Uniform Sampling for Timed Automata with Application to Language Inclusion Measurement

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joint work with
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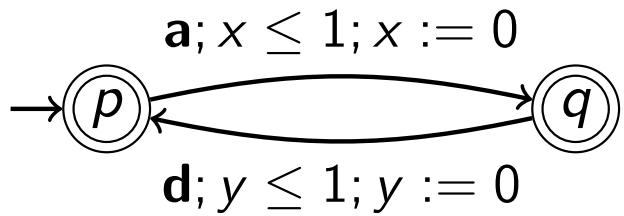
AVERTS, 14 July 2017

The context of timed automata theory

Introduced by Alur and Dill in 1990 to model and verify **real-time** properties of **embedded systems** (cars, phones, pacemakers...). Build by adding real time *clocks* to finite state automaton.



Since then many people worked on

- ▷ adding real-time to classical automata and verification theory.
- ▷ implementing model checking tools (e.g. UPPAAL).

Model checking timed regular properties.

- ▷ Model encoded as a timed automaton A.
- ▷ Specification encoded as a timed automaton B.
- ▷ Problem : is $L(A) \subseteq L(B)$?

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Our statistical model checking method

- ▷ Estimate statistically the proportion of runs of $L(A)$ that are in $L(B)$.
- ▷ If the estimation is 1 then answer **Yes** with high confidence;
- ▷ else answer **No** and exhibit a counter-example.

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Our method is based on

- volumetry of timed languages described in [Asarin, B., Degorre, Information & Computation 2015];
- uniform sampling of timed words (wrt. volumes);

Simulation of a hybrid system $\mathcal{H}$ whose inputs are given by a TA $\mathcal{A}$

- Draw inputs $w \in \mathcal{A}$ at random and check properties on the corresponding outputs.

Example

- $\mathcal{H}$ models a boiler with two modes Heating and Non-Heating
- $\mathcal{A}$ models the following input behaviours:

- the boiler is no more than 20 minutes in Heating mode before being switch to Non-Heating mode
- it is switched to Heating mode at least twice per hour.

- Possible question one may ask:
  - will the boiler have a problem?
  - will a problem appears in less than 0.01% of possible inputs?
  - what is the average energy consumption of the boiler?

Outline

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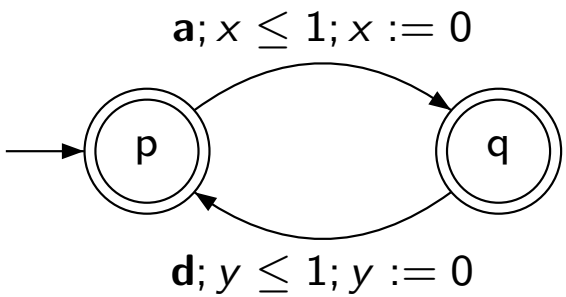

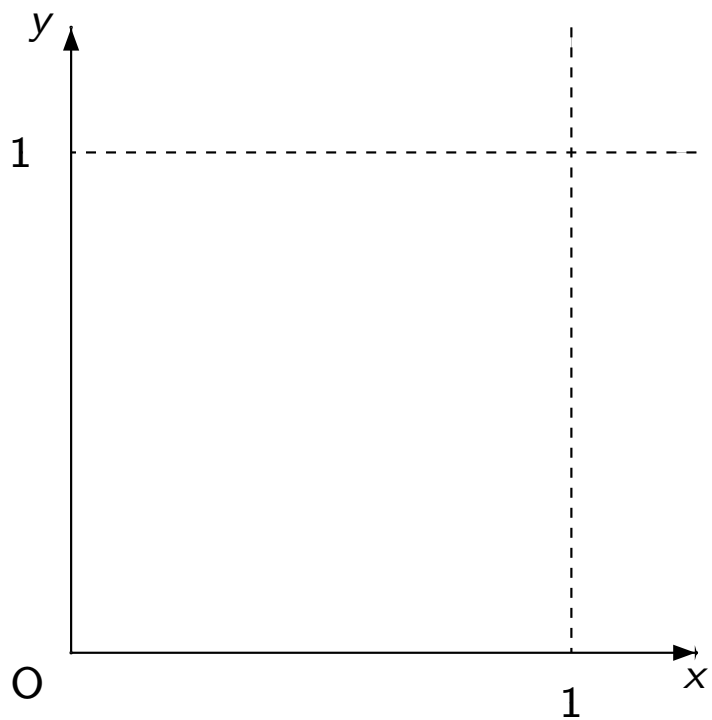

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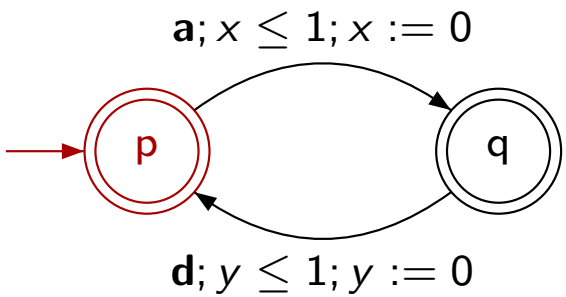

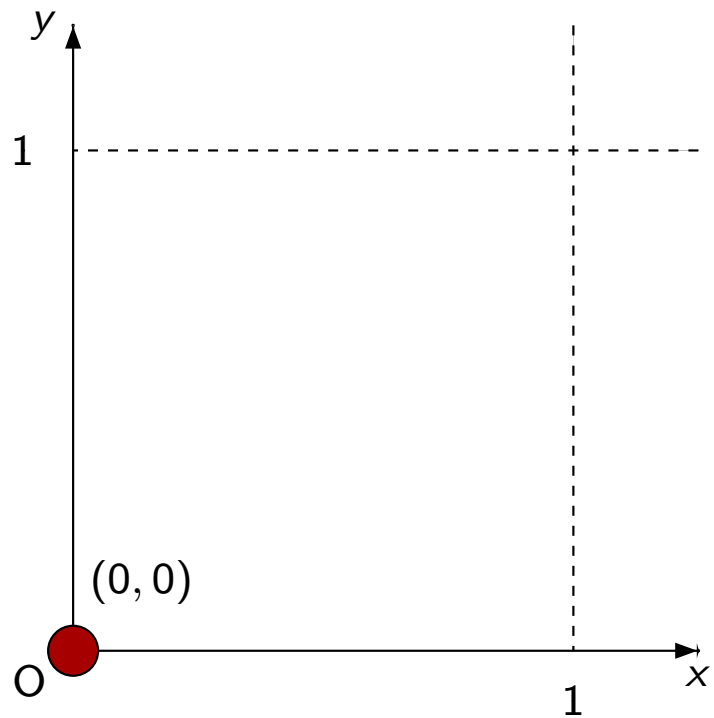

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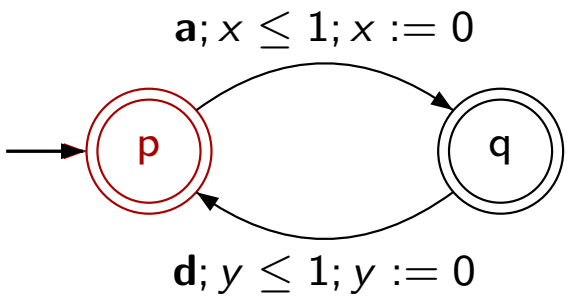

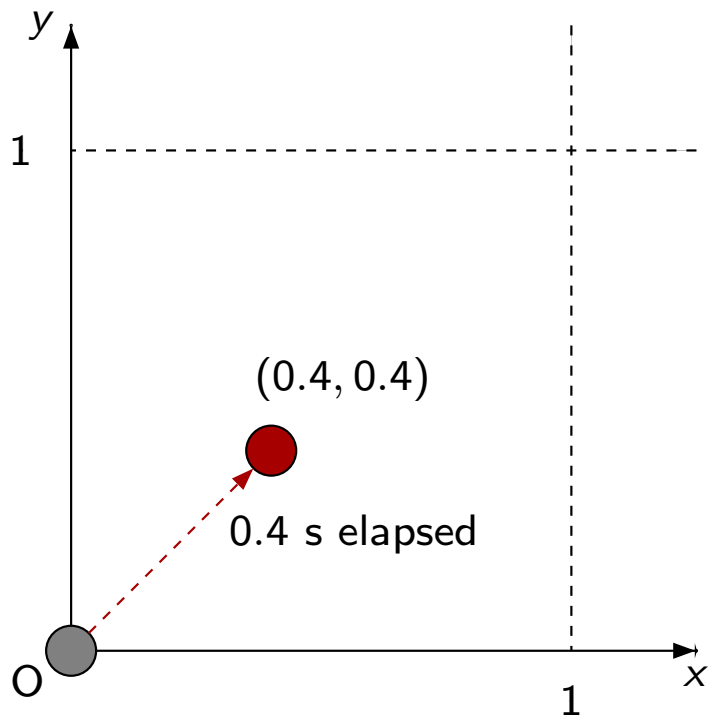

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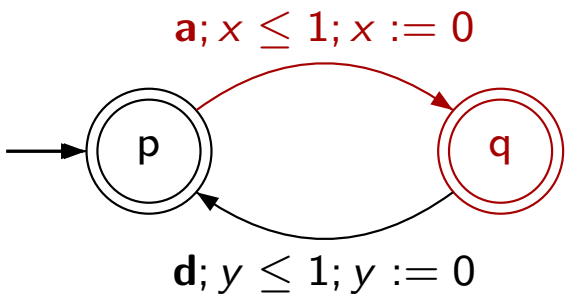

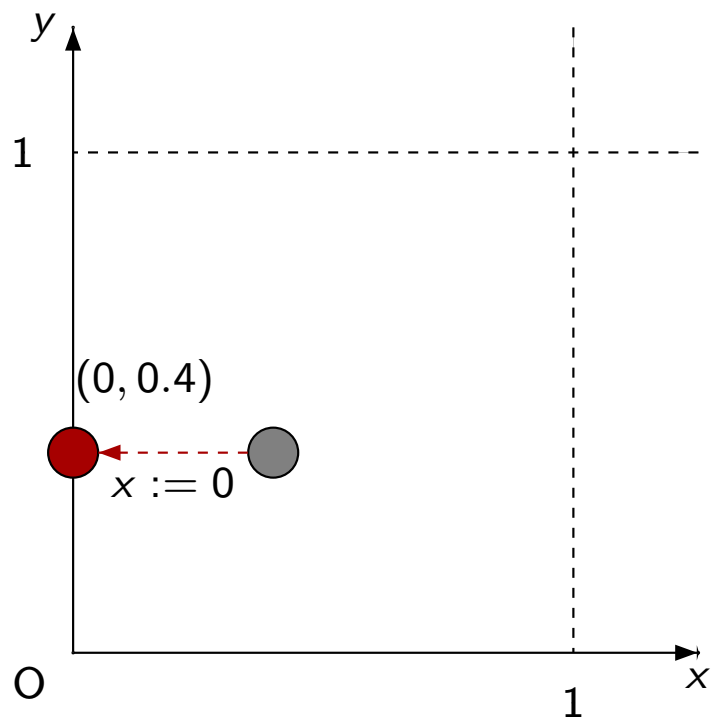

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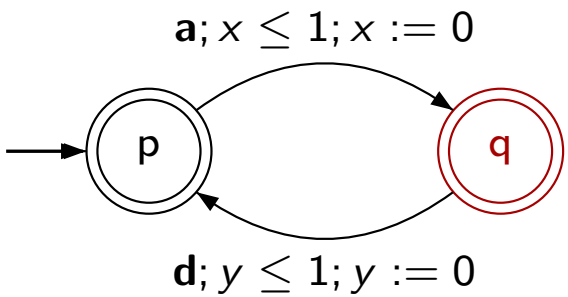

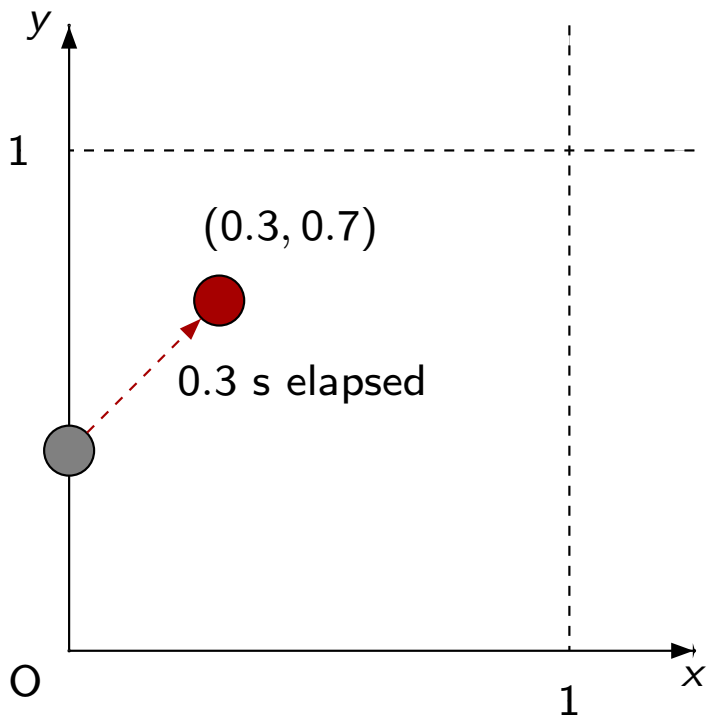

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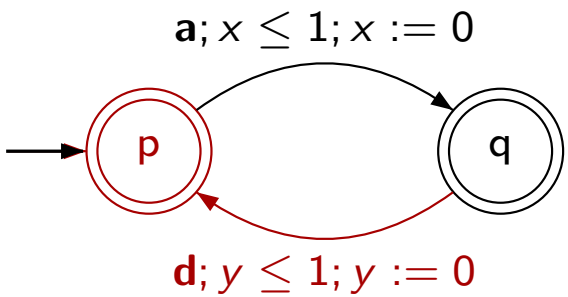

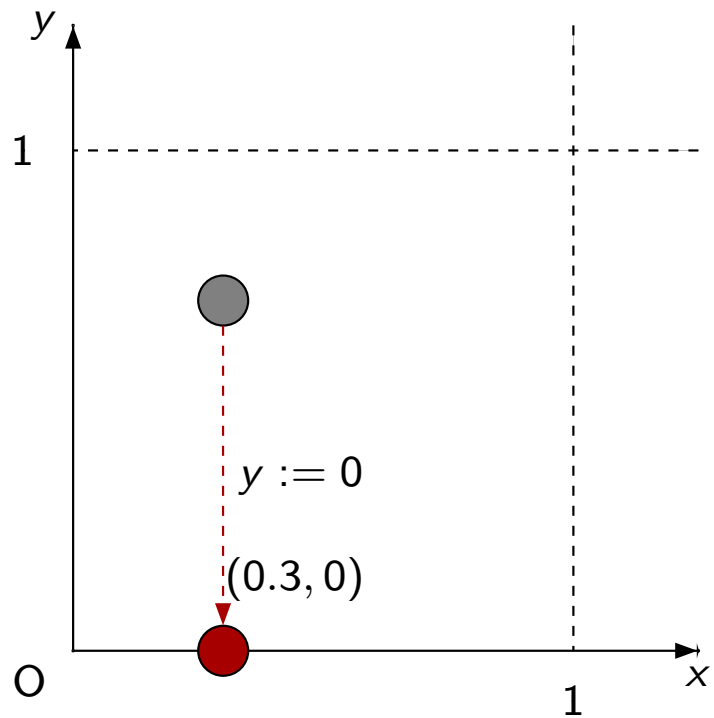

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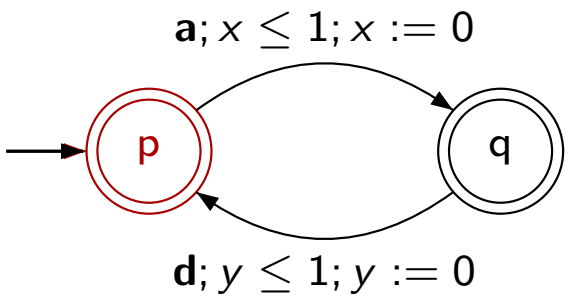

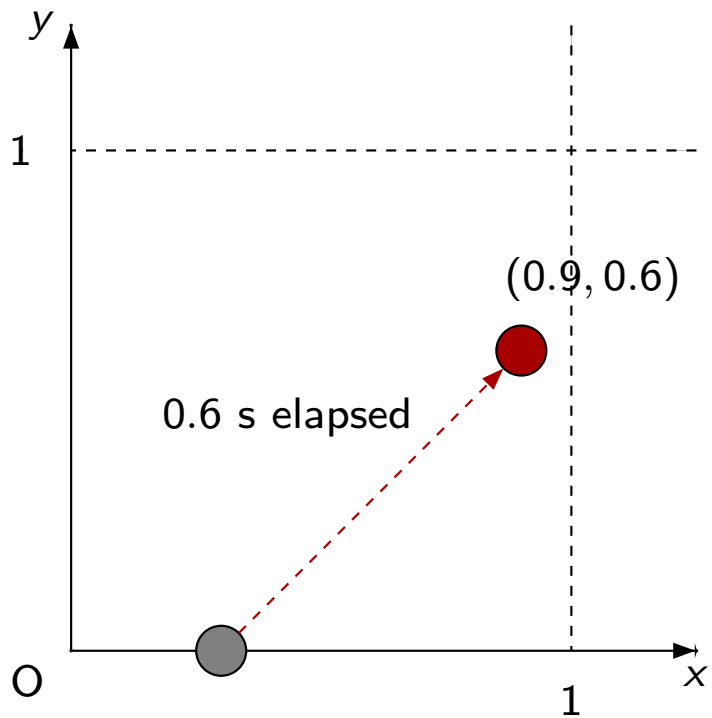

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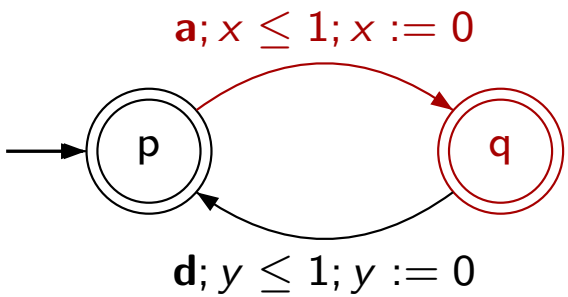
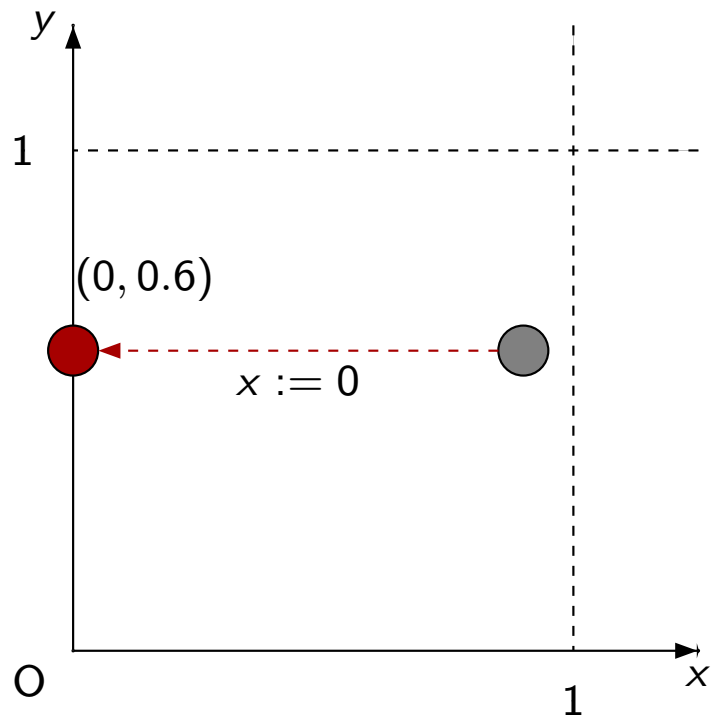
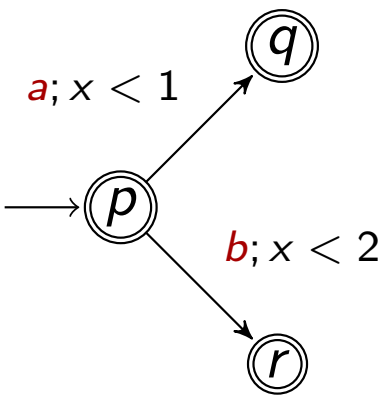




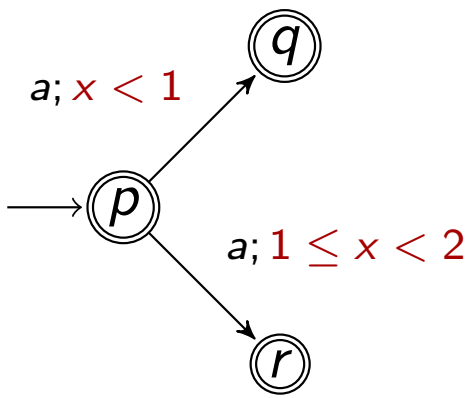
Some (restrictive) hypotheses we make

Restrictive hypotheses

- The clocks and the delays are bounded by a constant M.
- The timed automaton is deterministic (DTA)
  (we address non-deterministic timed automata (NTA) later).



Deterministic



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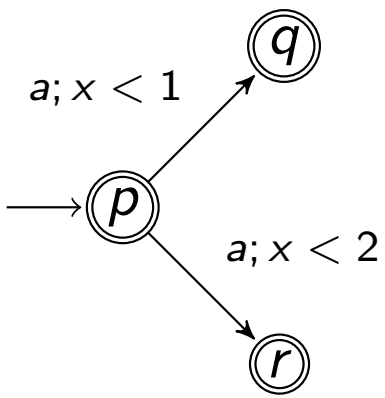
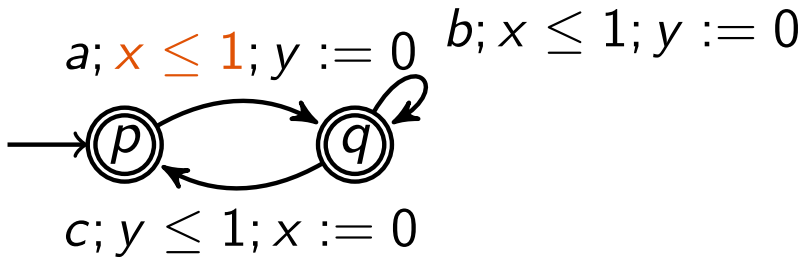
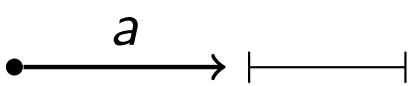


Non-deterministic

Measuring timed languages wrt. volumes

Constraints of timings along a path w = polytope P_w^L





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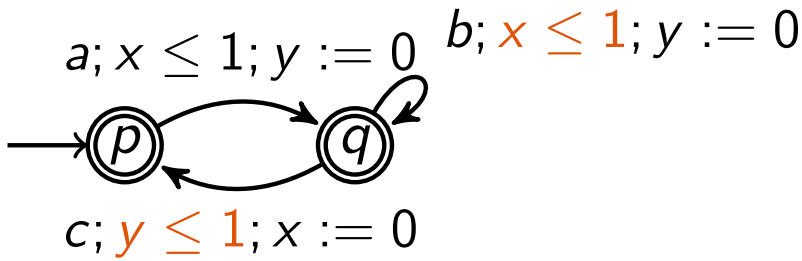

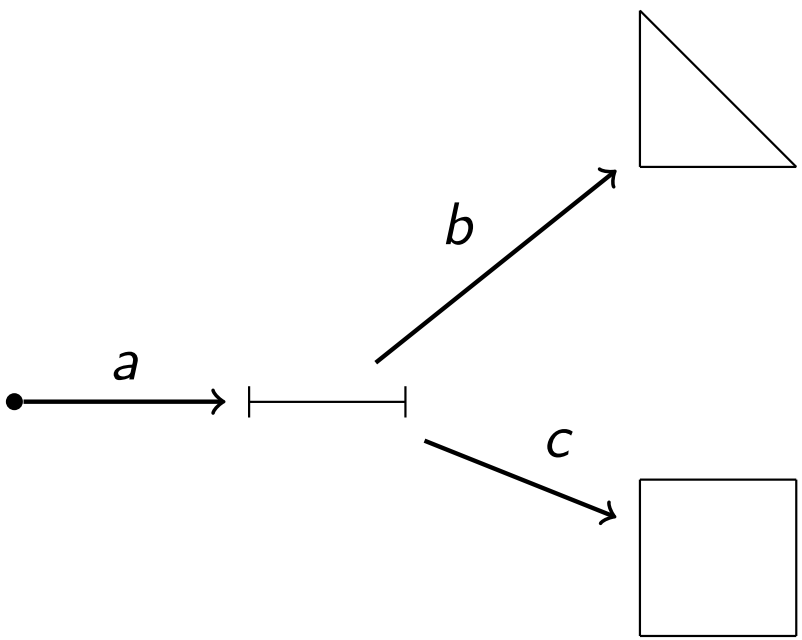

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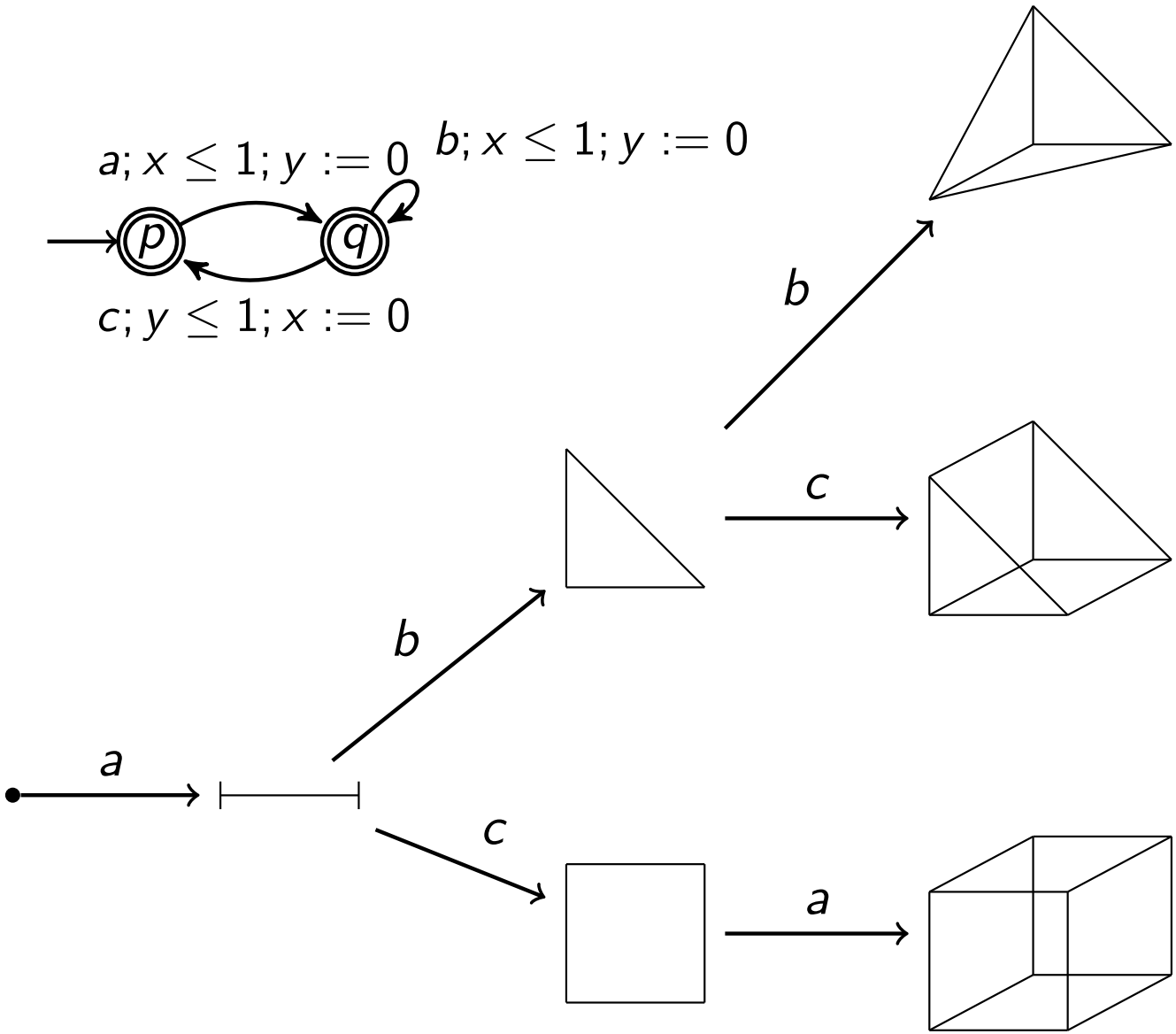

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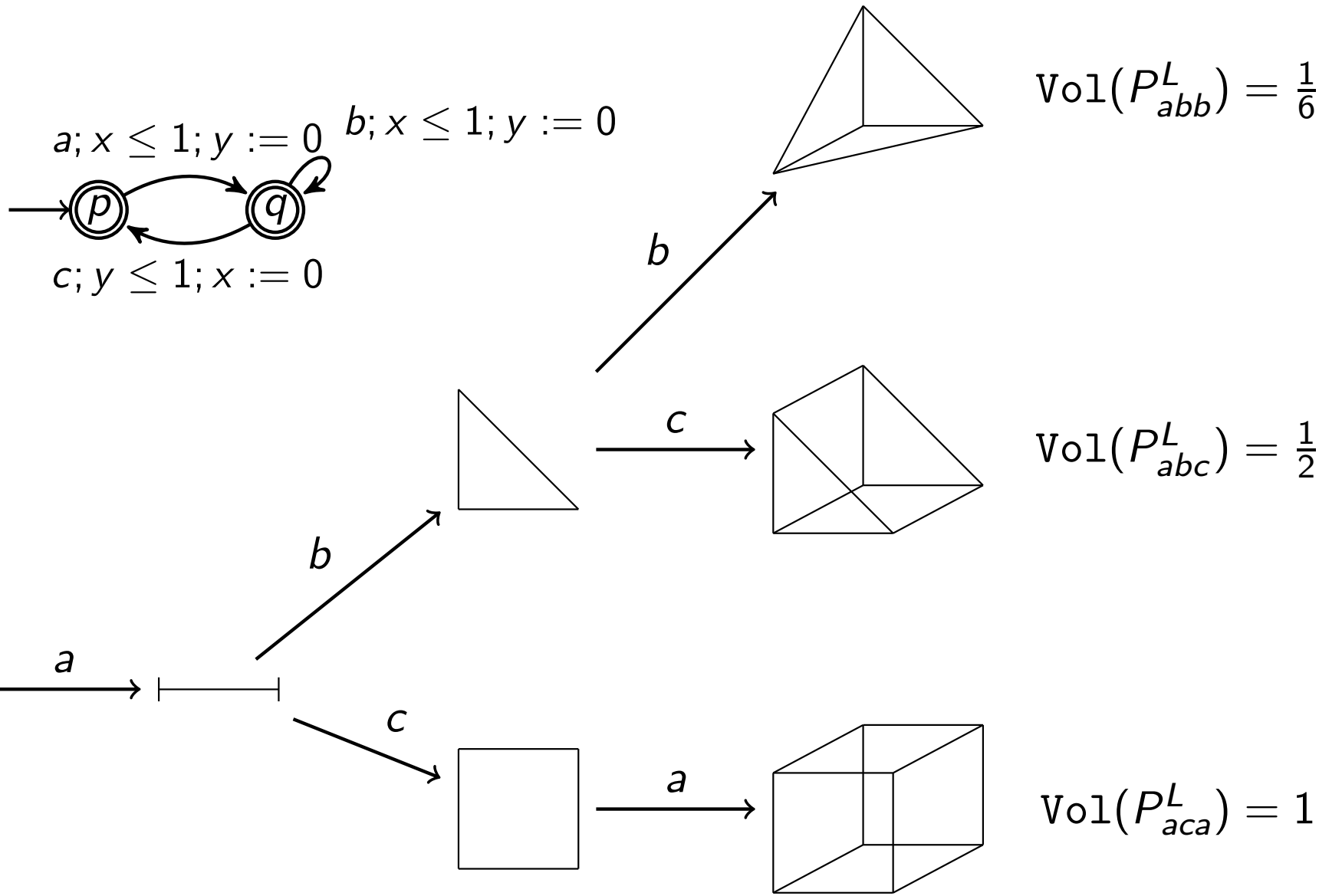

Computing the volumes

A recipe

- ▷ Define for $n \geq 0$ and reachable state $(q, \vec{x})$ the volume $v_n(q, \vec{x})$ of timed words of length n starting from $(q, \vec{x})$;
- ▷ write recursive equations on volume functions ;
- ▷ compute $v_n(q_0, \vec{0})$ the volume of the language from the initial state.

Problem : recursive equations are difficult to write and use a priori

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Previous solution : decompose the state space into regions. [Asarin, B., Degorre, Information & Computation 2015]

- the equations are easy to write ;
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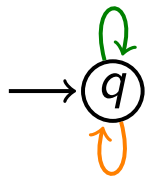
Problem : too many regions!!!
Our solution : use zones instead of regions.

A first decomposition into zones (the forward reachability timed automaton).

b,
$0 < x < 3$,
$0 < y < 2$,
$x := 0$



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$0 < x < 2$,
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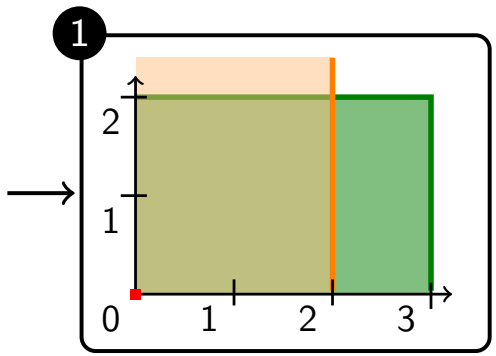

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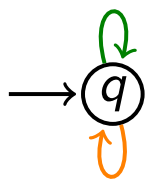

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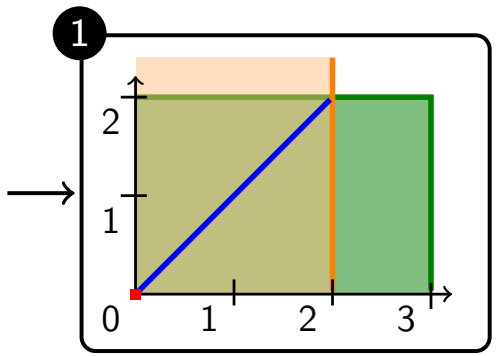

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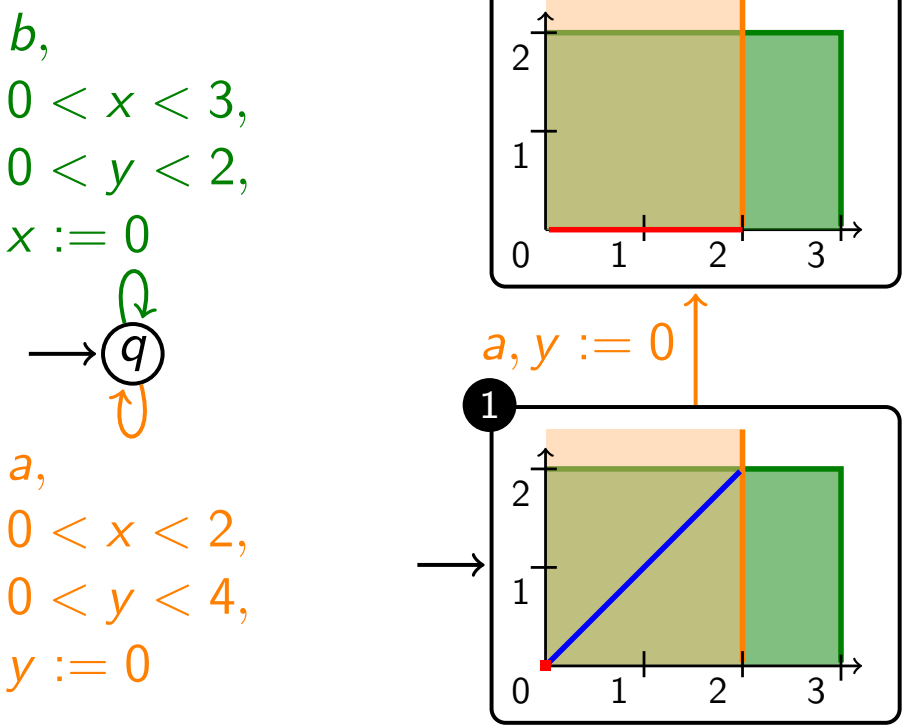

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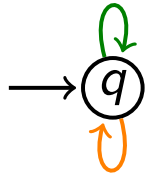

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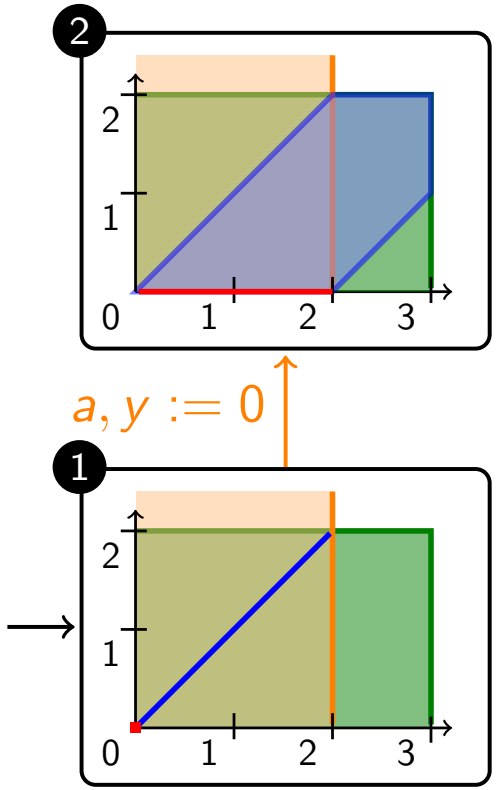

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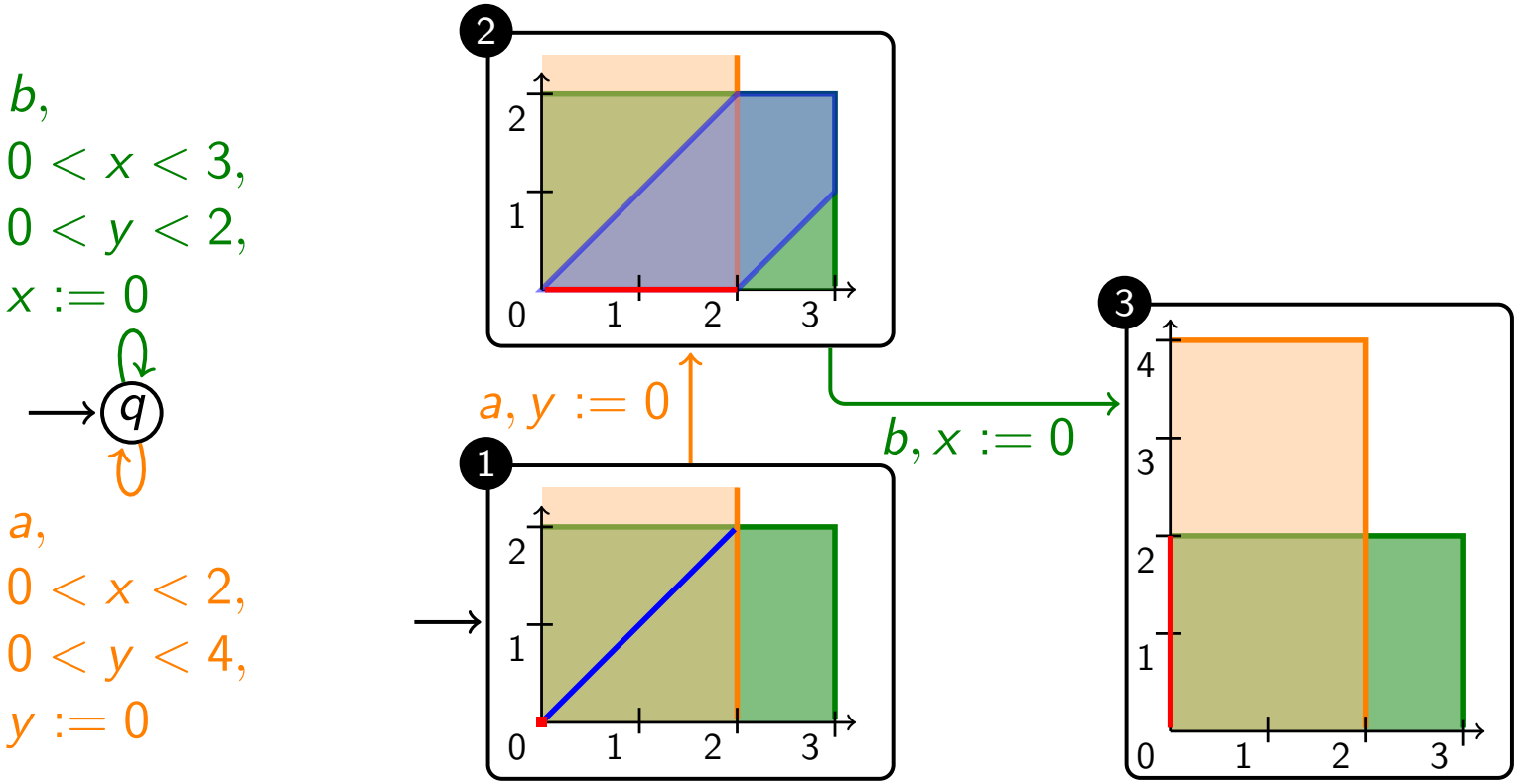

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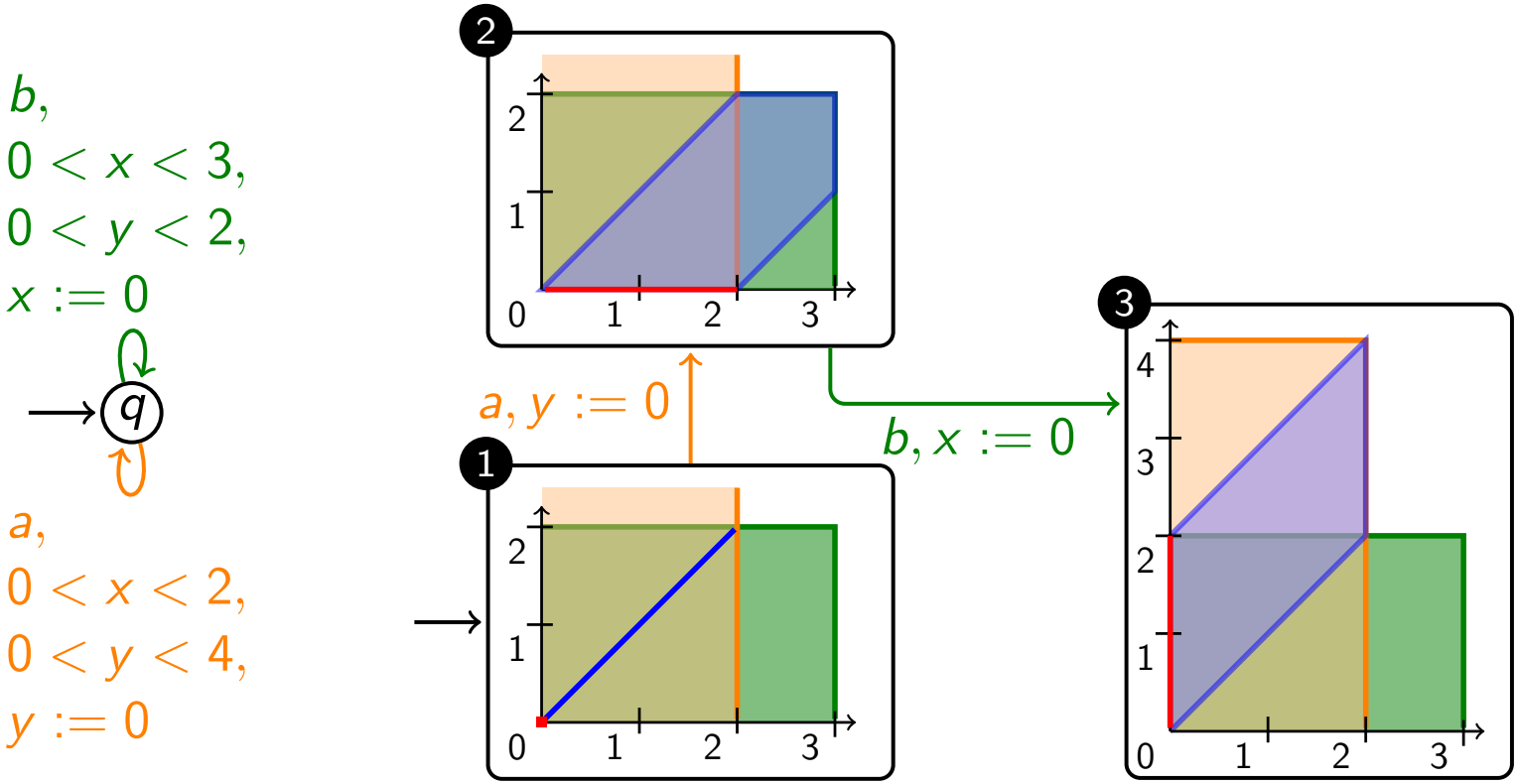

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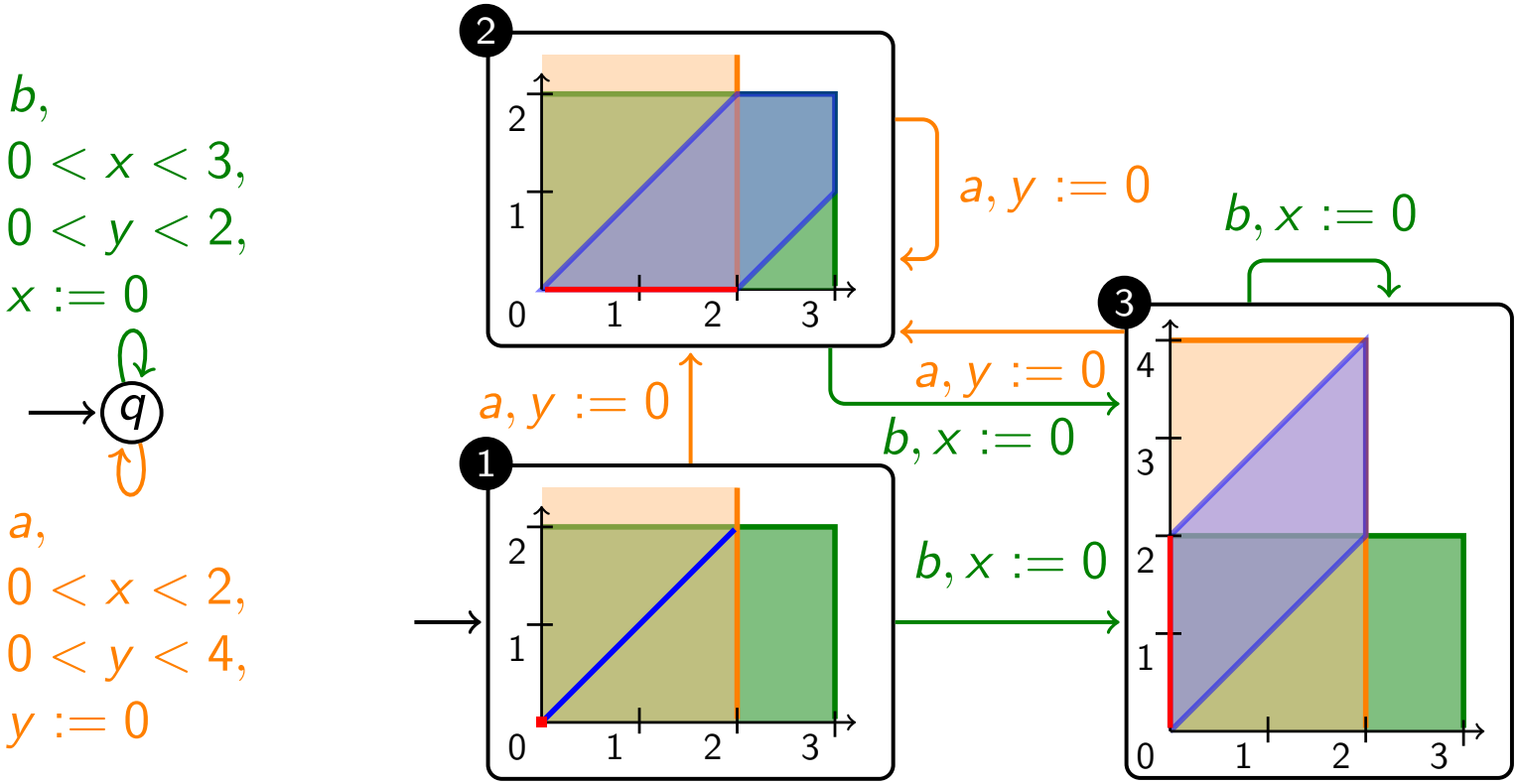

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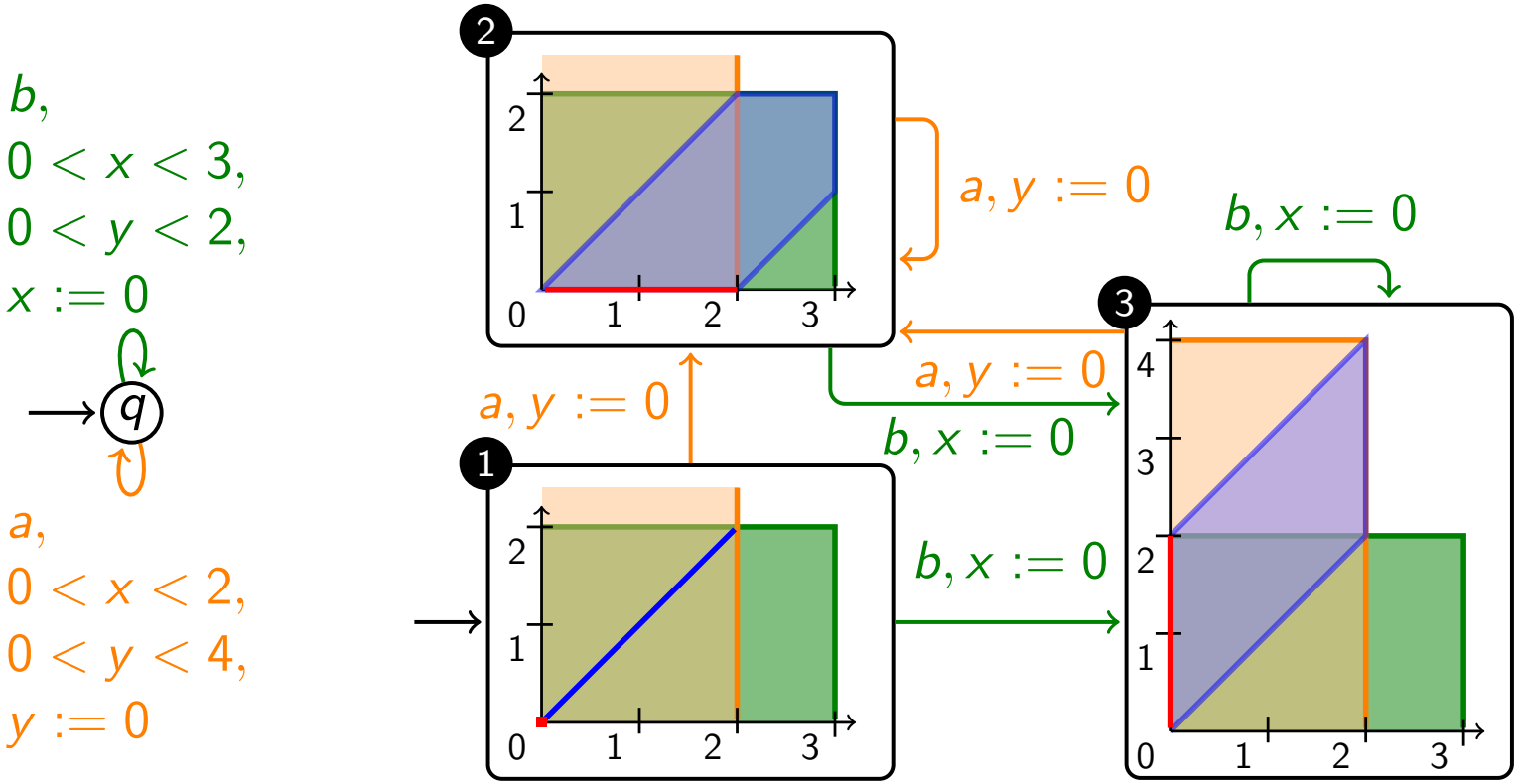

One of the recursive equations to compute volumes

$$v_{n+1}[❷,(x,0)] = \int_0^{2-x} v_n(❷,(x+t,0))\mathrm{d}t + \int_0^{\min(2,3-x)} v_n(❸,(0,t))\mathrm{d}t.$$

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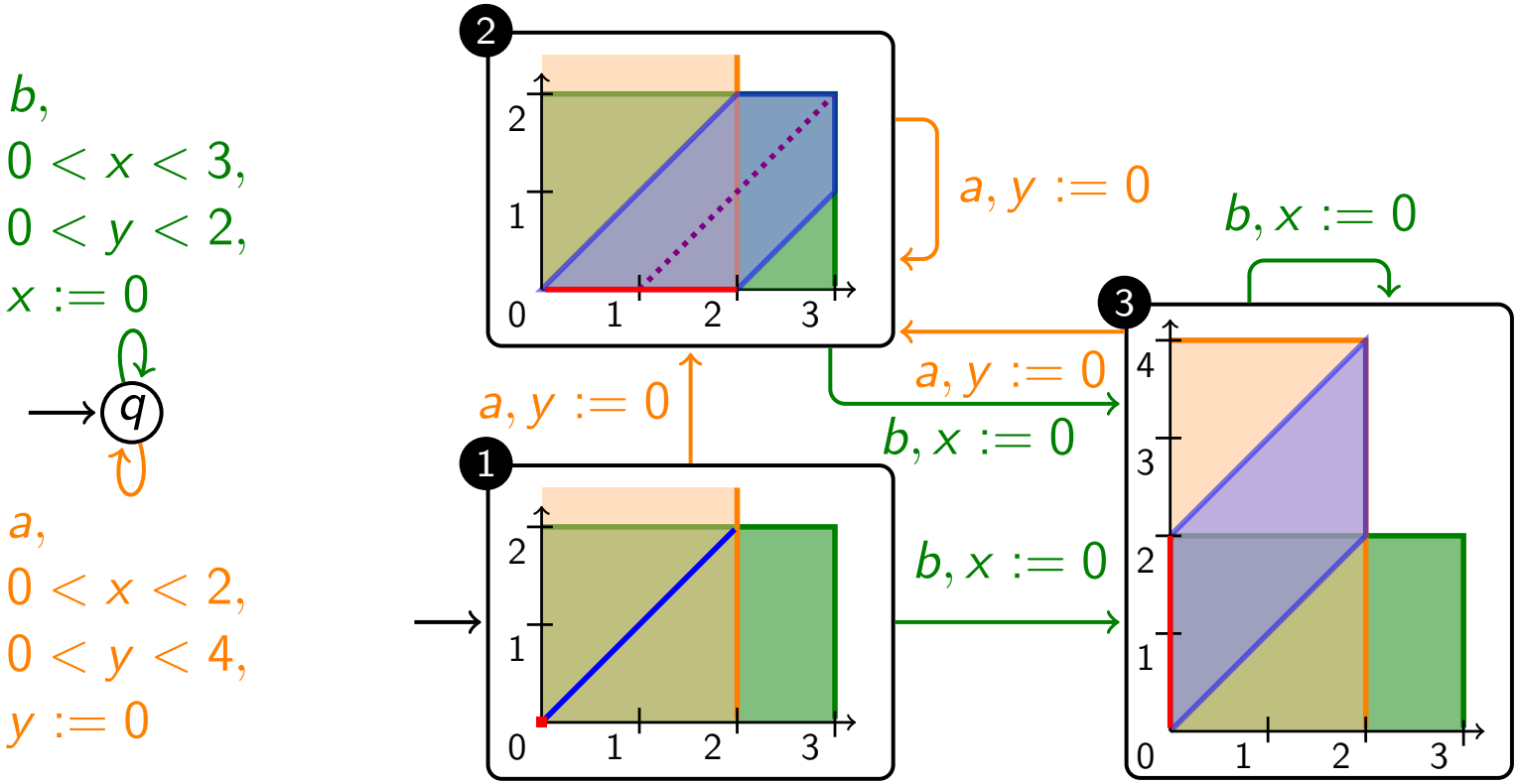

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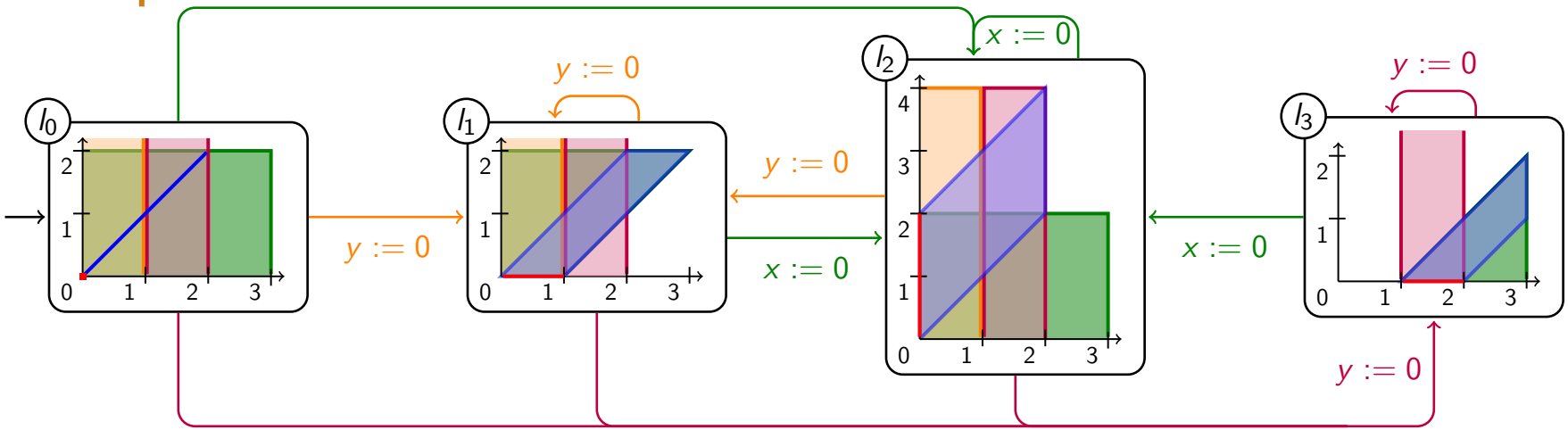
A further split is needed to simplify $\min(2, 3-x)$.

The split timed automaton



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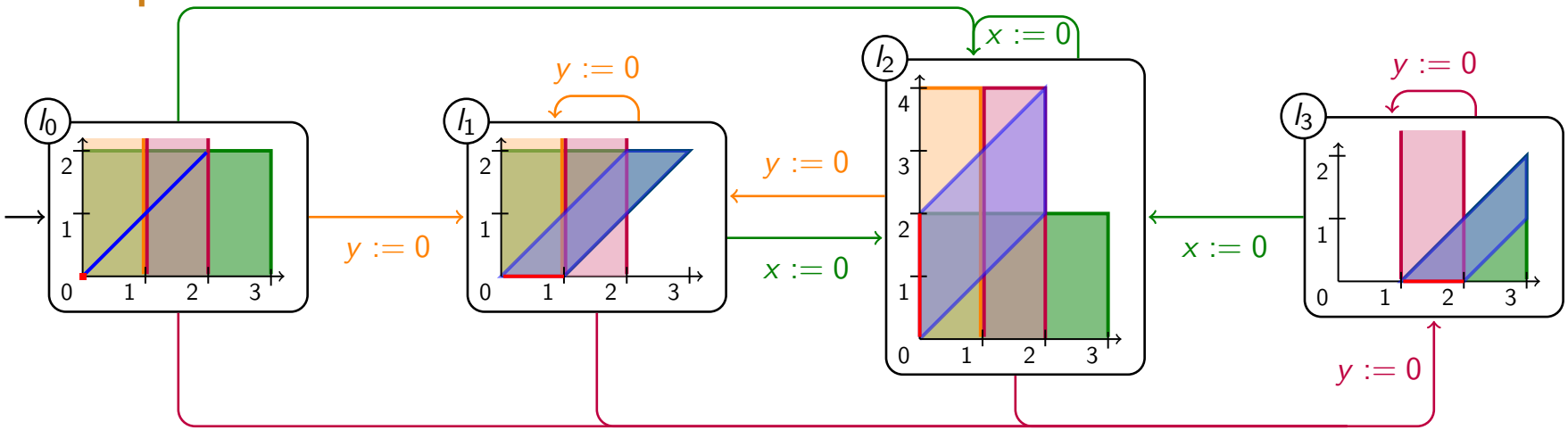


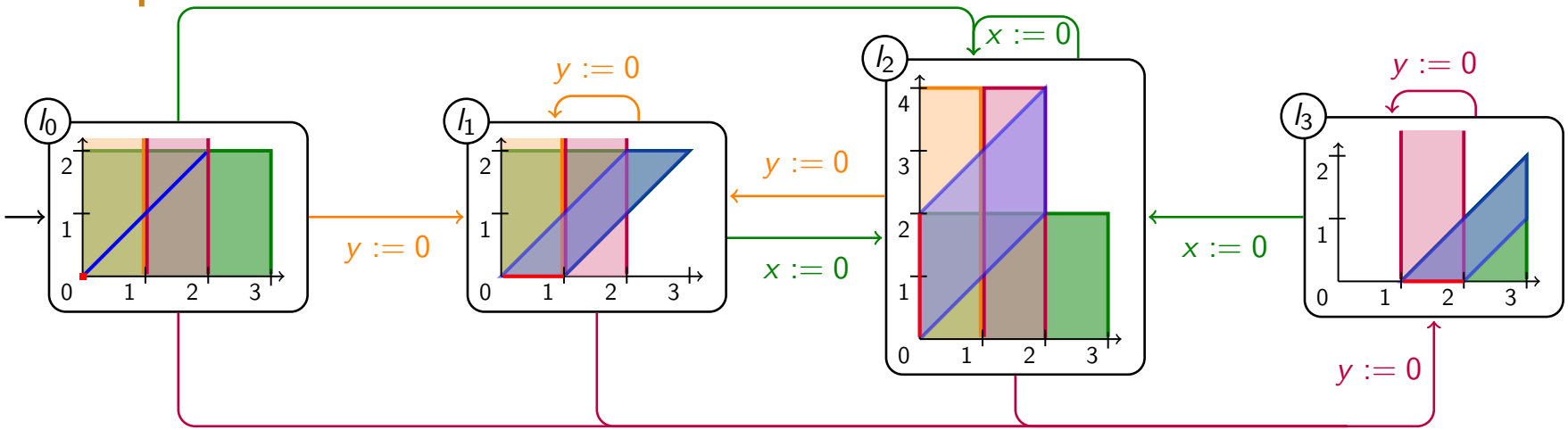
The previous equation is now split into two simpler equations

$$v_{n+1}[\textcircled{l_1}, (x, 0)] =$$

$$\int_0^{1-x} v_n(\textcircled{l_1}, (x+t, 0))\mathrm{d}t + \int_0^{2} v_n(\textcircled{l_2}, (0, t))\mathrm{d}t + \int_{1-x}^{2-x} v_n(\textcircled{l_3}, (x+t, 0))\mathrm{d}t;$$

$$v_{n+1}[\textcircled{l_3}, (x, 0)] = \int_0^{3-x} v_n(\textcircled{l_2}, (0, t))\mathrm{d}t + \int_0^{2-x} v_n(\textcircled{l_3}, (x+t, 0))\mathrm{d}t.$$

The split timed automaton



The previous equation is now split into two simpler equations

$$v_{n+1}[\textcircled{l_1}, (x,0)] =$$

$$\int_0^{1-x} v_n(\textcircled{l_1}, (x+t,0))\mathrm{d}t + \int_0^{2} v_n(\textcircled{l_2}, (0,t))\mathrm{d}t + \int_{1-x}^{2-x} v_n(\textcircled{l_3}, (x+t,0))\mathrm{d}t;$$

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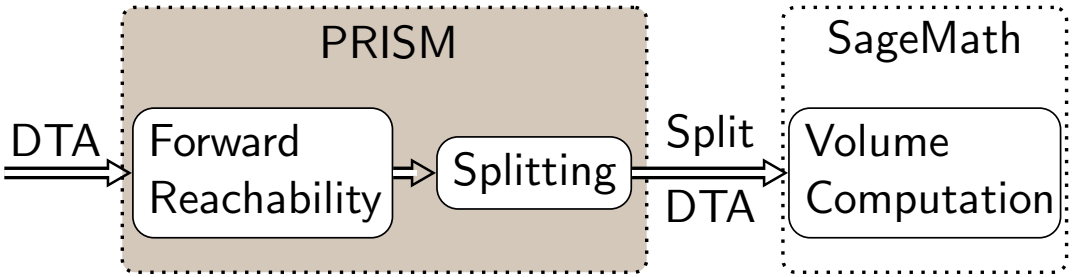
Volume functions are polynomials computable in polynomial time wrt. n.
e.g. $v_3[\textcircled{$l_3$}, (x,0)] = -\frac{1}{6}x^3 - \frac{1}{2}x^2 - 25x + \frac{133}{2}$.

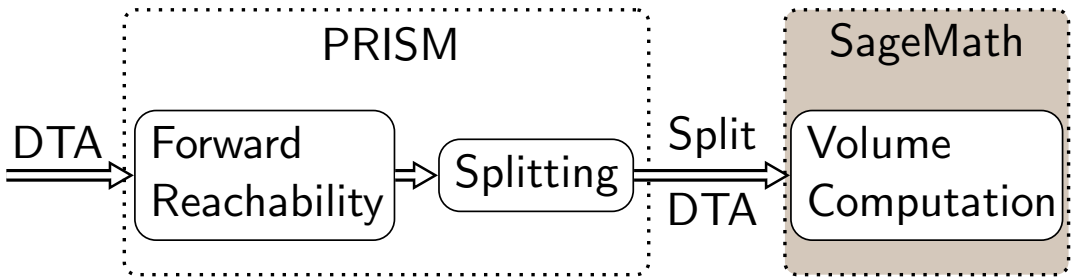
A first glimpse at our tool chain



PRISM (probabilistic model checker)

- ▷ Compute forward reachability zone graph
- ▷ Split the zone graph

A first glimpse at our tool chain



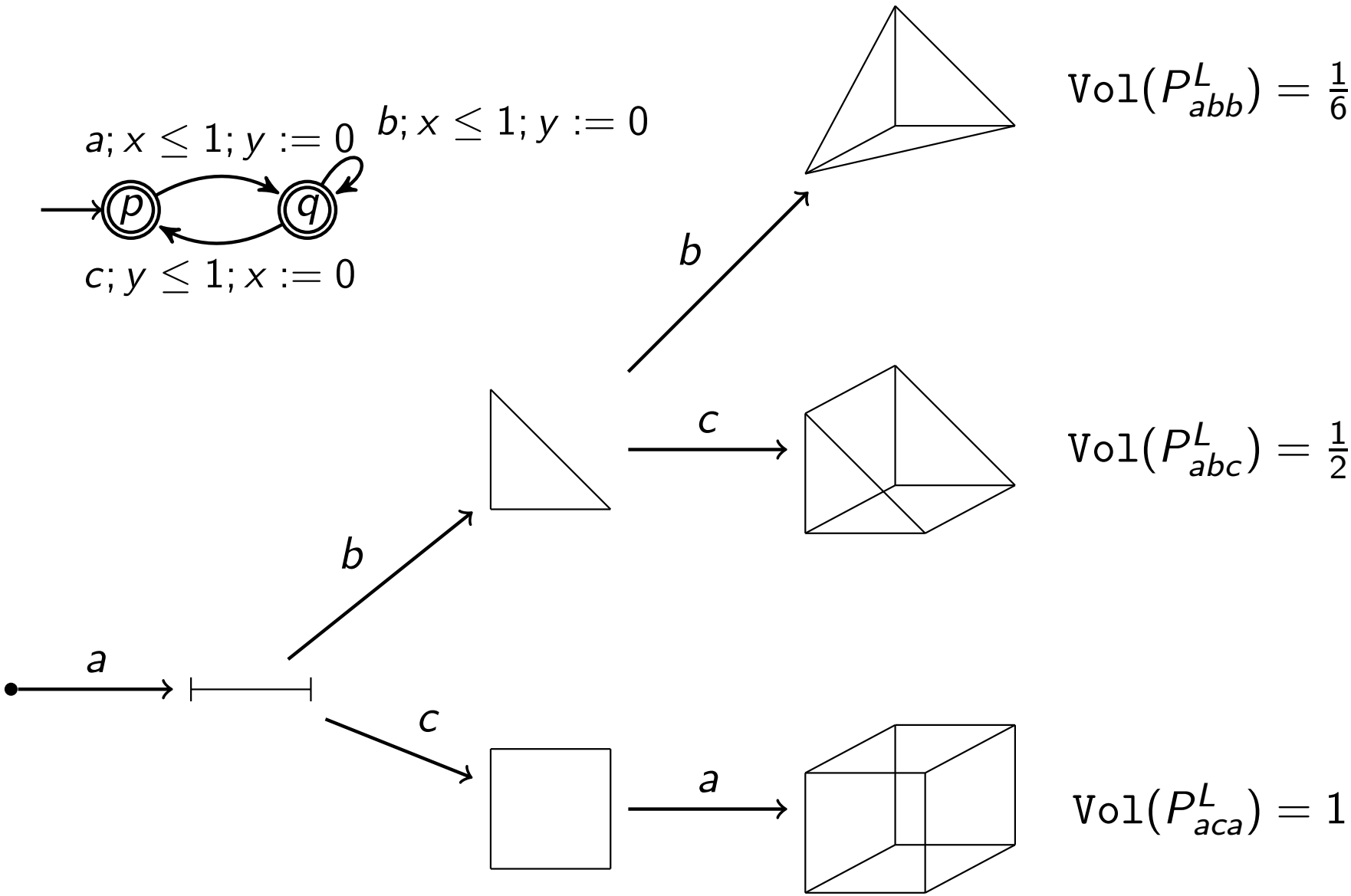
SageMath (open-source mathematics software)

- Compute volumes

Outline

Exact uniform sampling

Assign the same weight to every timed word of length n.



Several methods of (quasi)-uniform sampling

Recursive method

- ▷ At step k, the timed transition $s_k \xrightarrow{t_k, a_k} s_{k+1}$ is randomly picked with weight proportional to the volume of timed words of length $n - k$ from s_{k+1}.
- ▷ Drawback, necessitate to compute volume functions up to v_n.

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Receding horizon method (the one we implemented)

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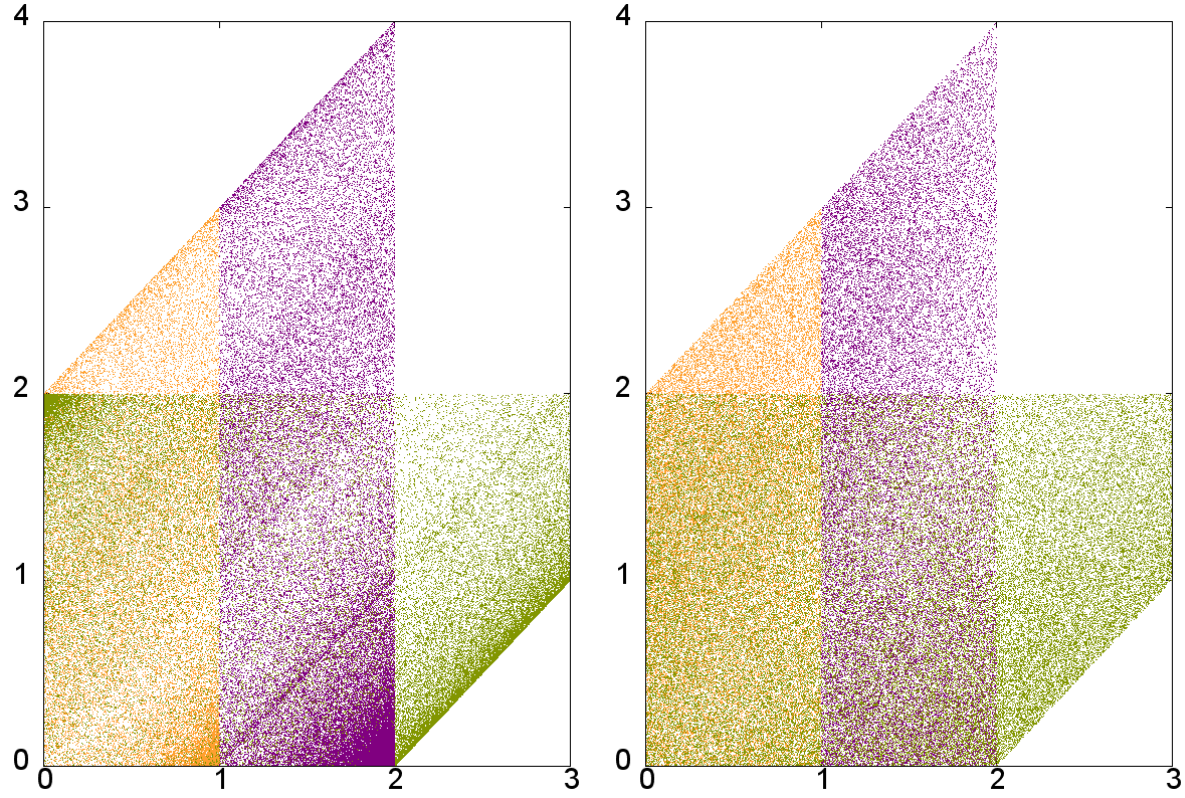
Maximal entropy (infinite receding horizon)

- ▷ When $n \to \infty$ (or $m \to \infty$), we obtain a maximal entropy stochastic process [B., Information and Computation 2015].

Comparing isotropic sampling and receding horizon sampling with m=9

Isotropic="by default"= every discrete transition available has the same weight, every delay available has the same weight.



Isotropic $m = 9$

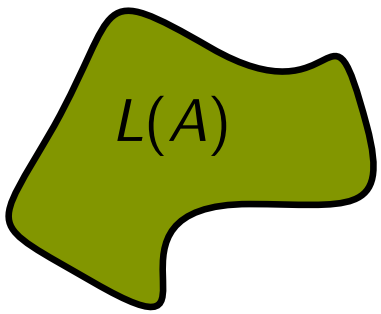
Generate a 200,000 steps trajectory

Outline

Statistical measure of general timed languages (1/2)

$L(A)$ a "complex" timed language

- A is a timed model, *possibly non-deterministic timed automaton*, *possibly stop-watch automaton*
- We only require to be able to check the membership of a word in the language



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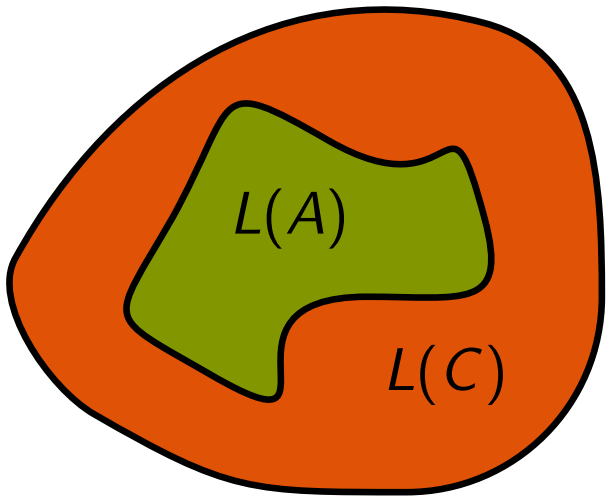
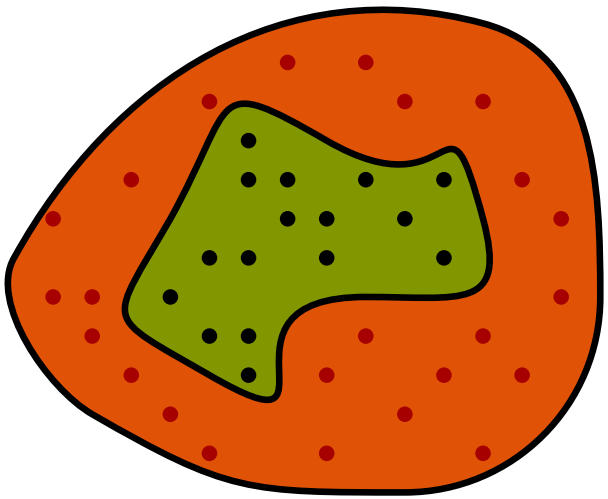
$L(C)$ a "simple" over-approximation: of the language $L(A) \subseteq L(C)$

- C is a deterministic time automaton
- We can compute the volume of its language
- We can sample uniformly from its language



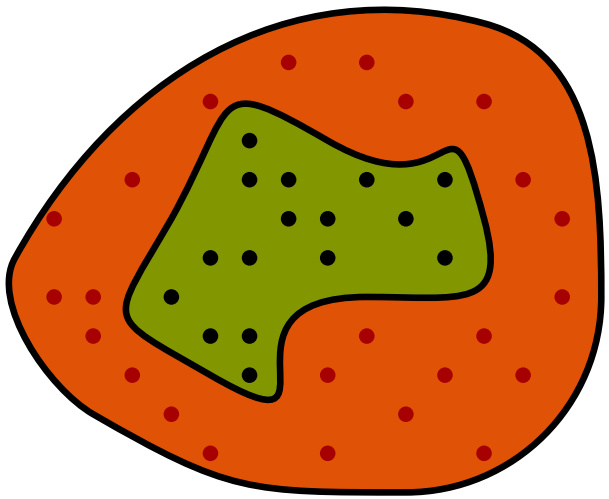
Statistical measure of general timed languages (2/2)



Sampling in the over-approximation

- ▷ Sample uniformly timed words of length n in the language $L_n(C)$ (by previous methods for DTAs).

Statistical measure of general timed languages (2/2)



Sampling in the over-approximation

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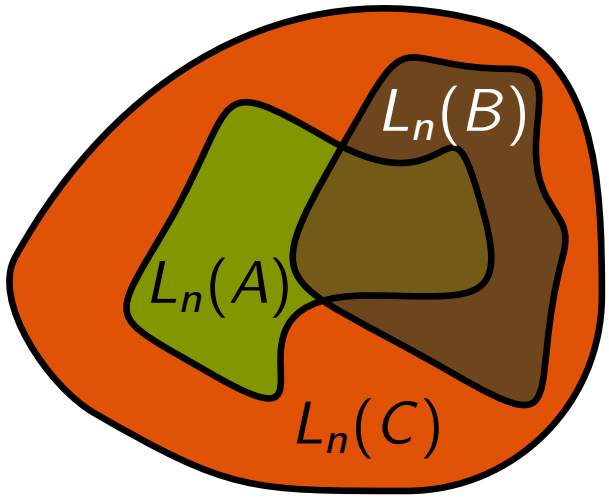
Filtering

- ▷ For each trajectory in $L_n(C)$ check the membership in $L_n(A)$
- ▷ Estimate the volume of $\texttt{Vol}_n(L_n(A))$ as $\frac{\#\text{trajectory in } L_n(A)}{\#\text{trajectory in } L_n(C)} \cdot \texttt{Vol}_n(L_n(C))$

Statistical language inclusion measurement

Two languages

- $L(A)$ and $L(B)$ are "complex" languages
- Use an over-approximation $L(C)$ with C a DTA.
- Using membership of automaton estimate
  $\mathtt{Vol}_n\left(L_n(A) \cap L_n(B)\right)$,
  $\mathtt{Vol}_n\left(L_n(A) \setminus L_n(B)\right)$,
  $\mathtt{Vol}_n\left(L_n(A) \,\Delta\, L_n(B)\right) \ldots$



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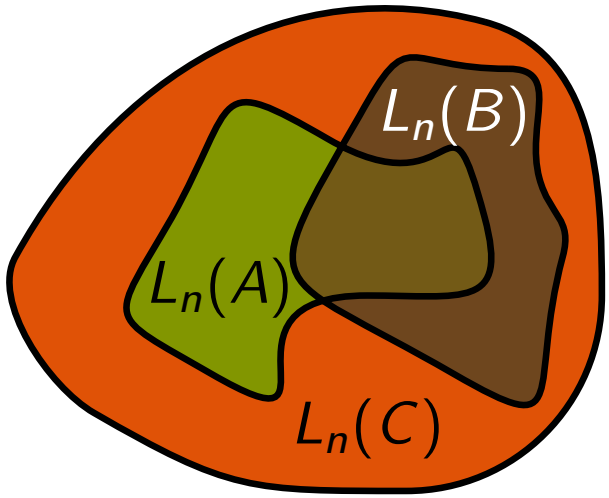
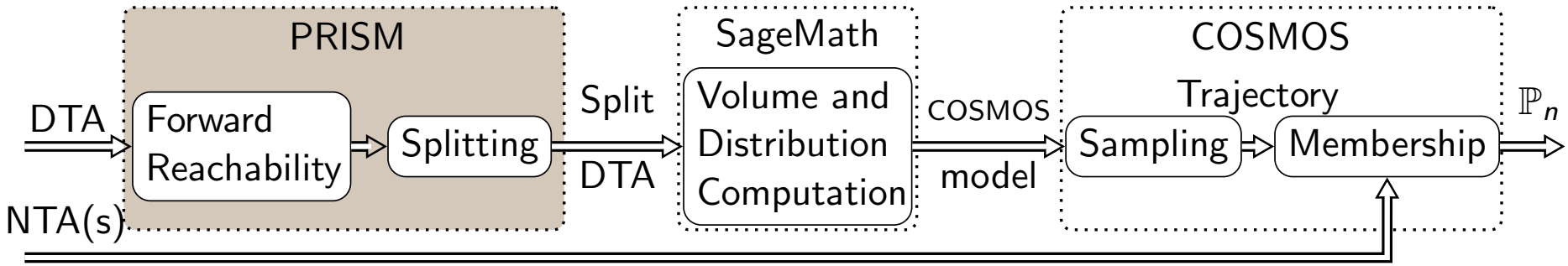


Checking $L_n(A) \subset L_n(B)$ (up to null volume measure)

$$\mathbb{P}_n(B|A) = \frac{\mathtt{Vol}_n\left(L_n(A) \cap L_n(B)\right)}{\mathtt{Vol}_n(L_n(A))} = 1$$

Toolchain



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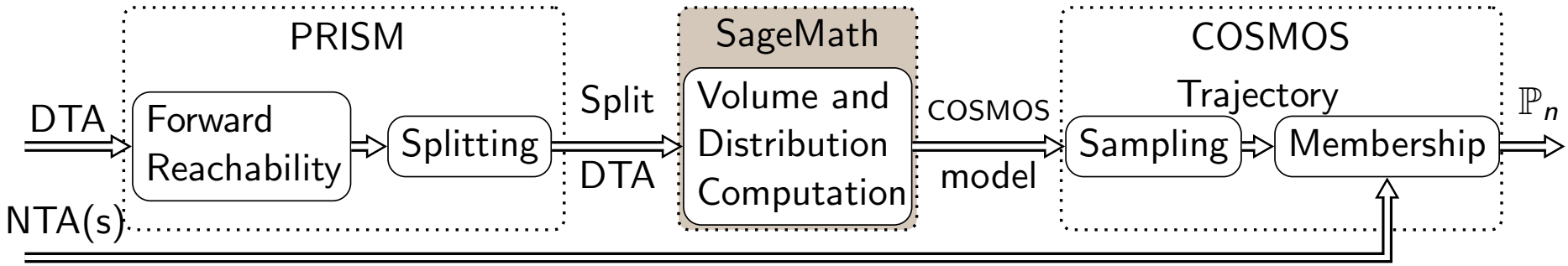
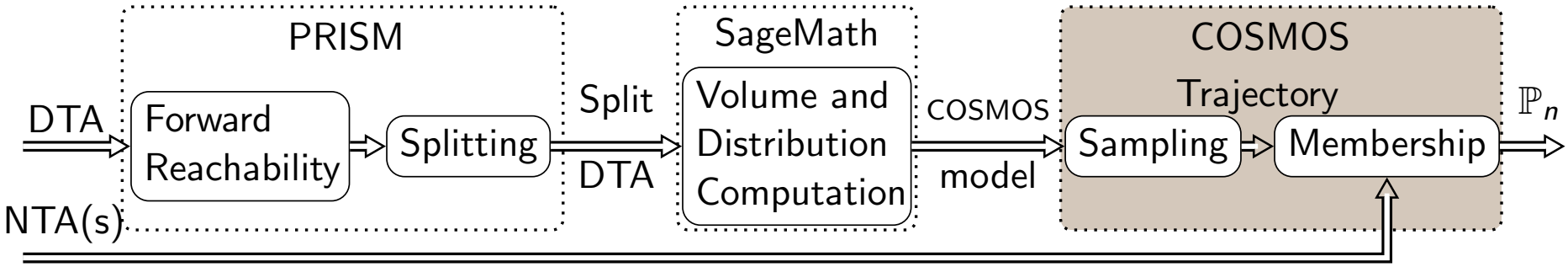


SageMath (open-source mathematics software)

- ▷ Compute volumes and *distributions*.
- ▷ Output a fully probabilistic COSMOS model.

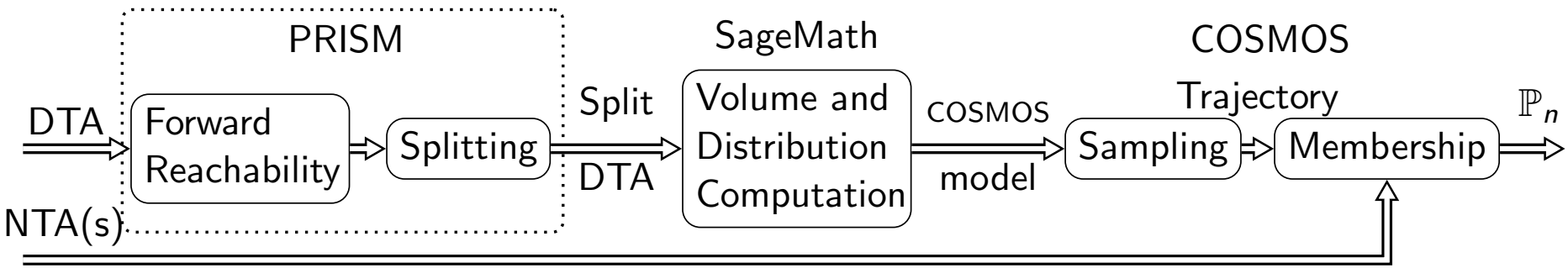
Toolchain



COSMOS (statistical model checker)

- Sampling of trajectories of the probabilistic model from SageMath.
- Output estimation of $\frac{\mathtt{Vol}_n(L_n(A))}{\mathtt{Vol}_n(L_n(C))}$ or estimation of $\frac{\mathtt{Vol}_n(L_n(A) \cap L_n(B))}{\mathtt{Vol}_n(L_n(A))}$ (for language inclusion).

Toolchain



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A case study of a repair failure model is available in [Barbot B. Beunardeau Kwiatkowska, Qest'16].

Outline

We have seen how to

- measure timed languages in terms of volumes;
- adopt a zone-based framework to compute efficiently such volumes;
- sample (quasi-)uniformly timed words for DTA;
- apply this to measure and sampling for general timed languages.

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- measure timed languages in terms of volumes;
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- apply this to measure and sampling for general timed languages.

What we plan to do:

- experiment more with the same theory
  - implement membership for more expressive timed languages (stopwatch, hybrid);
  - develop bigger case studies;
- extend the theory
  - develop random generation mehods uniform on timed words of same duration (as opposed to uniform on timed words of same length).
  - develop uniform random generation for networks of TAs.