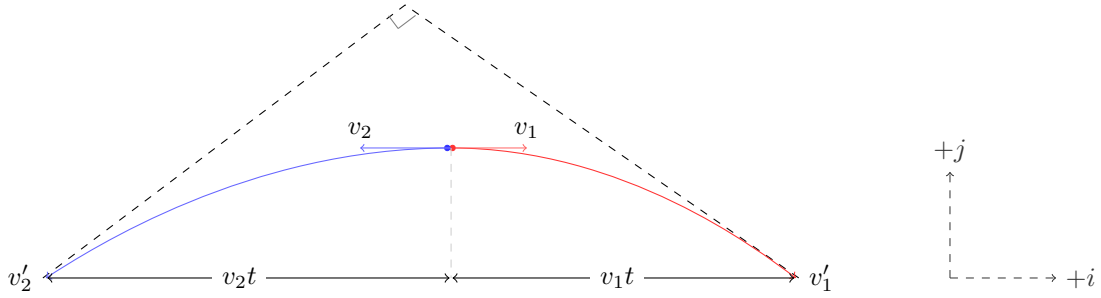
1.11 Two particles move in a uniform gravitational field with an accelaration g. At the initial moment the particles were located at one point and moved with velocities $v_1 = 3.0\text{ms}^{-1}$ and $v_2 = 4.0\text{ms}^{-1}$ horizontally in opposite directions. Find the distance between the particles at the moment when their velocities become mutually perpendicular.

Sol. We can visualize the situation as given below. Suppose that the velocities of two bodies become mutually perpendicular after time t. Let the velocities of the two bodies at that instant be v_1' and v_2', respectively. What can we say about these velocities?



Thought Process

- The velocity of each body can be resolved into horizontal and vertical component ($\hat{i}$ and $\hat{j}$).
- The accelaration due to gravity g acts only on the vertical component of velocity. The horizontal component remains unchanged for both bodies.
- Since the initial velocity in the vertical direction is 0 for both bodies, they travel the same distance in vertical direction. Hence, the distance between the bodies at t is given only by the horizontal distance between the bodies at time t.

We have,

$$v_1' = v_1\hat{i} - gt\hat{j}$$
$$v_2' = -v_2\hat{i} - gt\hat{j}$$

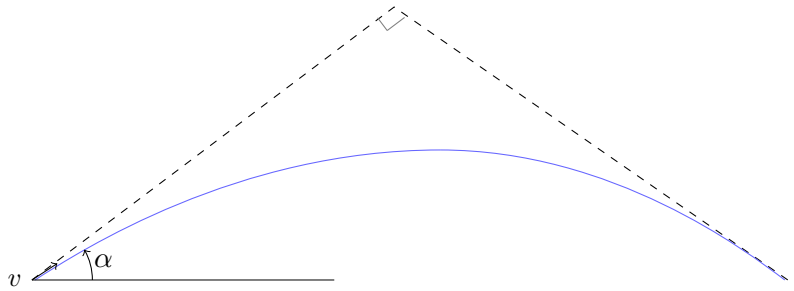
At time t, the vectors v_1' and v_2' are mutally perpendicular. Hence, their dot product must be 0.

$$\begin{aligned} v_1' \cdot v_2' &= 0 \\ (v_1\hat{i} - gt\hat{j}) \cdot (-v_2\hat{i} - gt\hat{j}) &= 0 \\ -v_1v_2\hat{i}\cdot\hat{i} - v_1gt\hat{i}\cdot\hat{j} - v_2gt\hat{i}\cdot\hat{j} + (gt)^2\hat{j}\cdot\hat{j} &= 0 \\ -v_1v_2 + (gt)^2 &= 0 \quad [\because \hat{i}\cdot\hat{i} = 1 \text{ and } \hat{i}\cdot\hat{j} = 0] \\ t &= \frac{\sqrt{v_1v_2}}{g} \end{aligned}$$

In time t, the bodies have moved v_1t and v_2t in opposite directions, therefore the distance between them is

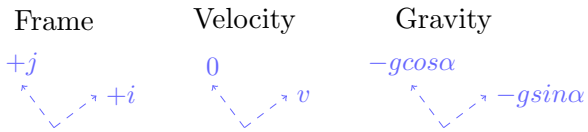
$$(v_1 + v_2)t = (v_1 + v_2)\frac{\sqrt{v_1v_2}}{g}$$

Related Problem A stone is thrown at an angle α to the ground with velocity v. After what time will the velocity vector be perpendicular to the initial velocity vector?



Method I Hint: Follow a similar procedure as that used in the earlier solution.

Method II Hint: Suppose we change the frame of reference by rotating it clockwise by α degrees. The velocity and acceleration due to gravity vectors in this frame of reference change accordingly(shown below). At time t when the velocity vector becomes perpendicular to the initial vector, what is the magnitude of velocity in the $\hat{i}$ direction?



Solve using this approach and verify the answer with the answer obtained from Method I.

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