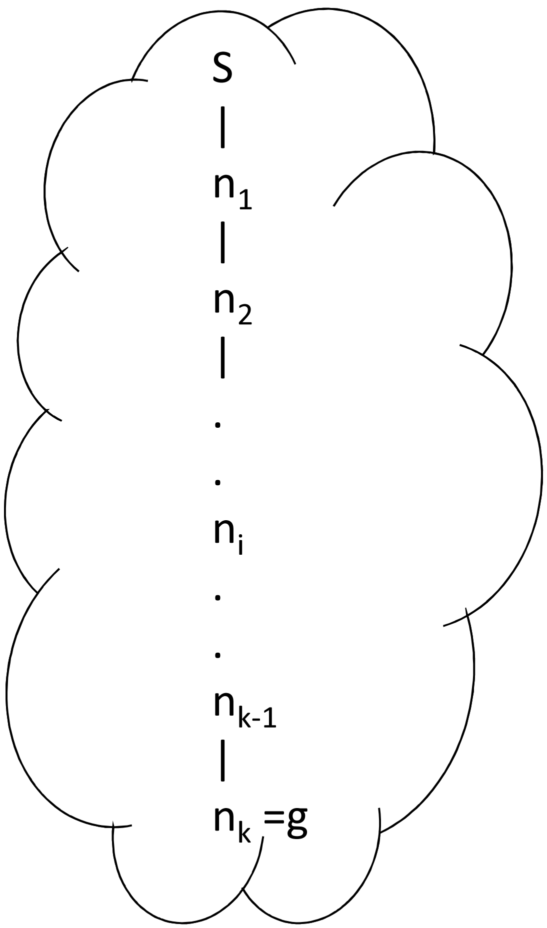
  Any time before A* terminates there exists on the OL, a node from the optimal path all whose ancestors in the optimal path are in the CL.

This means,

Э in the OL always a node 'n' s.t.

$$g(n) = g^*(n)$$

Key point about A* search



Statement:

Let S -n_1-n_2-n_3…n_i…-n_{k-1}-n_k(=G) be an optimal path.

At any time during the search:

1. There is a node n_i from the optimal path in the OL
2. For n_i all its ancestors S,n_1,n_2,…,n_{i-1} are in CL
3. $g(n_i) = g^*(n_i)$

Proof of the statement

Proof by induction on iteration no. j

Basis : j = 0, S is on the OL, S satisfies the statement

Hypothesis : Let the statement be true for j = p (p^{th} iteration)

Let n_i be the node satisfying the statement

Proof (continued)

<u>Induction</u> : Iteration no. $j = p+1$

<u>Case 1</u> : n_i is expanded and moved to the closed list

Then, n_{i+1} from the optimal path comes to the OL

Node n_{i+1} satisfies the statement

(note: if n_{i+1} is in CL, then n_{i+2} satisfies the property)

<u>Case 2</u> : Node $x \neq n_i$ is expanded

Here, n_i satisfies the statement

A* Algorithm- Properties

- **Admissibility**: An algorithm is called admissible if it always terminates and terminates in optimal path
- **Theorem**: A* is admissible.
- **Lemma**: Any time before A* terminates there exists on *OL* a node *n* such that $f(n) <= f^*(s)$
- **Observation**: For optimal path $s \rightarrow n_1 \rightarrow n_2 \rightarrow \ldots \rightarrow g$,
  1. $h^*(g) = 0, g^*(s)=0$ and
  2. $f^*(s) = f^*(n_1) = f^*(n_2) = f^*(n_3)\ldots = f^*(g)$

A* Properties *(contd.)*

$f^*(n_i) = f^*(s)$, $\quad n_i \neq s$ and $n_i \neq g$

Following set of equations show the above equality:

$$f^*(n_i) = g^*(n_i) + h^*(n_i)$$

$$f^*(n_{i+1}) = g^*(n_{i+1}) + h^*(n_{i+1})$$

$$g^*(n_{i+1}) = g^*(n_i) + c(n_i, n_{i+1})$$

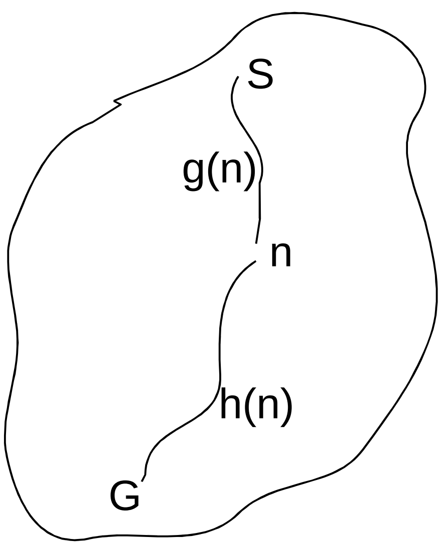
$$h^*(n_{i+1}) = h^*(n_i) - c(n_i, n_{i+1})$$

Above equations hold since the path is optimal.

Admissibility of A*

A* always terminates finding an optimal path to the goal if such a path exists.

Intuition



(1) In the open list there always exists a node *n* such that $f(n) <= f^*(S)$.

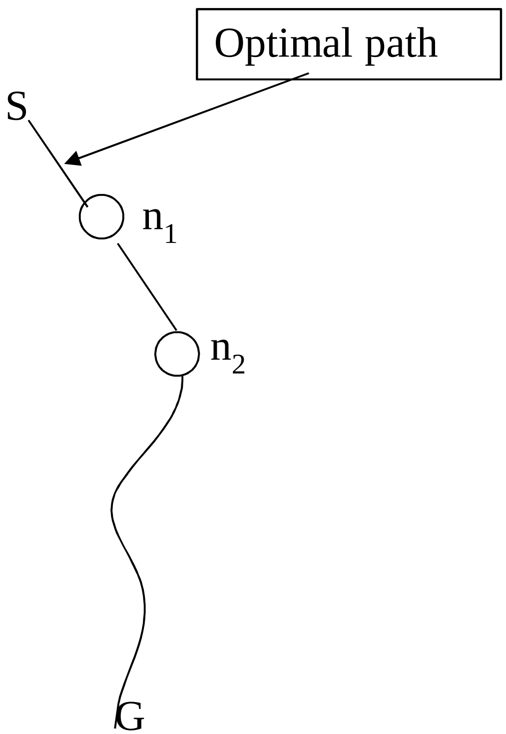
(2) If A* does not terminate, the *f* value of the nodes expanded become unbounded.

1) and 2) are together inconsistent

Hence A* must terminate

Lemma

Any time before A* terminates there exists in the open list a node *n'* such that $f(n') <= f^*(S)$



For any node n_i on optimal path,

$$f(n_i) = g(n_i) + h(n_i)$$
$$<= g^*(n_i) + h^*(n_i)$$

Also $f^*(n_i) = f^*(S)$

Let *n'* be the first node in the optimal path that is in OL. Since <u>all</u> parents of *n'* in the optimal have gone to CL,

$$g(n') = g^*(n') \text{ and } h(n') <= h^*(n')$$
$$=> f(n') <= f^*(S)$$

If A* does not terminate

Let *e* be the least cost of all arcs in the search graph.

Then *g(n) >= e.l(n)* where *l(n)* = # of arcs in the path from *S* to *n* found so far. If A* does not terminate, *g(n)* and hence *f(n) = g(n) + h(n) [h(n) >= 0]* will become unbounded.

This is not consistent with the lemma. So A* has to terminate.

2nd part of admissibility of A*

The path formed by A* is optimal when it has terminated

Proof

Suppose the path formed is not optimal

Let G be expanded in a non-optimal path.

At the point of expansion of G,

$f(G) = g(G) + h(G)$

$= g(G) + 0$

$> g^*(G) = g^*(S) + h^*(S)$

$= f^*(S)$ [$f^*(S)$ = cost of optimal path]

This is a contradiction

So path should be optimal

Summary on Admissibility

- 1. A* algorithm halts
- *2.* A* algorithm finds optimal path
- 3. If $f(n) < f^*(S)$ then node *n* has to be expanded before termination
- 4. If A* does not expand a node *n* before termination then $f(n) >= f^*(S)$

Exercise-1

Prove that if the distance of every node from the goal node is "known", then no "search:" is necessary

Ans:

- For every node *n*, *h(n)=h*(n).* The algo is A*.
- Lemma proved: any time before A* terminates, there is a node *m* in the OL that has *f(m) <= f*(S)*, *S*= start node (*m* is the node on the optimal path all whose ancestors in the optimal path are in the closed list).
- For *m, g(m)=g*(m)* and hence *f(m)=f*(S).*
- Thus at every step, the node with *f=f** will be picked up, and the journey to the goal will be completely directed and definite, with no "search" at all.
- Note: when *h=h**, *f* value of any node on the OL can never be less than *f*(S)*.

Exercise-2

If the *h* value for every node over-estimates the *h** value of the corresponding node by a constant, then the path found need not be costlier than the optimal path by that constant. Prove this.

Ans:

- Under the condition of the problem, $h(n) <= h^*(n) + c$.
- Now, any time before the algo terminates, there exists on the OL a node *m* such that $f(m) <= f^*(S)+c$.
- The reason is as follows: let *m* be the node on the optimal path all whose ancestors are in the CL (there *has to be* such a node).
- Now, $f(m)= g(m)+h(m)=g^*(m)+h(m) <= g^*(m)+h^*(m)+c = f^*(S)+c$
- When the goal *G* is picked up for expansion, it must be the case that
- $f(G)<= f^*(S)+c=f^*(G)+c$
- *i.e.,* $g(G)<= g^*(G)+c$, since $h(G)=h^*(G)=0$.

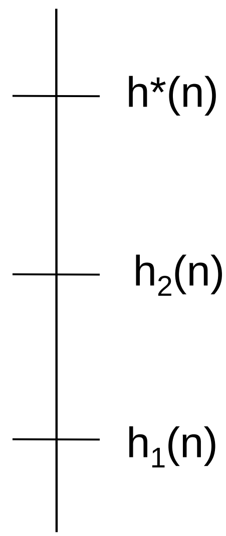
Better Heuristic Performs Better

Theorem

A version A_2* of A* that has a "better" heuristic than another version A_1* of A* performs at least "as well as" A_1*

Meaning of "better"
$h_2(n) > h_1(n)$ for all n

Meaning of "as well as"
A_1* expands at least all the nodes of A_2*



For all nodes n, except the goal node

Proof by induction on the search tree of A_2*.

A* on termination carves out a tree out of *G*

Induction

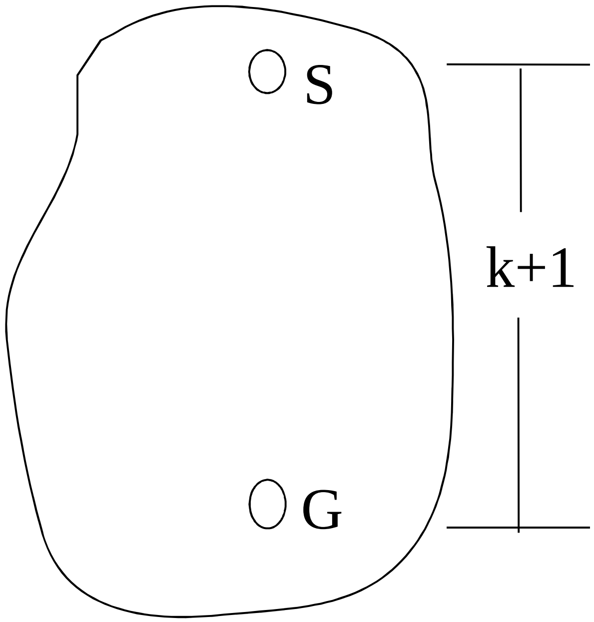
on the depth *k* of the search tree of A_2*. A_1* before termination expands all the nodes of depth *k* in the search tree of A_2*.

k=0. True since start node *S* is expanded by both

Suppose A_1* terminates without expanding a node *n* at depth *(k+1)* of A_2* search tree.

Since A_1* has seen all the parents of *n* seen by A_2*

$g_1(n) <= g_2(n)$ *(1)*



Since A_1* has terminated without expanding *n*,
$f_1(n) >= f^*(S)$ (2)

Any node whose *f* value is strictly less than $f^*(S)$ has to be expanded.
Since A_2* has expanded *n*
$f_2(n) <= f^*(S)$ (3)

From (1), (2), and (3)
$h_1(n) >= h_2(n)$ which is a contradiction. Therefore, A_1* has to expand all nodes that A_2* has expanded.

Exercise

If better means $h_2(n) > h_1(n)$ for some *n* and $h_2(n) = h_1(n)$ for others, then Can you prove the result ?

Lab assignment

- Implement A* algorithm for the following problems:
  - 8 puzzle
  - Missionaries and Cannibals
  - Robotic Blocks world
- Specifications:
  - Try different heuristics and compare with baseline case, *i.e.*, the breadth first search.
  - Violate the condition $h \leq h^*$. See if the optimal path is still found. Observe the speedup.

Resources

- Main Text:
  - Artificial Intelligence: A Modern Approach by Russell & Norvik, Pearson, 2003.
- Other Main References:
  - Principles of AI - Nilsson
  - AI - Rich & Knight
  - Knowledge Based Systems – Mark Stefik
- Journals
  - AI, AI Magazine, IEEE Expert,
  - Area Specific Journals e.g, Computational Linguistics
- Conferences
  - IJCAI, AAAI
- Imp “site”: *moodle.iitb.ac.in*